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Effects of boundary conditions on massless two-dimensional electrodynamics in a static bag*

Barry Freedman and Vladimir Krapchev

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We investigate the behavior of massless two-dimensional quantum electrodynamics when the fields are confined to a static string. Surface modifications of the equal-time commutation relations are studied, and the generating functional is constructed, consistent with the linear boundary conditions of the theory. It is shown that massive scalars exist independent of the size of the cavity. The boundary condition of the initial fields constrain the boson field to vanish at the end points of the bag. We further demonstrate that the short-distance behavior of the theory is unaffected by the introduction of linear boundary conditions.

I. INTRODUCTION

The MIT bag model of hadrons[1] treats a strongly interacting particle as a finite region of space to which quark and gluon fields are confined. Since an exact solution to the theory by determining the surface of the bag from the boundary conditions is prohibitively difficult even when neglecting the gluon interaction, one introduces fields confined to a cavity with static boundaries. This picture presents a solution only in the rest frame of the bag. The resulting model does not have translational invariance. Despite these obvious limitations it provides a solution for the spectrum of light hadrons,[2] where low-momentum behavior of the theory is important.

We study an exactly solvable two-dimensional model of massless fermion and gluon fields confined to a cavity with static boundary and interacting via gauge coupling. The equations of motion for the system are supplemented by boundary conditions, identical to the linear boundary conditions of the MIT bag model of hadrons. A cavity of certain length will provide a natural infrared cutoff to the Schwinger model[3] and it is not at all clear if the old results will hold. For ordinary two-dimensional QED only a massive scalar field is present and all degrees of freedom associated with the fermion field are lost.

As we show, the two-dimensional static bag model corresponds to an almost canonical field theory. The canonical commutation and anticommutation relations are modified to satisfy the boundary conditions and differ from the usual ones by surface terms. It is possible to find the exact generating functional for the system and in doing so we introduce the necessary modifications due to boundary conditions. Again as in the Schwinger model we find a massive boson field but with the constraint of a Dirichlet condition. The current algebra is modified only by surface terms.

This provides us with an example that the mechanism of dynamical mass generation is not affected by the introduction of fields confined to a cavity of fixed length, at least for the two-dimensional case.

In Sec. II we derive from a Lagrangian the appropriate equations of motion and boundary conditions for the system. We observe that the relevant object of our investigation is the time-ordered product of the electric field strength. We choose an axial gauge ($A_1=0$) and solve the theory in the charge-zero sector. In Sec. III we discuss the almost canonical anticommutation relation, which is derived for the free fermion fields and postulated for the interacting fields. We calculate the free fermion Green's function for the static bag. In Sec. IV we construct the generating functional. A central part in this calculation is to find the fermion Green's function in the presence of an external source. Section V presents a calculation of the vacuum expectation value of the time-ordered product of the electric field which is proportional to the propagator for a scalar field of mass squared g^2/π, where g is the gauge coupling constant. This scalar field obeys the Dirichlet boundary condition. We show the surface modifications to the current algebra and construct the creation (annihilation) operators for the massive boson field in terms of the currents. In Sec. VI we show that the Hamiltonian is in fact diagonal in the creation (annihilation) operators of the scalar field. For a better understanding of the cavity propagator we present a multiple-reflection expansion.

II. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS. AXIAL GAUGE.

We consider two-dimensional massless fermions and gluons interacting with a minimal coupling of strength g with a dimension of mass. The fields are confined to a cavity of fixed size. We use the metric $g^{00}=1$, $g^{11}=-1$. The Lagrangian density of the theory is

$$\mathcal{L}(\underline{x})=\theta_B(\underline{x})\{\tfrac{1}{2}i\bar{\psi}(\underline{x})\overleftrightarrow{\partial}\psi(\underline{x})-\tfrac{1}{4}F^{\mu\nu}(\underline{x})F_{\mu\nu}(\underline{x}) - g\bar{\psi}(\underline{x})A\!\!\!/(\underline{x})\psi(\underline{x}) + \tfrac{1}{2}\partial^{\mu}[n_{\mu}\bar{\psi}(\underline{x})\psi(\underline{x})]-B\}, \tag{2.1}$$

where

$$F^{\mu\nu}=\partial^{\nu}A^{\mu}-\partial^{\mu}A^{\nu}.$$

$\theta_B(\underline{x})$ is 1 inside the bag and zero elsewhere. B is a parameter of the theory and represents the constant energy per unit volume introduced by Chodos *et al.*[1] and it accounts for the confinement of the fields inside the cavity. n_μ is a normal to the surface of the bag pointing inwards, $n^2=-1$. The total divergence that we have added to the usual expression in the presence of $\theta_B(\underline{x})$ will introduce a surface term. When varying the action one should keep in mind that for the static cavity approximation $\delta\theta_B(\underline{x})=0$, and we do not get the second-order boundary condition characteristic of the MIT bag model. Inside the cavity we have the equations of motion.

$$[i\partial\!\!\!/-gA\!\!\!/(\underline{x})]\psi(\underline{x})=0, \tag{2.2}$$

$$\partial_{\nu}F^{\mu\nu}(\underline{x})=gj^{\mu}(\underline{x}), \tag{2.3a}$$

where

$$j^{\mu}(\underline{x})=:\bar{\psi}(\underline{x})\gamma^{\mu}\psi(\underline{x}):. \tag{2.3b}$$

On the surface we get

$$-in\!\!\!/\psi(\underline{x}_s)=\psi(\underline{x}_s), \tag{2.4}$$

$$n_{\mu}F^{\mu\nu}(\underline{x}_s)=0 . \tag{2.5}$$

It immediately follows from (2.4) that

$$n_{\mu}j^{\mu}(\underline{x}_s)=0 . \tag{2.6}$$

In two dimensions the em field tensor is simply related to the electric field

$$F^{\mu\nu}(\underline{x})=\epsilon^{\mu\nu}E(\underline{x}) , \tag{2.7}$$

where $\epsilon^{01}=-\epsilon^{10}=1$. From (2.3) we find

$$\partial_{\nu}\epsilon^{\mu\nu}E(\underline{x})=gj^{\mu}(\underline{x}) . \tag{2.8}$$

$E(\underline{x})$ is a scalar field and from (2.5) it follows that it obeys a Dirichlet boundary condition, i.e., it vanishes on the surface.

We take a cavity with end points at $x=\pm l$. For the normal to the surface $n^{\mu}=(0,n^1)$ we have $n^1(\pm l)=\mp 1$. One should remember that there are no photons in two dimensions and the Schwinger model is a theory of self-interacting massless fermions. To make this point clear we choose the axial gauge in which $A_1=0$. We have only a Coulomb potential present in the interaction and it is determined by the fermion charge density. The advantage is that only physical states exist in the theory and there are no photon degrees of freedom. In this noncovariant formalism we can write from (2.3)

$$\partial_1{}^2A^0(\underline{x})=-gj^0(\underline{x}) , \tag{2.9a}$$

$$\partial_0\partial_1A^0(\underline{x})=gj^1(\underline{x}) . \tag{2.9b}$$

From (2.5) A^0 obeys the Neumann boundary condition

$$n^1\partial_1A^0(\underline{x})|_{x=\pm l}=0 . \tag{2.10}$$

If we integrate (2.9a) over x between the end points of the cavity we get

$$\sum_i n^1\partial_1A^0(t,x_i)=gQ , \tag{2.11}$$

where Q is the charge operator and $\{x_i\}$ are the end points. For (2.10) and (2.11) to be compatible we must require that the charge operator acting on all physical states give zero, i.e., we are looking for a solution in the charge-zero sector:

$$Q\,|\text{physical state}\rangle=0. \tag{2.12}$$

We must stress that this is not a subclass of all solutions, but the only solution consistent with the boundary conditions of the static cavity. From (2.9a) we find for $A^0(\underline{x})$

$$A^0(\underline{x})=-g\int_{-l}^{l}V(x,y)j^0(t,y)dy , \tag{2.13}$$

where

$$V(x,y)=-\frac{1}{4l}\left\{\sum_{n=1}^{\infty}\frac{1}{2k_n{}^2}[f_n(x)f_n^*(y)+f_n^*(x)f_n(y)] -(x^2+y^2)\right\} \tag{2.14}$$

and

$$f_n(x)=e^{ik_nx}+(-1)^n e^{-ik_nx},\quad k_n=\frac{\pi n}{2l} . \tag{2.15}$$

The modes $f_n(x)$ from which $V(x,y)$ is constructed obey the Neumann boundary condition $\partial_1f_n(x)=0$ for $x=\pm l$. The Green's function $V(x,y)$ satisfies the equation

$$\partial_1{}^2V(x,y)=\delta_N(x,y)\equiv\delta(x-y)+\delta(|x+y|-2l) \tag{2.16}$$

and the boundary condition

$$n^1\partial_1V(x,y)|_{x=\pm l}=-\tfrac{1}{2} . \tag{2.17}$$

We have adopted the following prescription for handling the surface terms:

$$\int_{-l}^{l}=\lim_{\epsilon\to 0}\tfrac{1}{2}\left(\int_{-l+\epsilon}^{l+\epsilon}+\int_{-l-\epsilon}^{l-\epsilon}\right) . \tag{2.18}$$

We leave it to the reader to convince himself that (2.13) is also a solution to (2.9b). The identity which is helpful in proving this is

$$-\partial_x\partial_y V(x,y)=\delta_D(x,y)\equiv\delta(x-y)-\delta(|x+y|-2l). \tag{2.19}$$

Using the symmetry of the kernel $V(x,y)$ we can write the equation (2.2) in a form which explicitly shows the self-coupling:

$$\left[i\not\partial+g^2\gamma^0\int_{-l}^{l}dy\,j^0(t,y)V(y,x)\right]\psi(\underline{x})=0. \tag{2.20}$$

The interaction Hamiltonian is

$$H_I=-\tfrac{1}{2}g^2\int_{-l}^{l}\int_{-l}^{l}dx\,dy\,j^0(t,x)V(x,y)j^0(t,y). \tag{2.21}$$

III. ALMOST CANONICAL ANTICOMMUTATION RELATION AND THE FREE FERMION GREEN'S FUNCTION

We turn our attention to free massless fermions in a static cavity. We would like to find the proper modifications to the equal-time commutation relations (ETCR's) that arise in the presence of boundary conditions. Obviously the well-known canonical form

$$\{\psi(\underline{x}_1),\psi^\dagger(\underline{x}_2)\}_{t_1=t_2}=\delta(x_1-x_2)$$

does not meet the requirement (2.4). We shall find that the relation for fields in a cavity is also local but has additional surface terms. For the γ matrices we use the representation

$$\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \gamma^1=\begin{pmatrix}0&-1\\1&0\end{pmatrix}.$$

In our cavity with end points at $x\pm l$ and convention for the normal to the surface (Sec. II) we find the solution to (2.2) and (2.4) for $g=0$:

$$\psi^{(0)}(\underline{x})=\frac{1}{2\sqrt{l}}\sum_{n\geq 0}\left[b_n\begin{pmatrix}e^{-in'\pi(t-x)/2l}\\(-1)^n e^{-in'\pi(t+x)/2l}\end{pmatrix}+d_n^\dagger\begin{pmatrix}e^{in'\pi(t-x)/2l}\\(-1)^{n+1}e^{in'\pi(t+x)/2l}\end{pmatrix}\right], \tag{3.1}$$

where $n'=n+\frac{1}{2}$ and $d_n^\dagger$, b_n are the usual creation and annihilation operators. They obey the well-known algebra

$$\{b_n,b_k^\dagger\}=\{d_n,d_k^\dagger\}=\delta_{nk}. \tag{3.2}$$

All other anticommutators are zero. We assume the same algebra for the second-quantized amplitudes, but modify the first quantization to be consistent with the surface conditions imposed on the fields. For the ETCR we find

$$\{\psi^{(0)}(\underline{x}_1),\psi^{\dagger(0)}(\underline{x}_2)\}_{t_1=t_2}=\delta_\psi(x_1,x_2)\equiv\delta(x_1-x_2)+in^1\gamma^1\delta(|x_1+x_2|-2l). \tag{3.3}$$

One can check that this expression satisfies (2.4) for our cavity and we postulate the same equal-time anticommutation relation for the interacting fermion field. The free fermion Green's function is defined as

$$\langle 0|T\psi_\alpha^{(0)}(\underline{x}_1)\bar\psi_\beta^{(0)}(\underline{x}_2)|0\rangle=iS_{\alpha\beta}^{(0)}(\underline{x}_1,\underline{x}_2), \tag{3.4}$$

and from (3.3) one can check that it satisfies the equation

$$i\not\partial_{x_1}S^{(0)}(\underline{x}_1,\underline{x}_2)=\gamma^0\delta_\psi{}^2(\underline{x}_1,\underline{x}_2)\gamma^0, \tag{3.5}$$

where

$$\delta_\psi{}^2(\underline{x}_1,\underline{x}_2)=\delta(t_1-t_2)\delta_\psi(x_1,x_2).$$

From (3.1) and (3.4) we find for the matrix elements of the free Green's function

$$\begin{aligned}iS_{11}^{(0)}(\underline{x}_1,\underline{x}_2)=&\frac{1}{4l}\theta(t)e^{-i\pi(t-y)/4l}\left[\tfrac{1}{2}+\sum_{k=-\infty}^{\infty}\delta\left(1-\frac{t-y}{2l}-2k\right)+\tfrac{1}{2}i\cot\frac{\pi}{2}\left(1-\frac{t-y}{2l}\right)\right]\\&+\frac{1}{4l}\theta(-t)e^{i\pi(t-y)/4l}\left[\tfrac{1}{2}+\sum_{k=-\infty}^{\infty}\delta\left(1+\frac{t-y}{2l}-2k\right)+\tfrac{1}{2}i\cot\frac{\pi}{2}\left(1+\frac{t-y}{2l}\right)\right],\end{aligned} \tag{3.6}$$

where $t=t_1-t_2$, $y=x_1+x_2$.

From considerations of symmetry we get

$$S_{22}^{(0)}(t,y)=S_{11}^{(0)}(t,-y). \tag{3.7}$$

Similarly one also finds

$$iS_{12}^{(0)}(\underline{x}_1, \underline{x}_2) = \frac{1}{4l}\theta(t)e^{-i\pi(t-x)/4l}\left[\tfrac{1}{2} + \sum_{k=-\infty}^{\infty}\delta\left(\frac{t-x}{2l} - 2k\right) - \tfrac{1}{2}i\cot\pi\frac{t-x}{4l}\right]$$
$$- \frac{1}{4l}\theta(-t)e^{i\pi(t-x)/4l}\left[\tfrac{1}{2} + \sum_{k=-\infty}^{\infty}\delta\left(\frac{t-x}{2l} - 2k\right) + \tfrac{1}{2}i\cot\pi\frac{t-x}{4l}\right], \tag{3.8}$$

where $x = x_1 - x_2$. To find S_{21} one uses again the symmetry properties

$$S_{21}^{(0)}(t, x) = S_{12}^{(0)}(t, -x).$$

A formula helpful in deriving (3.6) to (3.9) is (3.9)

$$\sum_{n=0}^{\infty} e^{in\alpha} = \tfrac{1}{2} + \pi\sum_{k=-\infty}^{\infty}\delta(\alpha - 2\pi k) + \tfrac{1}{2}i\cot\tfrac{1}{2}\alpha. \tag{3.10}$$

Even though the expressions for the matrix elements of the Green's function look very complicated we show (see Appendix B) that the relevant equal-time and small-distance behavior for the free Green's function in a cavity is the same as the behavior for a free Green's function of pointlike particles. We shall find in the next section that this property turns out to be of fundamental importance for the theory to be solvable.

IV. THE EXACT GENERATING FUNCTIONAL OF THE THEORY

To find the generating functional we use the Symanzik construction. For a discussion of this method we refer the reader to Ref. 4. The functional is defined as

$$Z(\eta, \bar\eta, B) = \left\langle 0\left| T\left(\exp\left\{i\int_{\Omega_B} d\underline{x}[\bar\eta(\underline{x})\psi(\underline{x}) + \bar\psi(\underline{x})\eta(\underline{x}) - j^0(\underline{x})B(\underline{x})]\right\}\right)\right|0\right\rangle, \quad \int_{\Omega_B} d\underline{x} - \int_{-\infty}^{\infty} dt\int_{-l}^{l} dx \tag{4.1}$$

where ψ is a Heisenberg field obeying the equation of motion (2.20), ETCR (3.3), and the boundary condition (2.4). We have introduced external c-number sources $\eta, \bar\eta, B$. The sources $\eta, \bar\eta$ coupled to the fermion field anticommute both with each other and with $\psi, \bar\psi$. The charge density $j^0(\underline{x})$ is given by (2.3b). Using Eqs. (2.4), (2.20), and (3.3) we find for the functional $Z(\eta, \bar\eta, B)$

$$\left[i\partial\!\!\!/ \frac{\delta}{i\delta\bar\eta(\underline{x})} - \theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})B(\underline{y})\frac{\delta}{i\delta\bar\eta(\underline{y})}\right]Z(\eta, \bar\eta, B)$$
$$= \theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})\gamma^0\eta(\underline{y})Z(\eta, \bar\eta, B) + g\gamma^0\frac{\delta}{i\delta\bar\eta(\underline{x})}\int_{\Omega_B} d\underline{y}\,\bar V(\underline{x},\underline{y})\frac{\delta}{i\delta B(\underline{y})}Z(\eta, \bar\eta, B)$$
$$+ 2\sum_{i=1}^{2}\delta(x - x_i)\frac{\delta}{i\delta\bar\eta(\underline{x})}Z(\eta, \bar\eta, B), \quad x_i = \pm l \tag{4.2}$$

where $\bar V(\underline{x},\underline{y}) = \delta(x_0 - y_0)V(x,y)$ and the end-points convention (2.18) was adopted. The details of this calculation are presented in Appendix A. The solution to Eqs. (4.2) is

$$Z(\eta, \bar\eta, B) = N\exp\left[\tfrac{1}{2}ig^2\int_{\Omega_B} d\underline{x}\,d\underline{y}\frac{\delta}{i\delta B(\underline{x})}\bar V(\underline{x},\underline{y})\frac{\delta}{i\delta B(\underline{y})}\right]$$
$$\times Z_0(\eta, \bar\eta, B), \tag{4.3}$$

where N is a normalization constant such that $Z(0,0,0) = 1$. For the free functional

$$Z_0(\eta, \bar\eta, B) = Z(\eta, \bar\eta, B; g = 0)$$

we get

$$Z_0(\eta, \bar\eta, B) = Z_0(B)\exp\left[i\int_{\Omega_B} d\underline{z}\,d\underline{z}'\bar\eta(\underline{z})S^B(\underline{z},\underline{z}')\eta(\underline{z}')\right], \tag{4.4}$$

where $S^B(\underline{z},\underline{z}')$ is the fermion Green's function in the presence of an external $B(\underline{x})$ source satisfying the equation

$$i\partial\!\!\!/ S^B(\underline{x},\underline{y}) - \gamma^0\int_{\Omega_B} d\underline{z}\,\delta_\psi{}^2(\underline{x},\underline{z})B(\underline{z})S^B(\underline{z},\underline{y})$$
$$= \gamma^0\delta_\psi{}^2(\underline{x},\underline{y})\gamma^0 \tag{4.5}$$

and the boundary condition

$$\delta(x - x_i)[in^1\gamma^1 S^B(\underline{x},\underline{y}) - S^B(\underline{x},\underline{y})] = 0. \tag{4.6}$$

The equation for $Z_0(B)$ is

$$\frac{\delta}{\delta B(\underline{x})}\ln Z_0(B) = -\theta_B(x)\mathrm{Tr}[\gamma^0 S^B(\underline{x},\underline{x})]. \tag{4.7}$$

The solution of (4.5) and (4.6) will constitute the major part of our effort to construct an exact generating functional. Let us write $S^B(\underline{x},\underline{y})$ in the form

$$iS^B(\underline{x},\underline{y})=\langle 0|T\psi^B(\underline{x})\overline{\psi}^B(\underline{y})|0\rangle, \tag{4.8}$$

where the fermion field with an external source satisfies the equation

$$i\partial\!\!\!/\psi^B(\underline{x})-\gamma^0\int_{\Omega_B}d\underline{z}\,B(\underline{z})\delta_\psi{}^2(\underline{x},\underline{z})\psi^B(\underline{z})=0. \tag{4.9}$$

With $\psi^B(\underline{x})$ obeying the boundary condition (2.4) and the ETCR (3.3) one can check that S^B from (4.8) satisfies (4.5) and (4.6). We write $\psi^B(\underline{x})$ in the form

$$\psi^B(\underline{x})=e^{i\phi(\underline{x})}\psi^{(0)}(\underline{x}), \tag{4.10}$$

where

$$\phi(\underline{x})=\gamma_5\phi_1(\underline{x})+\phi_2(\underline{x}) \tag{4.11}$$

and $\psi^{(0)}(\underline{x})$ is the free Dirac field in the cavity. We use the identities

$$\psi^{(0)}(t,2l-x)=-i\gamma^1\psi^{(0)}(\underline{x}), \tag{4.12a}$$

$$\psi^{(0)}(t,-2l-x)=i\gamma^1\psi^{(0)}(\underline{x}), \tag{4.12b}$$

which follow from the solution (3.1). The boundary condition is a special case for $x=l$. With the help of (4.12a) and (4.12b) Eq. (4.9) can be written as

$$i\partial\!\!\!/\psi^B(\underline{x})-\gamma^0\int_{\Omega_B}d\underline{z}\,B(\underline{z})[\delta^2(\underline{x}-\underline{z})e^{i\phi(\underline{z})}-\gamma^1e^{i\phi(\underline{z})}\gamma^1\delta(x_0-z_0)\delta(|x+z|-2l)]\psi^{(0)}(\underline{x})=0. \tag{4.13}$$

One can show that from the condition

$$\{\psi^B(\underline{x}),\psi^{\dagger B}(\underline{y})\}_{x_0=y_0}=\delta_\psi(x,y)$$

and the ETCR for the free fermion field (3.3) one can get the consistency requirement

$$e^{i\phi(\underline{x})}\delta_\psi{}^2(\underline{x},\underline{y})e^{-i\phi(\underline{x})}=\delta_\psi{}^2(\underline{x},\underline{y}). \tag{4.14}$$

With the explicit form for δ_ψ from (3.3) we find that $\phi(\underline{x})$ must have the following properties:

$$\phi_1(t,\pm 2l-x)=-\phi_1(\underline{x}), \tag{4.15a}$$

$$\phi_2(t,\pm 2l-x)=\phi_2(\underline{x}). \tag{4.15b}$$

A special case of (4.15a) for $x=l$ is that ϕ_1 vanishes on the surface. With (4.15a) and (4.15b) we can write (4.13) in the local form

$$[i\partial\!\!\!/-\gamma^0\overline{B}(\underline{x})]\psi^B(\underline{x})=0, \tag{4.16}$$

where

$$\overline{B}(\underline{x})=\int_{\Omega_B}d\underline{z}[\delta^2(\underline{x}-\underline{z})+\delta(x_0-z_0)\delta(|x+z|-2l)B(\underline{z})]. \tag{4.17}$$

Using the representation (4.10) for $\psi^B(\underline{x})$ we find that (4.16) is equivalent to

$$[i\partial\!\!\!/-\gamma^0\overline{B}(\underline{x})]e^{i\phi(\underline{x})}=0,$$

which gives the simple equation

$$\partial\!\!\!/\phi(\underline{x})+\gamma^0\overline{B}(\underline{x})=0. \tag{4.18}$$

The solution to (4.18) is

$$\phi_1(\underline{x})=\partial_1\int_{\Omega_B}D_N(\underline{x},\underline{y})B(\underline{y})d\underline{y}, \tag{4.19a}$$

$$\phi_2(\underline{x})=-\partial_0\int_{\Omega_B}D_N(\underline{x},\underline{y})B(\underline{y})d\underline{y}. \tag{4.19b}$$

$D_N(\underline{x},\underline{y})$ is the propagator of a scalar field obeying Neumann boundary conditions in the cavity. It satisfies the equation

$$\Box D_N(\underline{x},\underline{y})=\delta_N{}^2(\underline{x},\underline{y})\equiv\delta^2(\underline{x}-\underline{y})+\delta(x_0-y_0)\delta(|x+y|-2l) \tag{4.20a}$$

and the boundary condition

$$\partial_x D_N(\underline{x},\underline{y})_{x=\pm l}=0. \tag{4.20b}$$

$D_N(\underline{x},\underline{y})$ is constructed out of Neumann modes in a similar manner to $\overline{V}(\underline{x},\underline{y})$. Its explicit form is

$$D_N(\underline{x},\underline{y})=-\frac{1}{8\pi l}\int_{-\infty}^{\infty}d\omega\,e^{-i\omega(x_0-y_0)}\left[\sum_{n=1}^{\infty}\frac{f_n^\omega(x)f_n^{*\omega}(y)}{\omega^2-k_n{}^2+i\epsilon}-i\frac{\omega(x_0-y_0)}{\omega^2+i\epsilon}\right], \tag{4.21}$$

where $f_n^\omega(x)=e^{i\omega x}+(-1)^ne^{-i\omega x}$.

One can easily check that (4.19a) and (4.19b) satisfy Eq. (4.18) as well as the functional relations (4.15a) and (4.15b). To prove this one finds that after one calculates the spectral integral in (4.21) the modes become

$$f_n(x)=e^{\pm ik_nx}+(-1)^ne^{\mp ik_nx},$$

which satisfies the identities

$$f_n(x)=f_n(\pm 2l-x),$$

$$\partial_xf_n(x)=-\partial_xf_n(\pm 2l-x).$$

From Eq. (4.7) and the equal-time, small-distance behavior of $S^B(\underline{x},\underline{y})$, which is discussed in Appendix B, we finally get for $Z_0(B)$

$$Z_0(B) = \exp\left[-\frac{i}{2\pi}\int_{\Omega_B} d\underline{x}\, d\underline{y}\, B(\underline{x})\partial_1{}^2(x) D_N(\underline{x},\underline{y}) B(\underline{y})\right], \tag{4.22}$$

where we have taken into account the symmetry of $D_N(\underline{x},\underline{y})$. It is worth noting before we close this section that $S^B(\underline{x},\underline{y})$ can be constructed as a periodic function for all x and y. The full periodic structure of $\phi(\underline{x})$ is given by $D_N(\underline{x},\underline{y})$ for points outside the cavity. Since by definition all particles exist in the interval $-l \le x \le l$ we need only consider as meaningful the behavior of the Green's function in that region.

V. TIME-ORDERED PRODUCT OF THE ELECTRIC FIELD AND CURRENT ALGEBRA

Knowing the exact functional of our model we can proceed to calculate the relevant Green's function. Using (2.7) and (2.13) one can write for the Green's function of the electric field

$$\langle 0|\, TE(\underline{x})\, E(\underline{y})\,|0\rangle = g^2 \partial_x \partial_y \int_{\Omega_B} d\underline{z}\, d\underline{w}\, \overline{V}(\underline{x},\underline{z})\, \overline{V}(\underline{y},\underline{w}) \times \langle 0|\, Tj^0(\underline{z})\, j^0(\underline{w})\,|0\rangle\ . \tag{5.1}$$

We take derivatives of the generating functional with respect to the external source $B(\underline{x})$. When we set all sources equal to zero we get the time-ordered product of $j^0(\underline{x})$

$$\theta_B(z)\,\theta_B(w)\langle 0|\, Tj^0(\underline{z})\, j^0(\underline{w})\,|0\rangle = \frac{\delta}{i\delta B(\underline{z})}\,\frac{\delta}{i\delta B(\underline{w})}\, Z(\eta,\overline{\eta},B)\Big|_{\eta=\overline{\eta}=B=0}. \tag{5.2}$$

In the limit where $\eta=\overline{\eta}=0$ derivatives with respect to $B(\underline{x})$ from the factor

$$\exp\left[\, i\int_{\Omega_B} d\underline{x}\, d\underline{y}\, \overline{\eta}(\underline{x})\, S^B(\underline{x},\underline{y})\,\eta(\underline{y})\right]$$

will vanish. The right-hand side of (5.2) then becomes

$$\frac{N}{i^2}\,\frac{\delta}{\delta B(\underline{z})}\,\frac{\delta}{\delta B(\underline{w})}\exp\left[-\tfrac{1}{2} i g^2 \int_{\Omega_B} d\underline{x}\, d\underline{y}\, \frac{\delta}{\delta B(\underline{x})}\,\overline{V}(\underline{x},\underline{y})\frac{\delta}{\delta B(\underline{y})}\right]\exp\left[-\frac{i}{2\pi}\int_{\Omega_B} d\underline{x}'\, d\underline{y}'\, B(\underline{x}')\,\partial_{x'}{}^2 D_N(\underline{x}',\underline{y}')\, B(\underline{y}')\right]. \tag{5.3}$$

This expression can be evaluated by a well-known trick. Let us write

$$\begin{aligned} F(\lambda) &\equiv \exp\left(-\tfrac{1}{2} i\lambda \int \frac{\delta}{\delta B}\,\overline{V}\,\frac{\delta}{\delta B}\right)\exp\left[\tfrac{1}{2} i \int B\left(-\frac{1}{\pi}\,\partial_1{}^2 D_N\right) B\right] \\ &= \exp\left[\tfrac{1}{2} i \int B\chi(\lambda)\, B + \phi(\lambda)\right], \end{aligned} \tag{5.4}$$

where $\chi(\lambda)$, $\phi(\lambda)$ do not depend on the source B and obey the boundary conditions

$$\chi(0) = -\frac{1}{\pi}\,\partial_1{}^2 D_N\ , \tag{5.5a}$$

$$\phi(0) = 0\ . \tag{5.5b}$$

We differentiate with respect to λ (5.4) and from the coefficients of B we get the pair of equations

$$\frac{d\chi(\underline{x},\underline{y}\,|\,\lambda)}{d\lambda} = \int_{\Omega_B} d\underline{x}'\, d\underline{y}'\, \chi(\underline{x},\underline{x}'\,|\,\lambda) \times \overline{V}(\underline{x}',\underline{y}')\,\chi(\underline{y}',\underline{y}\,|\,\lambda)\ , \tag{5.6}$$

$$\frac{d\phi(\lambda)}{d\lambda} = \tfrac{1}{2}\,\mathrm{Tr}[\,\overline{V}(\underline{x},\underline{y})\,\chi(\underline{y},\underline{x}\,|\,\lambda)]\ . \tag{5.7}$$

We are interested only in the solution for $\chi(\lambda)$ since $\phi(\lambda)$ is absorbed in the normalization factor N. The result given as an operator statement is

$$\chi(\lambda) = -\frac{1}{\pi}\,\partial_1{}^2 D_N\,\frac{1}{1+\lambda\overline{V}(1/\pi)\,\partial_1{}^2 D_N}\ . \tag{5.8}$$

From (5.8), (5.4), and (5.3) we get finally

$$\langle 0|\, TE(\underline{x})\, E(\underline{y})\,|0\rangle = \frac{ig^2}{\pi}\,\partial_x\partial_y \int_{\Omega_B} d\underline{z}\, d\underline{w}\, \overline{V}(\underline{x},\underline{z})\,\partial_1{}^2 D_N\,\frac{1}{1+(g^2/\pi)\,\overline{V}\partial_1{}^2 D_N}\,(\underline{z},\underline{w})\,\overline{V}(\underline{w},\underline{y}). \tag{5.9}$$

At this point it is useful to introduce the propagator of a massless scalar field obeying Dirichlet boundary conditions. It is constructed from modes vanishing at the end points

$$\eta_n(x) = e^{ik_n x} - (-1)^n e^{-ik_n x} , \quad (5.10)$$

where

$$k_n = \frac{\pi n}{2l} .$$

The spectral representation of the Dirichlet propagator is

$$\Delta_D(\underline{x}, \underline{y}) = -\frac{1}{8\pi l}\int_{-\infty}^{\infty} dk\, e^{-ik(x_0-y_0)} \sum_{n=1}^{\infty} \frac{\eta_n^k(x)\,\eta_n^{*k}(y)}{k^2 - k_n^2 + i\epsilon} \quad (5.11)$$

and

$$\eta_n^k(x) = e^{ikx} - (-1)^n e^{-ikx} .$$

This Green's function satisfies the equation

$$\Box\Delta_D(\underline{x}, \underline{y}) = \delta_D{}^2(\underline{x}, \underline{y}) \equiv \delta(x_0 - y_0)\,[\delta(x-y) - \delta(|x+y| - 2l)] . \quad (5.12)$$

We call the right-hand side of this equation the Dirichlet δ function—it vanishes on the boundary. We shall use the identity

$$\partial_x{}^2 D_N(\underline{x}, \underline{y}) = -\partial_x\,\partial_y\,\Delta_D(\underline{x}, \underline{y}) , \quad (5.13)$$

which is trivial to prove because

$$\partial_1 \eta_n^k(x) = ikf_n^k(x) .$$

Now let us formally expand the kernel in (5.9):

$$\partial_1{}^2 D_N \frac{1}{1 + (g^2/\pi)\,\overline{V}\partial_1{}^2 D_N} = \partial_1{}^2 D_N - \frac{g^2}{\pi}\,\partial_1{}^2 D_N\,\overline{V}\partial_1{}^2 D_N + \cdots . \quad (5.14)$$

From the expansion (5.14) and the identity (5.13) we get for the first term in (5.9)

$$I_1 = -\frac{ig^2}{\pi}\,\Delta_D(\underline{x}, \underline{y}). \quad (5.15)$$

The second term in the expansion has the form

$$I_2 = -i\left(\frac{g^2}{\pi}\right)^2 \int_{\Omega_B} d\underline{z}\, d\underline{z}'\, d\underline{w}\, d\underline{w}'\, \vec{\partial}_x\, \overline{V}(\underline{x}, \underline{z}) \times \partial_z{}^2 D_N(\underline{z}, \underline{z}')\, \overline{V}(\underline{z}', \underline{w}) \times \partial_w{}^2 D_N(\underline{w}, \underline{w}')\, V(\underline{w}', \underline{y})\,\overleftarrow{\partial}_y . \quad (5.16)$$

Here again we use (5.13), integrate by parts over $dz\,dz'\,dw\,dw'$, and get from (2.19)

$$I_2 = i\left(\frac{g^2}{\pi}\right)^2 \int_{\Omega_B} d\underline{z}\, d\underline{z}'\, d\underline{w}\, d\underline{w}'\, \delta_D(\underline{x}, \underline{z})\, \Delta_D(\underline{z}, \underline{z}')\, \delta_D(\underline{z}', \underline{w}) \times \Delta_D(\underline{w}, \underline{w}')\, \delta_D(\underline{w}', \underline{y}) = i\left(\frac{g^2}{\pi}\right)^2 \int_{\Omega_B} dz\, \Delta_D(\underline{x}, \underline{z})\, \Delta_D(\underline{z}, \underline{y}) . \quad (5.17)$$

The same procedure can be carried for all the terms in the expansion (5.14) and we can write

$$\langle 0|\, TE(\underline{x})\, E(\underline{y})\,|0\rangle = -\frac{ig^2}{\pi}\,\Delta_D\left(1 + \frac{g^2}{\pi}\,\Delta_D\right)^{-1}(\underline{x}, \underline{y}) . \quad (5.18)$$

It is easy to show that $\Delta_D^M \equiv \Delta_D[1 + (g^2/\pi)\Delta_D]^{-1}$ corresponds to the propagator of a scalar field of mass squared $M = g^2/\pi$ obeying Dirichlet boundary conditions. Formally we have

$$\Box\Delta_D^M = \Box\left[\Delta_D - \frac{g^2}{\pi}\,\Delta_D\Delta_D + \left(\frac{g^2}{\pi}\right)^2 \Delta_D\Delta_D\Delta_D - \cdots\right] .$$

In this expansion all the proper integrations are of course understood. From (5.12) we get

$$\Box\Delta_D^M = \delta_D - \frac{g^2}{\pi}\,\Delta_D + \left(\frac{g^2}{\pi}\right)^2 \Delta_D\Delta_D - \cdots = \delta_D - \frac{g^2}{\pi}\,\Delta_D\left(1 + \frac{g^2}{\pi}\,\Delta_D\right)^{-1}$$

or (5.19)

$$\left(\Box + \frac{g^2}{\pi}\right)\Delta_D^M = \delta_D.$$

But this is exactly the Green's function corresponding to a scalar particle of mass squared g^2/π. Obviously from the form of (5.18) Δ_D^M satisfies the Dirichlet boundary condition. As we stated in the Introduction the result is that only massive bosons with proper boundary conditions are present in the theory and their mass is the same as the well-known result for the Schwinger model,

$$\langle 0|\, TE(\underline{x})E(\underline{y})\,|0\rangle = -\frac{ig^2}{\pi}\,\Delta_D\left(\underline{x}, \underline{y}; \frac{g^2}{\pi}\right), \quad (5.20)$$

where

$$\Delta_D\left(\underline{x}, \underline{y}; \frac{g^2}{\pi}\right) = -\frac{1}{16\pi l}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(x_0-y_0)} \times \sum_{n=1}^{\infty} \frac{\eta_n(x)\eta_n^*(y) + \eta_n^*(x)\eta_n(y)}{\omega^2 - \omega_n{}^2 + i\epsilon} \quad (5.21)$$

and $\omega_n{}^2 = k_n{}^2 + g^2/\pi$. From (5.20) we can identify the massive scalar field obeying the Dirichlet boundary condition as

$$E(\underline{x}) = \frac{g}{\sqrt{\pi}}\,\phi_D\left(\underline{x}; \frac{g^2}{\pi}\right). \quad (5.22)$$

For a multiple-reflection expansion of (5.21) see Appendix C. It is quite remarkable that the results of the two-dimensional QED are independent of the introduction of an elementary length with appropriate boundary conditions. One might expect that there is a nontrivial effect on the screening mechanism and that only as $l\to\infty$ can the old results be recovered. However, it appears that since the small-distance behavior is not affected and the theory is almost canonical there are no additional terms contributing to the time-ordered product of the electric field strengths.

Now we shall show that the current algebra is modified only by surface terms, consistent with the boundary conditions. From (5.20) and (5.21) we can write

$$\langle 0|E(\underline{x})E(\underline{y})|0\rangle=\frac{g^2}{16\pi l}\sum_{n=1}^{\infty}\frac{1}{\omega_n}e^{-i\omega_n(x_0-y_0)}\times[\eta_n(x)\eta_n^*(y)+\eta_n^*(x)\eta_n(y)]. \tag{5.23}$$

To find the vacuum expectation value of the currents we use (2.8) and $\partial_1 f_n(x)=ik_n\eta_n(x)$ to get

$$\langle 0|j^0(\underline{x})j^1(\underline{y})|0\rangle=\frac{1}{16\pi l}\sum_{n=1}^{\infty}e^{-i\omega_n(x_0-y_0)}\times k_n[f_n(x)\eta_n^*(y)-f_n^*(x)\eta_n(y)]. \tag{5.24}$$

Similarly one can find also

$$\langle 0|j^1(\underline{x})j^0(\underline{y})|0\rangle=\frac{1}{16\pi l}\sum_{n=1}^{\infty}e^{-i\omega_n(x_0-y_0)}\times k_n[\eta_n(x)f_n^*(y)-\eta_n^*(x)f_n(y)]. \tag{5.25}$$

If we exchange $x\to y$ in (5.25) and subtract from (5.24) at equal times we have

$$\langle 0|[j^0(\underline{x}),j^1(\underline{y})]_{x_0=y_0}|0\rangle=\frac{1}{8\pi l}\sum_n k_n[f_n(x)\eta_n^*(y)-f_n^*(x)\eta_n(y)]. \tag{5.26}$$

By using the completeness relation for the Dirichlet modes we find for the current algebra

$$\langle 0|[j^0(\underline{x}),j^1(\underline{y})]_{x_0=y_0}|0\rangle=-\frac{i}{\pi}\partial_1(x)\delta_D(x,y). \tag{5.27}$$

The equal-time commutators for the same components of the currents are identically zero. One can observe that the Schwinger term is the same as in ordinary two-dimensional QED, but there are surface modifications in the δ function which make (5.27) consistent with the boundary condition: $j^1(\underline{y})$ vanishes on the surface. From (2.8) and (5.22) we identify

$$j^\mu(\underline{x})=\frac{\epsilon^{\mu\nu}}{\sqrt{\pi}}\partial_\nu\phi_D(\underline{x};g^2/\pi), \tag{5.28}$$

where

$$\partial_\mu j^\mu(\underline{x})=0,$$
$$\partial_\mu j^{5\mu}(\underline{x})\equiv\partial_1 j^0(\underline{x})+\partial_0 j^1(\underline{x})=\frac{g^2}{\pi^{3/2}}\phi_D(\underline{x}).$$

One can immediately verify that the charge

$$Q=\frac{1}{\sqrt{\pi}}[\phi_D(x=l,t)-\phi_D(x=-l,t)]$$

is zero, consistent with our initial discussion in Sec. II.

From the current-algebra relation (5.27) we find that $\phi_D(\underline{x})$ satisfies the almost canonical commutation relation, appropriate for a field obeying the Dirichlet boundary condition,

$$[\dot{\phi}_D(\underline{x}),\phi_D(\underline{y})]_{x_0=y_0}=-i\delta_D(x,y). \tag{5.29}$$

The usual expansion for $\phi_D(\underline{x})$ in terms of second-quantized amplitudes is

$$\phi_D(\underline{x})=\tfrac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2l\omega_n)^{1/2}}[a_n e^{-i\omega_n t}\eta_n(x)+a_n^\dagger e^{i\omega_n t}\eta_n^*(x)]. \tag{5.30}$$

From (5.29) it is trivial to show that the amplitudes obey the usual algebra

$$[a_n,a_k^\dagger]=\delta_{nk}, \tag{5.31}$$

with all other commutators zero. In terms of the currents we get for the amplitude a_n

$$2i\left(\frac{2l\omega_n}{\pi}\right)^{1/2}a_n=\int_{-l}^{l}dx\left[\eta_n^*(x,t)j^1(x,t)+\frac{\omega_n}{k_n}f_n^*(x,t)j^0(x,t)\right], \tag{5.32}$$

where

$$\eta_n(x,t)=e^{-i\omega_n t}\eta_n(x),$$
$$f_n(x,t)=e^{-i\omega_n t}f_n(x).$$

The relation (5.31) can be checked directly from (5.27). $a_n, a_n^\dagger$ play a role similar to the "plasmon" operators for the conductive string (see Ref. 5).

VI. DISCUSSION AND SUMMARY

The Hamiltonian of the system is diagonal in the Fock space of the creation and annihilation operators of the massive scalar field. Since it gives the physical spectrum we can assert that only massive scalar particles obeying a Dirichlet boundary condition exist in the spectrum. To show this we use the relations of time-translational invariance

$$[H, j^\mu(\underline{x})] = -i\partial_0 j^\mu(\underline{x}). \tag{6.1}$$

The right-hand side of this equation can be rewritten using current conservation and the equation for the axial-vector current. From (5.28), (5.30), (5.32), and the orthogonality relations of the Dirichlet and Neumann modes $\eta_n(x)$ and $f_n(x)$ we get

$$[H, a_n] = -w_n a_n. \tag{6.2}$$

The free and interacting parts of the Hamiltonian separately are not diagonal with respect to $a_n, a_n^\dagger$, but the total Hamiltonian is diagonal. For a clear formulation of the ordinary two-dimensional QED in terms of the currents see Ref. 6.

From (5.28), (5.22), and (5.20) one can write

$$\langle 0 | Tj^\mu(\underline{x}) j^\nu(\underline{y}) | 0\rangle = -\frac{i}{\pi}\epsilon^{\mu\alpha}\epsilon^{\nu\beta}\partial_{x\alpha}\partial_{y\beta}\Delta_D\left(\underline{x},\underline{y};\frac{g^2}{\pi}\right). \tag{6.3}$$

For the time-ordered product of the currents constructed from the solution for the free fermion field (3.1) we have

$$\langle 0 | Tj^{(0)\mu}(\underline{x}) j^{(0)\nu}(\underline{x}) | 0\rangle = -\frac{i}{\pi}\epsilon^{\mu\alpha}\epsilon^{\nu\beta}\partial_\alpha(x)\partial_\beta(y)\Delta_D(\underline{x},\underline{y}), \tag{6.4}$$

where $\Delta_D(\underline{x},\underline{y})$ is the propagator for a massless particle (5.11). The multiple-reflection expansion (C7) shows that the leading singularity on the light cone is the same for $\Delta_D(\underline{x},\underline{y}; g^2/\pi)$ and $\Delta_D(\underline{x},\underline{y})$—it is independent of the mass of the particle. If we probe the static bag with virtual photons of high momentum we observe the leading light-cone behavior for free quark partons in the cavity, even though they exist in pairs with strong spin-zero correlation. This is consistent with our basic intuition that the short-distance behavior of the model should be unaffected by boundary conditions which remove the long wavelengths of the theory.

This example gives us hope that even in the case of fields confined to a bag in four dimensions it may be possible for the masses of fermions and vector mesons to arise spontaneously, without the presence of primary scalar fields in the Lagrangian. In the ordinary field theory such a possibility was examined by various authors, for example see Ref. 8. We believe that a further investigation along these lines for particles with structure might prove illuminating.

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APPENDIX A

Here we present a derivation of (4.2), (4.3), (4.4), and (4.7). Let us denote by $\mathcal{L}_{\text{ext}}(\underline{z})$

$$\mathcal{L}_{\text{ext}}(\underline{z}) = \overline{\eta}(\underline{z})\psi(\underline{z}) + \overline{\psi}(\underline{z})\eta(\underline{z}) - j^0(\underline{z})B(\underline{z}). \tag{A1}$$

Using (2.20) and (2.4) we can write for the generating functional (4.1)

$$\begin{aligned} i\partial\!\!\!/ \frac{\delta}{i\delta\overline{\eta}(\underline{x})} Z(\eta,\overline{\eta},B) &= i\partial\!\!\!/ \left\langle 0 \middle| T\exp\left[i\int_{-\infty}^{t}\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}(\underline{z})\right]\theta_B(x)\psi(\underline{x})\exp\left[i\int_t^\infty\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}(\underline{z})\right]\middle| 0\right\rangle \\ &= -\left\langle 0\middle| T\exp\left(i\int_{-\infty}^{t}\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\theta_B(x)g^2\gamma^0\int_{\Omega_B} j^0(\underline{y})\,\overline{V}(\underline{y},\underline{x})d\underline{y}\,\psi(\underline{x})\exp\left(i\int_t^\infty\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\middle| 0\right\rangle \\ &\quad + \sum_{i=1}^{2}\left\langle 0\middle| T\exp\left(i\int_{-\infty}^{t}\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\psi(t,x_i)\delta(x-x_i)\exp\left(i\int_t^\infty\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\middle| 0\right\rangle \\ &\quad - \theta_B(x)\gamma^0\left\langle 0\middle| T\exp\left(i\int_{-\infty}^{t}\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\int_{-l}^{l} dz'\,[\psi(\underline{x}),\mathcal{L}_{\text{ext}}(z',t)]\exp\left(i\int_t^\infty\int_{-l}^{l} d\underline{z}\,\mathcal{L}_{\text{ext}}\right)\middle| 0\right\rangle. \end{aligned} \tag{A2}$$

To calculate the commutator of ψ and $\mathcal{L}_{\text{ext}}$ we use the relation

$$[a, bc] = \{a, b\}c - b\{a, c\}. \tag{A3}$$

Recalling the ETCR (3.3) and the properties of the sources $\eta,\overline{\eta}$ we can write

$$[\psi(\underline{x}), \mathcal{L}_{\text{ext}}(z',t)] = \delta_\psi(x,z')[-B(\underline{z}')\psi(z') + \gamma^0\eta(\underline{z}')]. \tag{A4}$$

The equation (A2) becomes

$$i\not\partial \frac{\delta}{i\delta\bar{\eta}(\underline{x})} Z(\eta,\bar{\eta},B) - \theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi^2(\underline{x},\underline{y})B(\underline{y})\frac{\delta}{i\delta\bar{\eta}(y)} Z(\eta,\bar{\eta},B)$$
$$= \theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})\gamma^0\eta(\underline{y})Z(\eta,\bar{\eta},B)$$
$$+2\sum_{i=1}^{2}\delta(x-x_i)\frac{\delta Z(\eta,\bar{\eta},B)}{i\delta\bar{\eta}(\underline{x})} + \theta_B(x)g^2\gamma^0\frac{\delta}{i\delta\bar{\eta}(\underline{x})}\int_{\Omega_B} d\underline{y}\,\bar{V}(\underline{x},\underline{y})\frac{\delta}{i\delta B(\underline{y})} Z(\eta,\bar{\eta},B), \tag{A5}$$

where we have used again the convention for integrating around the surface [see (2.18)]. The solution of (A5) for $g=0$ is given by (4.4), where $S^B(\underline{x},\underline{y})$ satisfies Eq. (4.5) and the boundary condition (4.6). To calculate $Z_0(B)$ we write

$$\frac{\delta}{i\delta\eta_\alpha(\underline{x})}\gamma^0_{\alpha\beta}\frac{\delta}{i\bar{\eta}_\beta(\underline{x})}Z_0(\eta,\bar{\eta},B) = -\frac{\delta}{i\delta B(\underline{x})}Z_0(\eta,\bar{\eta},B). \tag{A6}$$

With $Z_0(\eta,\bar{\eta},B)$ given by (4.4) in the limit when the external sources $\eta,\bar{\eta}$ vanish we get

$$Z_0(B)\mathrm{Tr}[\gamma^0 S^B(\underline{x},\underline{x})] = -\frac{\delta}{\delta B(\underline{x})}Z_0(B). \tag{A7}$$

This leads to Eq. (4.7), where the bag confinement is explicitly shown with $\theta_B(x)$. The form of the solution for the interacting functional is derived from the free functional with the usual ansatz (see Ref. 4)

$$Z(\eta,\bar{\eta},B) = F\left(\frac{\delta}{i\delta\eta}A\frac{\delta}{i\delta\bar{\eta}},\frac{\delta}{i\delta B}\right)Z_0(\eta,\bar{\eta},B). \tag{A8}$$

If we set

$$F=\exp\left[ig^2\int_{\Omega_B} d\underline{x}\,d\underline{y}\,\frac{\delta}{i\delta\eta(\underline{x})}\gamma^0\frac{\delta}{i\delta\bar{\eta}(\underline{x})}\,\bar{V}(\underline{x},\underline{y})\frac{\delta}{i\delta B(\underline{y})}\right], \tag{A9}$$

we find the following equation for the generational functional:

$$\left[i\not\partial\frac{\delta}{i\delta\bar{\eta}(\underline{x})} - \theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})B(\underline{y})\frac{\delta}{i\delta\bar{\eta}(\underline{y})}\right]Z(\eta,\bar{\eta},B)$$
$$=\theta_B(x)\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})\gamma^0\eta(\underline{y})Z(\eta,\bar{\eta},B) + 2\sum_{i=1}^{2}\delta(x-x_i)\frac{\delta}{i\delta\bar{\eta}(\underline{x})}Z(\eta,\bar{\eta},B)$$
$$+\theta_B(x)g^2\int_{\Omega_B} d\underline{y}\,\gamma^0\delta_\psi{}^2(\underline{x},\underline{y})\frac{\delta}{i\delta\bar{\eta}(\underline{y})}\int_{\Omega_B} d\underline{z}\,\bar{V}(\underline{y},\underline{z})\frac{\delta}{i\delta B(\underline{z})}Z(\eta,\bar{\eta},B). \tag{A10}$$

By using the property

$$\theta_B(x)\int_{\Omega_B} d\underline{y}\,\delta_\psi{}^2(\underline{x},\underline{y})f(\underline{y})\psi(\underline{y}) = \theta_B(x)f(\underline{x})\psi(\underline{x})$$

(where f is an arbitrary scalar function), we recover Eq. (A5). From (A8) and (A9) we can write

$$\frac{\partial Z(\eta,\bar{\eta},B)}{\partial g^2} = i\int_{\Omega_B} d\underline{x}\,d\underline{y}\,\frac{\delta}{i\delta\eta(\underline{x})}\gamma^0\frac{\delta}{i\delta\bar{\eta}(\underline{x})}$$
$$\times\bar{V}(\underline{x},\underline{y})\frac{\delta}{i\delta B(\underline{y})}Z(\eta,\bar{\eta},B). \tag{A11}$$

This leads to (4.3), where we have taken into consideration the symmetry of the expression to account for the $\frac{1}{2}$ factor.

APPENDIX B

We shall calculate the behavior of $\mathrm{Tr}[\gamma^0 S^B(\underline{x},\underline{y})]$ at equal times and small distances. We set $x_0=y_0$ and perform symmetrically the limit in space $x\to x+\frac{1}{2}\epsilon$, $y\to x-\frac{1}{2}\epsilon$, and $\epsilon\to\pm 0$ symmetrically from both directions. From (4.8), (4.10), and (4.11) we have

$$S^B(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon) = [e^{i\phi_2}(\cos\phi_1 + i\gamma_5\sin\phi_1)]_{x+\epsilon/2}$$
$$\times S^0(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon)$$
$$\times[e^{-i\phi_2}(\cos\phi_1 + i\gamma_5\sin\phi_1)]_{x-\epsilon/2}. \tag{B1}$$

Since $e^{i\phi}$ has only elements on the diagonal and γ^0 is off-diagonal in our representation we find that only the off-diagonal elements of $S^0(x+\frac{1}{2}\epsilon, x-\frac{1}{2}\epsilon)$ will contribute to $\mathrm{Tr}[\gamma^0 S^B(\underline{x}, \underline{x})]$. Using the explicit form (3.11) we get

$$S^0_{12}(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon) = \frac{1}{8l}\frac{1}{\sin(\pi\epsilon/4l)}. \tag{B2}$$

Similarly from (3.12) we have

$$S^0_{21}(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon) = -\frac{1}{8l}\frac{1}{\sin(\pi\epsilon/4l)}. \tag{B3}$$

Notice that the free propagator for a theory without boundary conditions has the small-distance behavior

$$S^0_F \sim \gamma^1 \frac{1}{\epsilon}.$$

This behavior is essentially the same as the one given by (A2) and (A3). Now the relevant contribution becomes

$$\begin{aligned}\overline{S}^B(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon) &= -\frac{1}{2\pi\epsilon}(1+i\tfrac{1}{2}\epsilon\,\partial_1\phi_2)^2[\cos\phi_1(x+\tfrac{1}{2}\epsilon)+i\gamma_5\sin\phi_1(x+\tfrac{1}{2}\epsilon)]\gamma^1[\cos\phi_1(x-\tfrac{1}{2}\epsilon)+i\gamma_5\sin\phi_1(x-\tfrac{1}{2}\epsilon)]\\ &= -\frac{1}{2\pi\epsilon}(1+i\tfrac{1}{2}\epsilon\,\partial_1\phi_2)^2\{\cos[\phi_1(x+\tfrac{1}{2}\epsilon)-\phi_1(x-\tfrac{1}{2}\epsilon)]+i\gamma_5\sin[\phi_1(x+\tfrac{1}{2}\epsilon)-\phi_1(x-\tfrac{1}{2}\epsilon)]\}.\end{aligned}$$

$\overline{S}$ means that we take into account only the off-diagonal elements important for $\mathrm{Tr}(\gamma^0 S^B)$. By keeping only the terms at the most linear in ϵ we can write

$$\overline{S}^B(x+\tfrac{1}{2}\epsilon, x-\tfrac{1}{2}\epsilon) = -\frac{1}{2\pi\epsilon}(1+i\epsilon\,\partial_1\phi_2)(1+i\gamma_5\epsilon\,\partial_1\phi_1)\gamma^1. \tag{B4}$$

If we take the limit $\epsilon \to \pm 0$ symmetrically the singular term will go away and we are left with

$$\overline{S}^B(x,x) = -\frac{i}{2\pi}(\partial_1\phi_2+\gamma_5\partial_1\phi_1)\gamma^1. \tag{B5}$$

Finally we get

$$\mathrm{Tr}[\gamma^0 S^B(\underline{x},\underline{x})] = \frac{i}{\pi}\partial_1\phi_1(\underline{x}). \tag{B6}$$

This is the same result as in the Schwinger model, except that $\phi_1(\underline{x})$ satisfies the boundary conditions [see (4.15a)].

APPENDIX C

Here we shall derive a multiple-reflection expansion of the propagator for a massive scalar field obeying a Dirichlet boundary condition (5.21) in terms of Feynman propagators.[9] Using the representation

$$\frac{i}{\omega^2-\omega_n{}^2+i\epsilon} = \int_0^\infty d\alpha\, e^{i\alpha(\omega^2-\omega_n{}^2+i\epsilon)}, \tag{C1}$$

we find for $\Delta_D(\underline{x},\underline{y})$ [see (5.21)]

$$\Delta_D(\underline{x},\underline{y}) = -\frac{i}{8\pi l}\int_{-\infty}^{\infty} d\omega \int_0^\infty d\alpha\, e^{i\alpha(\omega^2-g^2/\pi+i\epsilon)}e^{-i\omega(x_0-y_0)}\sum_{n=-\infty}^{\infty}[e^{ik_n(x-y)}e^{-i\alpha k_n{}^2}-(-1)^n e^{ik_n(x+y)}e^{-i\alpha k_n{}^2}]. \tag{C2}$$

By rotating the contour by $\pi/4$ we have the formula

$$\sqrt{\pi\alpha}\, e^{i\pi/4}e^{-i\alpha k_n{}^2} = \int_{-\infty}^{\infty} ds\, e^{(i/\alpha)s^2}e^{2isk_n}. \tag{C3}$$

We sum the exponentials over n and perform the ω integration by rotating the contour by $\pi/4$. The result for (C2) is

$$\Delta_D(\underline{x},\underline{y}) = -\frac{i}{8\pi l}\int_0^\infty \frac{d\alpha}{\alpha}\, e^{-i\alpha g^2/\pi}\int_{-\infty}^{\infty} ds\, e^{(i/4\alpha)s^2}e^{-it^2/4\alpha}\sum_{n=-\infty}^{\infty}\left[\delta\left(\frac{x-y}{2l}+\frac{s}{2l}-2m\right)-\delta\left(\frac{x+y}{2l}+\frac{s}{2l}-2m+1\right)\right]. \tag{C4}$$

For the Feynman propagator of a massive scalar field in two dimensions we have

$$\Delta_F(\underline{x}, \underline{y}) = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk \frac{e^{-ik(\underline{x}-\underline{y})}}{k^2 - m^2 + i\epsilon}. \tag{C5}$$

With the rotation $k_0 = -ik_0'$, the representation (C1), and after performing the integration $dk_0' dk_1$, we get

$$\Delta_F(\underline{x}, \underline{y}) = -\frac{i}{4\pi} \int_0^{\infty} \frac{d\alpha}{\alpha} e^{-i\alpha m^2} e^{-(i/4\alpha)(\underline{x}-\underline{y})^2}. \tag{C6}$$

If we compare the expression (C6) with (C4) we get the multiple-reflection expansion

$$\Delta_D\left(\underline{x}, \underline{y}; \frac{g^2}{\pi}\right) = \sum_{n=-\infty}^{\infty} \left[\Delta_F\left(Z_n^2; \frac{g^2}{\pi}\right) - \Delta_F\left(\overline{Z}_n^2; \frac{g^2}{\pi}\right)\right], \tag{C7}$$

where

$$Z_n^2 = (x_0 - y_0)^2 - (4nl - x + y)^2,$$

$$\overline{Z}_n^2 = (x_0 - y_0)^2 - [2(2n-1)l - x - y]^2.$$

The leading light-cone singularity inside the cavity is seen to be given by the $n = 0$ term in the expression. In addition, one can verify that when one approaches either end point of the cavity there is a cancellation between corresponding terms in the multiple-reflection series, consistent with the vanishing of $\Delta_D(\underline{x}, \underline{y}; g^2/\pi)$ on the surface.

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