Faster-than-light particles and T violation

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The properties of the interactions of faster-than-light particles (tachyons) with ordinary matter are discussed both in the framework of a classical closed causal cycle and in a field-theoretical formalism constrained by the reinterpretation principle. It is concluded that such interactions, if they exist, most likely violate time-reversal invariance. As a consequence it is suggested that tachyons might play a role in CP violation.

I. INTRODUCTION

For a long time the possibility that physical particles traveling faster than light (tachyons) might exist was not taken seriously. The main reason for such a neglect lay in apparent contradictions with naive causality, i.e., the notion that an effect should be preceded by a cause. Consider a tachyon which when seen from a particular frame has positive energy and travels forward in time, i.e., from A to B with $t_B > t_A$. Then there always are other frames where the particle is seen traveling backward in time, i.e., $t'_B < t'_A$, so that cause would appear to be preceded by effect.

A great step forward in resolving these difficulties was taken in 1962 by Bilaniuk, Deshpande, and Sudarshan,[1] who noticed that when the time order is reversed, the sign of the energy of the tachyon is also reversed. They then introduced the switching (or reinterpretation) principle according to which a negative-energy tachyon traveling backward in time is interpreted as a positive-energy tachyon traveling forward in time; causality thereby is recovered. The reinterpretation principle removes all serious objections against tachyonic phenomena involving one observer. However, some difficulties remained because, considering two or more observers in relative motion and imagining sequences of events which preserve the cause-effect relation in each frame, one may nevertheless be led to contradictory logical chains.

Closed causal cycles involving more than one observer and leading to logical paradoxes were devised by a number of authors. They all contain essentially the same ingredients. The simplest example is probably the following one[2]: Let A and B be two instruments in relative motion (see Fig. 1). The internal dynamics of the instrument B is set up in such a way that it will emit a tachyon at x_{B_2} if and only if no tachyon was emitted at x_{B_1}. On the other hand, the instrument A is set up to emit a tachyon at x_{A_2} if and only if it emitted a tachyon at x_{A_1}.

Denote by B_1, B_2, A_1, A_2 the events "emission of tachyons" at $x_{B_1}, x_{B_2}, x_{A_1}, x_{A_2}$ and by $\overline{B}_1, \overline{B}_2, \overline{A}_1, \overline{A}_2$ the events "no emission" at the same points. Denoting by $\alpha \Rightarrow \beta$ the statement "α implies β" one can then write the following two logical chains:

$$\overline{B}_1 \Rightarrow B_2 \Rightarrow A_1 \Rightarrow A_2 \Rightarrow B_1, \tag{1a}$$

$$B_1 \Rightarrow \overline{B}_2 \Rightarrow \overline{A}_1 \Rightarrow \overline{A}_2 \Rightarrow \overline{B}_1. \tag{1b}$$

In both cases an event, B_1 or $\overline{B}_1$, is seen to imply its opposite, $\overline{B}_1$ or B_1, hence the logical contradiction.

One should notice that in the chains (1a) and (1b) there are two distinct types of logical links. On the one hand there are statements such as $B_2 \Rightarrow A_1$ and $A_2 \Rightarrow B_1$, which depend only on the kinematics of tachyon propagation and on the ability of the instruments to register that an emission has taken place. On the other hand statements such as $\overline{B}_1 \Rightarrow B_2$ or $A_1 \Rightarrow A_2$ are more critically dependent on the internal dynamics of the instruments. This must be such that each one of the instruments must be able to trigger an event a certain time after some other event is observed. This kind of correlation is quite consistent with the behavior of ordinary matter clocks (which at least in this part of the universe and at this age we seem to have no trouble to construct). However, a second impor-

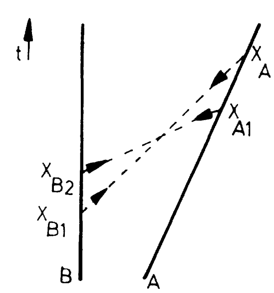

FIG. 1. The causal cycle as seen in the rest frame of B. The arrows in the tachyon lines are meant to emphasize that in its own rest frame the instrument A will also interpret the events as tachyon emissions.

tant point is that the instruments should be able to force emission of tachyons at x_{B_2} and x_{A_2}, i.e., the interactions between tachyons and ordinary matter should be controllable. If only spontaneous emission, as at x_{B_1} and x_{A_1}, can take place, then there will be no way to ensure that the links $\overline{B}_1 \Rightarrow B_2$ and $A_1 \Rightarrow A_2$ are implemented and no contradiction will arise.

Is "no controllable emission" equivalent to "no interaction between tachyons and ordinary matter"? Not really. However, it certainly means that such interactions would have a completely random nature, for otherwise one could in principle use any observed regularities to construct a controllable emitter.

To avoid the logical difficulty Rolnick has concluded in his paper[2] that tachyons even if they exist can never interact with nontachyons. As pointed out by Csonka,[3] who did a very careful analysis of this paradox, this is too strong a conclusion. As this author has shown, the difficulties associated with this thought experiment only demonstrate that the boundary conditions given for the instruments A and B are not compatible with each other.

In standard physical theories given the dynamical laws of a system and a set of initial conditions on a spacelike surface the latter history of the system is uniquely determined. In this thought experiment what we know is some part of its history, namely that instruments A and B should arrive at x_{A_1} and x_{B_1} in good working conditions to trigger their internal clocks, that at x_{A_2} and x_{B_2} they should be ready to emit tachyons, etc. Then to try to set up this experiment in a consistent manner what one should do would be to choose a particular frame, the rest frame of A, for example, and to try to set up initial conditions in the surface $t_A = 0$, i.e., to fix the initial coordinates and internal variables of the instruments as well as the coordinates of any other onjects in such a way that no other object but B can interact with the tachyons emitted by A at x_{A_1} and x_{A_2}.

The fact that the experiment cannot take place with all the instruments working as required at the appropriate instants then means that there is no set of initial conditions consistent with the dynamics and with what is known about the future history of the system. Because, as pointed out before, once the dynamical laws and the initial conditions are known the history is completely determined, there is nothing unusual about the contradiction of a set of initial conditions and the constraints on the future history of the system. The only reason why the "paradox" was disturbing was because both the required history and the way of specifying initial conditions (just checking that they preserve the cause-effect relation in two separate frames) used to be consistent with the dynamics before tachyons were introduced. The next logical step is to ask: What are the peculiarities of tachyon interaction dynamics that forbid us to set up initial conditions in a way that, without tachyons, used to be quite consistent?

Insight into this question is provided by looking at the time-reversed process (see Fig. 2). The instrument B will now interpret the events B_1 and B_2 as absorption of tachyons emitted by A, and A will interpret the events A_1 and A_2 as absorption of tachyons emitted by B. Thus for the time-reversed process the events where external tachyon propagation connects with internal dynamics are all absorptions in the rest frames of the participating instruments.

Now the reason we have previously been able to close the contradictory logical chains is the experimentally supported belief that any useful emitter should be able to force an emission to occur and thus initiate a propagation of some entity. However, one knows of no instrument that can cause absorption of something that is not present to begin with. Hence in this case there is no conceivable internal dynamics forcing us to write $\overline{B}_2 \Rightarrow B_1$ or $A_2 \Rightarrow A_1$, and without these critical links the contradictory chains cannot be closed and the process of Fig. 2 can occur without implying any logical contradiction. Of course this does not imply that it is necessarily an allowed process. The same dynamical mechanism that is required to forbid the process of Fig. 1 by either rendering inconsistent the required set of initial conditions or making the apparatus fail in its preassigned role might just as well forbid the process of Fig. 2. However, this is an unpleasant situation because it would seriously restrict the possibility of tachyon–ordinary-matter interactions.

The alternative possibility is that although the process of Fig. 1 is forbidden the time-reversed process is allowed, because it does not lead to any logical contradiction. In this case controllable interactions of tachyons with ordinary matter

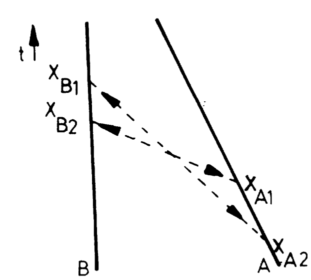

FIG. 2. The time-reversed process as seen in the rest frame of B.

would be possible, but these interactions would violate time-reversal invariance.

Seen in this light, the "paradox" that we have been discussing should not be more disturbing than, for example, the fact that although K capture from a $J=0$ to a $J=1$ negative-helicity state is possible, the parity-transformed process where the recoil nucleus would have positive helicity is never observed.

In the next section a field-theoretical argument will be constructed which makes quite plausible the possibility that the interactions of tachyons with ordinary matter violate either CPT or T invariance. It is, nevertheless, amusing to find that such a behavior of tachyonic interactions is already suggested by a careful analysis of a classical objection against tachyonic phenomena.

II. TACHYON FIELDS AND DISCRETE SYMMETRIES

In this section we will discuss, in the framework of Lorentz-invariant quantum field theory, the problem of implementing the discrete symmetry transformations in tachyon space. We will see that by choosing these transformations in the most natural way consistent with the reinterpretation principle [as defined by Eq. (2)], one concludes that the interactions between tachyons and ordinary matter should be expected to be either CPT-violating or if CPT-invariant to be T-violating (and CT-invariant).

Several types of quantum theories of tachyon fields have been proposed by different authors. For a survey of these different methods we refer to the excellent review of Kamoi and Kamefuchi.[4] As pointed out by these authors the method of Arons and Sudarshan,[5] who define the quantum field in terms of annihilation operators only, is probably the only consistent way of forming a quantum field theory for superluminal particles. This is because it is the one that leads to a relativistically covariant theory for free tachyon fields with nonvanishing Fourier components only for $|\vec{p}|\geq\mu$. The only finite-spin representations of the little group O(2, 1) associated to spacelike four-vectors are spin-zero representations. We will, thus, restrict ourselves to a discussion of spinless tachyons.

Choose for the tachyon states the covariant normalization

$$\langle p'|p\rangle=\epsilon 2p^0\delta^3(\vec{p}\,'-\vec{p})\delta_{\epsilon',\epsilon}\,,$$

where ϵ is the sign of the energy. The use of covariant normalization for tachyonic states seems quite appropriate because the zero-energy states for which the reinterpretation principle would be ambiguous then become spurious (zero-norm) states, and provide a natural separation between tachyon and antitachyon states.

Define creation operators $a^\dagger(p)$ by

$$|p\rangle=a^\dagger(p)|0\rangle\,.$$

The local quantum field $\Phi(x)$ is then defined by

$$\Phi(x)=(2\pi)^{-3/2}\int d^4p\,\delta(p^2+\mu^2)a(p)e^{-ip\cdot x},$$

with $\mu^2>0$.

The commutators for free fields are

$$[\Phi^\dagger(x),\Phi^\dagger(y)]=[\Phi(x),\Phi(y)]=0,$$

$$[\Phi(x),\Phi^\dagger(y)]=(2\pi)^{-3}\int_{|\vec{p}|\geq\mu}\frac{d^3p}{\omega(p)}\,e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\cos\omega(p)t$$

$$=\Sigma^{(1)}(x-y),$$

where $\omega(p)=+(|\vec{p}|^2-\mu^2)^{1/2}$. The function $\Sigma^{(1)}(x-y)$ is the only invariant function (for $p^2<0$) with nonvanishing Fourier components only for $|\vec{p}|\geq\mu$.

The space associated to the $\Phi(x)$ field will contain both positive- and negative-energy states. The negative-energy states will be dealt with by the reinterpretation principle which states that in any physical process an incoming (outgoing) negative-energy tachyon state shall be interpreted by an ordinary-matter observer as an outgoing (incoming) positive-energy antitachyon state.

To formalize the reinterpretation principle we will have to deal with the following two distinct vector spaces:

V space is the space generated by the $\phi(x)$ field and contains both positive- and negative-energy states.

$\tilde{V}$ space is the reinterpreted space which contains only positive energies and two species of tachyonic states (which may eventually be identical). Conventionally we will refer to them as tachyons and antitachyons.

Let S and $\tilde{S}$ be the scattering operators in V and $\tilde{V}$ space. Using the notation of Kamoi and Kamefuchi,[4] the reinterpretation principle then states

$$\left|\left\langle\begin{matrix}\vec{p}_1\cdots\vec{p}_k\\ \vec{q}_1\cdots\vec{q}_l\end{matrix}\right|S\left|\begin{matrix}\vec{r}_1\cdots\vec{r}_m\\ \vec{s}_1\cdots\vec{s}_n\end{matrix}\right\rangle\right|=\left|\left\langle\left\langle\begin{matrix}\vec{p}_1\cdots\vec{p}_k\\ -\vec{s}_1\cdots-\vec{s}_n\end{matrix}\right|\tilde{S}\left|\begin{matrix}\vec{r}_1\cdots\vec{r}_m\\ -\vec{q}_1\cdots-\vec{q}_l\end{matrix}\right\rangle\right\rangle\right|,\qquad(2)$$

where on the left-hand side the quantum numbers in the upper (lower) row are the momenta of positive- (negative-) energy states, and on the right-hand side they are the momenta of tachyons (antitachyons). The reinterpreted $\tilde V$ state is the one directly related to experiment and should be a subspace of the whole physical space containing tachyons, antitachyons, and ordinary matter, all with positive energy. The discrete transformations P, C, and T operating in this space should then have their usual properties. Namely, invariance under any one of these operations should imply the transformation properties for the $\tilde S$ matrix

$$\tilde P^\dagger \tilde S \tilde P = \tilde S, \tag{3a}$$

$$\tilde C^\dagger \tilde S \tilde C = \tilde S, \tag{3b}$$

$$\tilde T^\dagger \tilde S^\dagger \tilde T = \tilde S, \tag{3c}$$

i.e., in $\tilde V$ space, $\tilde P$ and $\tilde C$ should be unitary and $\tilde T$ antiunitary.

One has now to specify the transformation properties under $\tilde P$, $\tilde C$, and $\tilde T$ of the tachyon quantum numbers in $\tilde V$ space. For $\tilde C$ the only possibility consistent with the interpretation of the upper and lower quantum numbers as particle and antiparticle momenta is

$$\tilde C\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle\!\!\rangle=\tilde\eta_c\left|\begin{matrix}\vec{q}\\ \vec{p}\end{matrix}\right\rangle\!\!\rangle. \tag{4}$$

For $\tilde P$ and $\tilde T$, the physical interpretation of these operations requires a change in the sign of the particle three-momenta

$$\tilde P\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle\!\!\rangle=M_P\left|\begin{matrix}-\vec{p}\\ -\vec{q}\end{matrix}\right\rangle\!\!\rangle, \tag{5}$$

$$\tilde T\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle\!\!\rangle=M_T\left|\begin{matrix}-\vec{p}\\ -\vec{q}\end{matrix}\right\rangle\!\!\rangle, \tag{6}$$

where M_P and M_T are matrices in the tachyon-antitachyon space. Requiring $[\tilde P,\tilde C]=[\tilde P,\tilde T]=0$ the following two independent solutions are obtained for the matrices:

$$M_1=\eta\begin{pmatrix}1&0\\0&1\end{pmatrix}\ \text{and}\ M_2=\eta'\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

If tachyon and antitachyon are to have a well-defined parity the first solution should be chosen for M_P. However, for time reversal, because $\tilde T$ is antiunitary in $\tilde V$ space, there is no quantum number involved, so that the two alternatives are in principle possible; i.e., we could have either

$$\tilde T^{(1)}\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle\!\!\rangle=\tilde\eta_T\left|\begin{matrix}-\vec{p}\\ -\vec{q}\end{matrix}\right\rangle\!\!\rangle \tag{7a}$$

or

$$\tilde T^{(2)}\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle\!\!\rangle=\tilde\eta_T\left|\begin{matrix}-\vec{q}\\ -\vec{p}\end{matrix}\right\rangle\!\!\rangle. \tag{7b}$$

We will now use Eqs. (4)–(7) and the reinterpretation principle, Eq. (2), to find the properties of the operators that represent the discrete transformations in V space.

Space inversion. Using Eq. (3a) and Eq. (5) with M_P taken to be a diagonal matrix one obtains

$$\left|\left\langle\begin{matrix}\vec{p}_f\\ \vec{q}_f\end{matrix}\right|S\left|\begin{matrix}\vec{p}_i\\ \vec{q}_i\end{matrix}\right\rangle\right|=\left|\langle\!\!\left\langle\begin{matrix}\vec{p}_f\\ -\vec{q}_i\end{matrix}\right|\tilde P^\dagger\tilde S\tilde P\left|\begin{matrix}\vec{p}_i\\ -\vec{q}_f\end{matrix}\right\rangle\!\!\rangle\right|$$

$$=\left|\langle\!\!\left\langle\begin{matrix}-\vec{p}_f\\ \vec{q}_i\end{matrix}\right|\tilde S\left|\begin{matrix}-\vec{p}_i\\ \vec{q}_f\end{matrix}\right\rangle\!\!\rangle\right|$$

$$=\left|\left\langle\begin{matrix}-\vec{p}_f\\ -\vec{q}_f\end{matrix}\right|S\left|\begin{matrix}-\vec{p}_i\\ -\vec{q}_i\end{matrix}\right\rangle\right|.$$

This implies that

$$P\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle=\eta_P\left|\begin{matrix}-\vec{p}\\ -\vec{q}\end{matrix}\right\rangle \tag{8}$$

in V space and that P is a unitary operator.

Charge conjugation. We have

$$\left|\left\langle\begin{matrix}\vec{p}_f\\ \vec{q}_f\end{matrix}\right|S\left|\begin{matrix}\vec{p}_i\\ \vec{q}_i\end{matrix}\right\rangle\right|=\left|\langle\!\!\left\langle\begin{matrix}\vec{p}_f\\ -\vec{q}_i\end{matrix}\right|\tilde C^\dagger\tilde S\tilde C\left|\begin{matrix}\vec{p}_i\\ -\vec{q}_f\end{matrix}\right\rangle\!\!\rangle\right|$$

$$=\left|\langle\!\!\left\langle\begin{matrix}-\vec{q}_i\\ \vec{p}_f\end{matrix}\right|\tilde S\left|\begin{matrix}-\vec{q}_f\\ \vec{p}_i\end{matrix}\right\rangle\!\!\rangle\right|$$

$$=\left|\left\langle\begin{matrix}-\vec{q}_i\\ -\vec{p}_i\end{matrix}\right|S\left|\begin{matrix}-\vec{q}_f\\ -\vec{p}_f\end{matrix}\right\rangle\right|.$$

The only way to satisfy the above equality is to define the C operator in V space in such a way that

$$C\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle=\eta_C\left|\begin{matrix}-\vec{q}\\ -\vec{p}\end{matrix}\right\rangle \tag{9}$$

and $C^\dagger S^\dagger C=S$, i.e., the operator should reverse the signs of both the energy and the momentum and *be antiunitary*.

Time reversal. For the time-reversal transformation we will use Eq. (3c) for the transformation properties of $\tilde{S}$ and we will explore the two possibilities for the transformation properties of the states in $\tilde{V}$ space described by Eqs. (7a) and (7b). In the first case [Eq. (7a)],

$$\left|\left\langle \begin{matrix}\vec{p}_f\\ \vec{q}_f\end{matrix}\right| S \left|\begin{matrix}\vec{p}_i\\ \vec{q}_i\end{matrix}\right\rangle\right| = \left|\left\langle\left\langle \begin{matrix}\vec{p}_f\\ -\vec{q}_i\end{matrix}\right| \tilde{T}^\dagger \tilde{S}^\dagger \tilde{T} \left|\begin{matrix}\vec{p}_i\\ -\vec{q}_f\end{matrix}\right\rangle\right\rangle\right|$$

$$= \left|\left\langle\left\langle \begin{matrix}-\vec{p}_i\\ \vec{q}_f\end{matrix}\right| \tilde{S} \left|\begin{matrix}-\vec{p}_f\\ \vec{q}_i\end{matrix}\right\rangle\right\rangle\right|$$

$$= \left|\left\langle \begin{matrix}-\vec{p}_i\\ -\vec{q}_i\end{matrix}\right| S \left|\begin{matrix}-\vec{p}_f\\ -\vec{q}_f\end{matrix}\right\rangle\right| ,$$

one is led to

$$T^{(1)}\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle = \eta_T \left|\begin{matrix}-\vec{p}\\ -\vec{q}\end{matrix}\right\rangle \tag{10a}$$

and to the conclusion that T should be an antiunitary operator in V space. However, for the second case [Eq. (7b)]

$$\left|\left\langle \begin{matrix}\vec{p}_f\\ \vec{q}_f\end{matrix}\right| S \left|\begin{matrix}\vec{p}_i\\ \vec{q}_i\end{matrix}\right\rangle\right| = \left|\left\langle\left\langle \begin{matrix}\vec{p}_f\\ -\vec{q}_i\end{matrix}\right| \tilde{T}^\dagger \tilde{S}^\dagger \tilde{T} \left|\begin{matrix}\vec{p}_i\\ -\vec{q}_f\end{matrix}\right\rangle\right\rangle\right|$$

$$= \left|\left\langle\left\langle \begin{matrix}\vec{q}_f\\ -\vec{p}_i\end{matrix}\right| \tilde{S} \left|\begin{matrix}\vec{q}_i\\ -\vec{p}_f\end{matrix}\right\rangle\right\rangle\right|$$

$$= \left|\left\langle \begin{matrix}\vec{q}_f\\ \vec{p}_f\end{matrix}\right| S \left|\begin{matrix}\vec{q}_i\\ \vec{p}_i\end{matrix}\right\rangle\right| .$$

Then

$$T^{(2)}\left|\begin{matrix}\vec{p}\\ \vec{q}\end{matrix}\right\rangle = \eta_T \left|\begin{matrix}\vec{q}\\ \vec{p}\end{matrix}\right\rangle \tag{10b}$$

and in this case T will be a *unitary* operator in V space. From Eqs. (8), (9), and (10) the following transformation properties for the tachyon operators will follow:

$$Pa^\dagger(p^0,\vec{p})P^\dagger = \eta_P a^\dagger(p^0,-\vec{p}), \tag{11a}$$

$$Ca^\dagger(p^0,\vec{p})C^\dagger = \eta_C a^\dagger(-p^0,-\vec{p}), \tag{11b}$$

$$T^{(1)}a^\dagger(p^0,\vec{p})T^{(1)\dagger} = \eta_T a^\dagger(p^0,-\vec{p}), \tag{11c}$$

$$T^{(2)}a^\dagger(p^0,\vec{p})T^{(2)\dagger} = \eta_T a^\dagger(-p^0,\vec{p}). \tag{11d}$$

For the transformation properties of the field $\phi(x)$ one uses the above relations plus the unitary or antiunitary properties of the operators:

$$P\Phi(x^0,\vec{x})P^\dagger = \eta_P \Phi(x^0,-\vec{x}), \tag{12a}$$

$$C\Phi(x^0,\vec{x})C^\dagger = \eta_C \Phi(x^0,\vec{x}), \tag{12b}$$

$$T^{(1)}\Phi(x^0,\vec{x})T^{(1)\dagger} = \eta_T \Phi(-x^0,\vec{x}), \tag{12c}$$

$$T^{(2)}\Phi(x^0,\vec{x})T^{(2)\dagger} = \eta_T \Phi(-x^0,\vec{x}). \tag{12d}$$

Notice that although $T^{(1)}$ is antiunitary and $T^{(2)}$ is unitary because of the different transformation properties of the creation operators $\Phi(x)$ will transform in the same way in both cases.

So far we have shown that there exists a set of transformation relations of the tachyon fields under the discrete operations C, P, and T which is consistent with the reinterpretation principle. Therefore, we should have no trouble to construct in a consistent manner interaction Lagrangians for tachyon-tachyon interactions that preserve all the discrete symmetries. However, for interaction terms that involve tachyon fields and ordinary-matter fields the situation is quite different. Let us first analyze the case of C invariance.

An ordinary-matter charged spin-zero field, for example,

$$\psi(x) = (2\pi)^{-3/2}\int \frac{d^3p}{2p^0}\,[b_+(p)e^{-ip\cdot x} + b_-^\dagger(p)e^{ip\cdot x}], \tag{13}$$

will transform under charge conjugation as

$$C\psi(x)C^\dagger = \eta_c'\psi(x).$$

In this space C is a unitary operator. Had we chosen it to be antiunitary and preserved the transformation properties of creation and annihilation operators, namely

$$Cb_\pm(p)C^\dagger = \eta_c' b_\mp(p),$$

the result would have been

$$C\psi(x)C^\dagger = \eta_c'\psi^\dagger(-x);$$

i.e., the field would have been transformed into a field defined at a different space-time point.

To study the C-conjugation properties of a term $\mathcal{L}_{\rm int}(\Phi(x),\psi(x))$ describing interactions between a tachyon and an ordinary-matter field, one should find how it transforms under the tensor product operator

$$C = C_{\rm tach}\otimes C_{\rm matter}.$$

By definition $C_{\rm tach}$ will not disturb the creation and annihilation operators associated to the $\psi(x)$ field. However, since $C_{\rm tach}$ is antiunitary it will not commute with the exponentials $e^{\pm ip\cdot x}$ in the definition of $\psi(x)$ because they are not real numbers. Hence

$$C\mathcal{L}_{\rm int}(\Phi(x),\psi(x))C^\dagger \sim \mathcal{L}_{\rm int}(\Phi(x),\psi(-x))$$

and the action $\int\mathcal{L}_{\rm int}\,d^4x$ will not remain invariant because the coordinates are only changed in part of the integrand. The conclusion is that in the framework of ordinary Lagrangian field theory one is unable to construct C-invariant terms involving interactions between tachyons and ordinary matter. No such problem will occur, of course, for terms that involve only tachyon fields.

If one chooses the first definition $T^{(1)}$ for the time-reversal transformation this C-violation effect will never be compensated and the theory may be P- and T-invariant, but will necessarily violate CPT. However, choosing the second definition ($T^{(2)}$) for time reversal, one finds that the CT operation is antiunitary both in the tachyon and the ordinary-matter spaces so that CT-invariant terms may now be constructed in a consistent manner. The theory might then be CT- and P-invariant, but, of course, would violate C and T separately.

This completes our argument. The reader should have noticed that the reinterpretation principle played an essential role in our proof. Without requiring explicit consistency with this principle as expressed in Eq. (2) and because in V space the energy is not positive-definite one has no other reliable way to find what the nature of P, C, and T in V space should be. However, as pointed out in the beginning of Sec. I without the reinterpretation principle one is bound to run into all kinds of serious objections against tachyonic phenomena. Explicit consistency with this principle appears then to be a quite reasonable requirement to impose on any quantum model containing tachyons.

Our choice of $\tilde{S}$ rather than S to implement the discrete symmetries in tachyon space is indeed the most natural one if Eq. (2) is used as our definition of the reinterpretation principle. Without this equation our results break down, but because there is always the possibility that nature may choose to implement dynamics in tachyon space without an explicit representation of the reinterpretation principle at the S-matrix level, our argument, however compelling, should not be taken as an absolute proof of CPT or T violation of tachyon–ordinary-matter interactions.

The reader should also be reminded that the tachyon fields we are dealing with in this paper are Fourier transforms of the physically realizable entities that can carry information across spacelike intervals, i.e., particles with $|\vec{p}| \geq \mu$. For spacelike virtual states containing other Fourier components, as arise for example in general solutions of the Klein-Gordon equation for negative mass squared, the situation might be quite different.

That in a consistent quantum theory of tachyons one finds restrictions on the dynamics should come as no surprise because contrary to some people's opinion,[6] and as pointed out and justified by Rolnick[7] "... the only successful refutations of the causal anomalies have introduced constraints which do not allow emission of some tachyons," i.e., "tachyons can exist only if certain experiments cannot be performed.... "

A classical reasoning pointing to a way out of the causal cycle anomaly, different from the one we proposed in Sec. I, is the reasoning of Bilaniuk and Sudarshan,[8] which involves the consideration of cosmological boundary conditions. Some consequences of the reasoning of these authors will now be analyzed because such consequences may bear some relation to the new possibility, discovered in this section, of having the contradictory causal cycle broken by CPT violation in tachyon–ordinary-matter interactions. These authors consider a frame S_0, where the tachyon background (from distant sources) is zero. An instrument (absorber-emitter of tachyons) at rest in S_0 can emit any number of tachyons of velocity $v > c$. However, another absorber-emitter moving with a velocity ω ($\omega < c$) should not be allowed to emit tachyons of velocity exceeding $1/\omega$ in its rest frame, for otherwise some emissions of the second instrument would be seen in S_0 as absorptions of tachyons coming from distant sources, thus contradicting the no-background requirement. In fact, from this author's point of view, the discussion of the tachyon background amounts simply to a motivation to impose a prescription to inhibit the flow of information in a direction opposed to the direction of motion. It is easy to see that the prescription is equivalent to the requirement that the amplitudes in S_0, for the emission of a tachyon of momentum k by an instrument of momentum p, be of the form

$$M(p, k) = \theta(p\cdot k) f(p, k)\ , \qquad (14)$$

i.e., vanish for $p\cdot k < 0$. This condition seems to have a bearing on discrete symmetry considerations for if $M(p, k)$ is a boundary value of an analytic function Eq. (14) implies that it vanishes everywhere (no interactions between tachyons and ordinary matter). If it does not vanish it cannot be the boundary value of a master analytic function and without such a function the CPT theorem cannot be proved.

That the existence of a symmetry cannot be proved does not necessarily imply that it is violated. However, we feel that these considerations are suggestive enough to the effect that the argument of Bilaniuk and Sudarshan might be the classical analog of the CPT part of our quantum theory result in the same way as the classical argument given at the end of Sec. I corresponds to the T-violating part.

III. TACHYONS AND *CP* VIOLATION

We have seen that when constructing Lagrangian terms for the interactions of tachyons and ordinary matter, the maximum amount of discrete symmetry that can be built in, in a consistent manner, is CT and P invariance. These interaction terms will then naturally lead to CP-violating effects, which in this context appear as a consequence of the impossibility to implement a discrete transformation. CP violation would then exhibit a nature similar to P violation in weak interactions, where P cannot be implemented in a space containing left-handed neutrinos only.

As an example, let us consider an ordinary-matter complex spinless field $\psi(x)$ interacting with a tachyon field $\Phi(x)$. For $\psi(x)$, defined by Eq. (13), we take the following transformation properties:

$$P(\psi)P^\dagger = -\psi(x^0,-\vec{x}),$$
$$C\psi(x)C^\dagger = -\psi^\dagger(x),$$
$$T\psi(x)T^\dagger = \psi(-x^0,\vec{x}).$$

For the linear combinations

$$\psi_1(x) = \frac{1}{\sqrt{2}}\,[\psi(x)+\psi^\dagger(x)],$$
$$\psi_2(x) = \frac{1}{\sqrt{2}}\,[\psi(x)-\psi^\dagger(x)]$$

the relevant properties are

$$\psi_1^\dagger(x) = +\psi_1(x), \quad \psi_2^\dagger(x) = -\psi_2(x), \qquad (15a)$$
$$CP\psi_1(x)P^\dagger C^\dagger = +\psi_1(x^0,-\vec{x}), \quad CP\psi_2(x)P^\dagger C^\dagger = -\psi_2(x^0,-\vec{x}), \qquad (15b)$$
$$CT\psi_1(x)T^\dagger C^\dagger = -\psi_1(-x^0,\vec{x}), \quad CT\psi_2(x)T^\dagger C^\dagger = +\psi_2(-x^0,\vec{x}). \qquad (15c)$$

The free Lagrangian is

$$\mathcal{L} = \tfrac{1}{2}:(\partial_\mu\psi_1\partial^\mu\psi_1 - \partial_\mu\psi_2\partial^\mu\psi_2): - \tfrac{1}{2}m_1^2:\psi_1\psi_1: + \tfrac{1}{2}m_2^2:\psi_2\psi_2: + :\partial_\mu\Phi^\dagger\partial^\mu\Phi: + \mu^2:\Phi^\dagger\Phi:.$$

We will now show that the most general interaction term, bilinear in $\psi(x)$ and linear in $\phi(x)$, that preserves CT and P invariance leads in second order to a nonvanishing matrix element between ψ_1 and ψ_2, hence to CP violation.

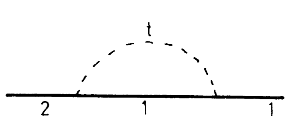

FIG. 3. $2\rightarrow 1$ transitions by virtual-tachyon effects.

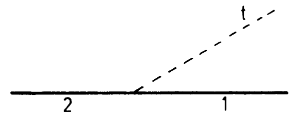

FIG. 4. First-order diagram for a $2\rightarrow 1$ transition.

The conditions for an interaction term to preserve CT and P invariance, without requiring C invariance, are[9]

$$CT\mathcal{L}_I(\psi(x),\Phi(x))T^\dagger C^\dagger = \mathcal{L}_I^\dagger(\psi(-x^0,\vec{x}),\Phi(-x^0,\vec{x})), \qquad (16a)$$

$$P\mathcal{L}_I(\psi(x),\Phi(x))P^\dagger = \mathcal{L}_I(\psi(x^0,-\vec{x}),\Phi(x^0,-\vec{x})). \qquad (16b)$$

From the transformation properties of the fields and their complex conjugation properties it then follows that any one of the interaction terms

$$\psi_1\psi_2(\Phi+\Phi^\dagger), \quad \psi_1\psi_1(\Phi+\Phi^\dagger), \quad \psi_2\psi_2(\Phi+\Phi^\dagger)$$

multiplied by real constants will satisfy conditions (16a) and (16b), provided that $\eta_P = \eta_C\eta_T = +1$.

A combination of the first term with any one of the others will of course lead to $2\rightarrow 1$ transitions by virtual tachyon effects (see Fig. 3). For the first-order diagram (see Fig. 4) to be observed as a real process, μ should satisfy

$$\mu \leq [(\Delta m)^2 + 2m_1\Delta m]^{1/2},$$

where $\Delta m = m_2 - m_1$. For the K_2-K_1 case, μ would in fact have to fall in a very narrow range, namely $\mu \lesssim 70$ eV.

In the example we have been discussing we have treated the spinless bosons that interact with the tachyons as fundamental fields. For the neutral-kaon system, it is probably more realistic to consider a system of fundamental spin-$\frac{1}{2}$ fields interacting with tachyons and to obtain the spinless bosons as composite states. Such a model is at present under investigation.

IV. CONCLUSION

The results discussed in this paper suggest that tachyons may very well be to the CP-violating interaction in much the same relation as neutrinos are to the weak interaction. Had it not been for weak interactions, the neutrinos might exist in large quantities and remain forever undetected. Similarly, because the only microscopic T-violating interaction that we know has an extremely weak intensity, we might even imagine a situation where a great number of tachyons would coexist with ordinary matter, but except for effects in the immensely precise natural interferrometer provided by the K^0-$\bar{K}^0$ system, they would remain practically undetectable.[10]

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[9]These equations can easily be checked by writing the S matrix in perturbation theory

$$S = T\left(\exp\left[-i\int d^4x\, H_I(\phi_t, \psi, \ldots)\right]\right)$$

and using the transformation properties of the S matrix.

[10]If the tachyons have gravitational interactions they might also lead to long-range T-violating effects.