Linearized Hartree formulation of the photon pairing problem*

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I describe and discuss a linearized, self-consistent extended Hartree formulation of the pairing problem for photons interacting, through electron vacuum polarization forces, in a general background metric.

Ladder sums for photon-photon scattering through electron vacuum polarization forces, in a general conformally flat metric, show weak-coupling singularities with intriguing structural resemblances to graviton-exchange amplitudes. This observation motivates the speculation that gravitation may be associated with a photon pairing phenomenon.[1] As noted in Ref. 1, the way to follow up on this speculation is presumably to avoid summation approximations which single out particular classes of diagrams, but rather to reanalyze the photon effective Lagrangian using the extended Hartree-Fock approximation. In this note[2] I give such a formulation (but not a solution) of the photon pairing problem in a general background metric, discuss certain of its features, and briefly comment on its relation to other ideas (notably Sakharov's) on the microscopic origin of gravitation.

The starting point for the discussion is the effective Lagrangian for a self-interacting electromagnetic field in a general background metric $g_{\mu\nu}$,

$$\mathcal{L} = (-g)^{1/2}[-\mathfrak{F} + \tfrac{1}{2}\kappa(4\mathfrak{F}^2 + 7\mathfrak{G}^2)], \tag{1}$$

$$g = \det(g_{\mu\nu}), \quad \kappa = \frac{4\alpha^2}{45m^4},$$

$$\mathfrak{F} = \tfrac{1}{4}\, g^{\mu\lambda} g^{\nu\sigma} F_{\mu\nu} F_{\lambda\sigma}, \quad \mathfrak{G} = \tfrac{1}{8}\epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma},$$

with α the fine-structure constant and m the electron mass. The interaction term in Eq. (1) provides a correct description of electron vacuum polarization induced photon-photon scattering for photon wavelengths appreciably larger than the electron Compton wavelength m^{-1}; for photon wavelengths much smaller than m^{-1}, the vacuum polarization interaction vanishes rapidly. In formulating a linearized photon pairing problem, I adopt a strictly local point of view. Taking an arbitrary point in spacetime as the coordinate origin, one can choose a freely falling inertial frame in which local gravitational fields vanish and where, in Riemann normal coordinates, the metric takes the form[3]

$$g_{\mu\nu} = \eta_{\mu\nu} - \tfrac{1}{3} R_{\mu\alpha\nu\beta} x^\alpha x^\beta - \frac{1}{3!} R_{\mu\alpha\nu\beta;\gamma} x^\alpha x^\beta x^\gamma + \frac{1}{5!}(-6R_{\mu\alpha\nu\beta;\gamma\delta} + \tfrac{16}{3} R_{\alpha\mu\beta}{}^{\rho} R_{\gamma\nu\delta\rho}) x^\alpha x^\beta x^\gamma x^\delta + \cdots, \tag{2a}$$

with the curvature tensor and its covariant derivatives evaluated at the origin. Since the Einstein equations do not involve the covariant derivatives of $R_{\mu\alpha\nu\beta}$, in seeking a connection between photon pairing and gravitation it is natural to assume that the third-and higher-order terms in Eq. (2a) can be neglected relative to the first two terms, and thus to take for the metric the simplified form

$$g_{\mu\nu} = \eta_{\mu\nu} - \tfrac{1}{3} R_{\mu\alpha\nu\beta} x^\alpha x^\beta, \tag{2b}$$

with the 20 curvature components $R_{\mu\alpha\nu\beta}$ given constants. The extended Hartree approximation is introduced by defining a "pairing amplitude" $P_{\mu\nu\lambda\sigma}$,

$$P_{\mu\nu\lambda\sigma} = \text{symmetrical} \lim_{x\to x'} \langle \tfrac{1}{2}\{F_{\mu\nu}(x), F_{\lambda\sigma}(x')\}\rangle, \tag{3}$$

with $\{\ ,\}$ and anticommutator and $\langle\ \rangle$ a vacuum expectation, and then linearizing the Lagrangian with respect to $P_{\mu\nu\lambda\sigma}$. Before proceeding with the linearization, I note that as defined in Eq. (3), $P_{\mu\nu\lambda\sigma}$ has 21 independent components; to reduce the number to 20 (the number of independent components of $R_{\mu\alpha\nu\beta}$), I assume the constraint

$$\epsilon^{\mu\nu\lambda\sigma} P_{\mu\nu\lambda\sigma} = 0, \tag{4a}$$

or equivalently, the vanishing of the vacuum expectation of the pseudoscalar invariant $\mathfrak{G}$,

$$\langle \mathfrak{G} \rangle = 0. \tag{4b}$$

In other words, the vacuum in the pairing problem is assumed to respect parity invariance.[4] Equation (4a), together with the symmetries

$$P_{\mu\nu\lambda\sigma} = P_{\lambda\sigma\mu\nu} = -P_{\nu\mu\lambda\sigma} = -P_{\mu\nu\sigma\lambda} \tag{5}$$

which are implicit in the definition of Eq. (3), implies that

$$P_{\mu\nu\lambda\sigma}+P_{\mu\sigma\nu\lambda}+P_{\mu\lambda\sigma\nu}=0\,, \tag{6}$$

so that $P_{\mu\nu\lambda\sigma}$ has all of the symmetries of the Riemann curvature tensor. Proceeding now to linearize the Lagrangian with respect to $P_{\mu\nu\lambda\sigma}$, using the symmetries of Eqs. (5) and (6) to simplify the results, gives

$$\begin{aligned}\mathcal{L}_{\text{lin}}&=(-g)^{1/2}(-\tfrac{1}{4}F_{\lambda\sigma}F^{\lambda\sigma}+\kappa F^{\mu\nu}F^{\lambda\sigma}N_{\mu\nu\lambda\sigma}\\&\qquad-\tfrac{1}{2}\kappa P^{\mu\nu\lambda\sigma}N_{\mu\nu\lambda\sigma})\\&=\langle\mathcal{L}_{\text{lin}}\rangle+{:}\mathcal{L}_{\text{lin}}{:}\,,\end{aligned} \tag{7}$$

$$\begin{aligned}{:}\mathcal{L}_{\text{lin}}{:}&=\mathcal{L}_{\text{lin}}-\langle\mathcal{L}_{\text{lin}}\rangle\,,\\ \langle\mathcal{L}_{\text{lin}}\rangle&=(-g)^{1/2}(-\tfrac{1}{4}P+\tfrac{1}{2}\kappa P^{\mu\nu\lambda\sigma}N_{\mu\nu\lambda\sigma})\,,\end{aligned}$$

with

$$\begin{aligned}N_{\mu\nu\lambda\sigma}&=-\tfrac{3}{8}P_{\mu\nu\lambda\sigma}+\tfrac{7}{8}(P_{\mu\lambda}g_{\nu\sigma}+P_{\nu\sigma}g_{\mu\lambda}\\&\qquad-P_{\nu\lambda}g_{\mu\sigma}-P_{\mu\sigma}g_{\nu\lambda})\\&\quad-\tfrac{5}{16}P(g_{\mu\lambda}g_{\nu\sigma}-g_{\nu\lambda}g_{\mu\sigma})\,,\end{aligned} \tag{8}$$

$$P_{\mu\lambda}=g^{\nu\sigma}P_{\mu\nu\lambda\sigma}\,,\quad P=g^{\mu\lambda}P_{\mu\lambda}\,.$$

Equations (2b), (3), (4a), (7), and (8) are the basic relations defining the linearized, self-consistent photon pairing problem.

In doing calculations with this system of equations, it is helpful to reexpress the problem in terms of Green's functions. Introducing the electromagnetic potential A by writing

$$F_{\mu\nu}=D_\mu A_\nu-D_\nu A_\mu\,, \tag{9}$$

with D the covariant derivative, and rearranging the kinetic term of Eq. (7) in the standard way[5] gives

$$\begin{aligned}&\int d^4x(-g)^{1/2}(-\tfrac{1}{4})F_{\lambda\sigma}F^{\lambda\sigma}\\&\quad=\int d^4x(-g)^{1/2}[-\tfrac{1}{2}(D_\mu A_\nu)^2+\tfrac{1}{2}(D^\mu A_\mu)^2-\tfrac{1}{2}A_\mu R^{\mu\nu}A_\nu]\,.\end{aligned} \tag{10}$$

In order to work in the covariant Lorentz gauge

$$D^\mu A_\mu=0\,, \tag{11}$$

one adds to Eq. (10) the gauge-breaking and scalar-ghost terms[5]

$$\int d^4x(-g)^{1/2}[-\tfrac{1}{2}(D^\mu A_\mu)^2-\bar{c}D_\mu D^\mu c]\,, \tag{12}$$

with c the complex massless scalar ghost field. Since c couples only to the background metric, but not to the electromagnetic field, it appears only in vacuum diagram calculations, and plays no role in determining the connected photon Green's functions which are of interest in the pairing problem. Combining Eqs. (7), (10), and (12), and dropping the ghost terms, one finds[6]

$$\begin{aligned}&\int d^4x\mathcal{L}_{\text{lin}}=\int d^4x\langle\mathcal{L}_{\text{lin}}\rangle+\int d^4x{:}\mathcal{L}_{\text{lin}}{:}\,,\\&\int d^4x{:}\mathcal{L}_{\text{lin}}{:}=\int d^4x\tfrac{1}{2}{:}A_\alpha\mathfrak{F}^{\alpha\beta}A_\beta{:}\,,\\&\mathfrak{F}^{\alpha\beta}=(-g)^{1/2}(D_\mu D^\mu g^{\alpha\beta}-R^{\alpha\beta}-8\kappa D_\mu N^{\mu\alpha\lambda\beta}D_\lambda),\\&\int d^4x\langle\mathcal{L}_{\text{lin}}\rangle=\int d^4x\tfrac{1}{2}\langle A_\alpha\mathfrak{F}^{\alpha\beta}A_\beta\rangle\\&\qquad-\int d^4x(-g)^{1/2}\tfrac{1}{2}\kappa P^{\mu\nu\lambda\sigma}N_{\mu\nu\lambda\sigma}\\&\qquad=\int d^4x\tfrac{1}{4}\langle A_\alpha\mathfrak{F}^{\alpha\beta}A_\beta\rangle-\int d^4x(-g)^{1/2}\tfrac{1}{8}P\,.\end{aligned} \tag{13}$$

Introducing the Feynman Green's function G, the Hadamard[7] "elementary solution" $G^{(1)}$, and the commutator function $\bar{G}$ by writing

$$\begin{aligned}G_{\gamma\delta'}(x,x')&=i\langle T(A_\gamma(x)A_{\delta'}(x'))\rangle/\langle 1\rangle\\&=\tfrac{1}{2}iG^{(1)}_{\gamma\delta'}(x,x')+\bar{G}_{\gamma\delta'}(x,x')\,,\\G^{(1)}_{\gamma\delta'}(x,x')&=\langle\{A_\gamma(x),A_{\delta'}(x')\}\rangle/\langle 1\rangle\,,\\\bar{G}_{\gamma\delta'}(x,x')&=\tfrac{1}{2}i\epsilon(x^0-x'^0)\langle[A_\gamma(x),A_{\delta'}(x')]\rangle/\langle 1\rangle\,,\end{aligned} \tag{14}$$

the equations of motion and canonical commutation relations obtained from the action of Eq. (13) may be combined to give the equation of motion for the Feynman Green's function

$$\mathfrak{F}^{\alpha\beta}G_{\beta\delta'}(x,x')=-\delta^\alpha_{\delta'}\,\delta^4(x-x')\,. \tag{15}$$

The pairing amplitude of Eq. (3) can then be expressed in terms of the coincidence limit of the Feynman or Hadamard Green's functions,

$$\begin{aligned}P_{\mu\nu\lambda\sigma}&=-i\ \underset{x\to x'}{\text{symmetrical limit}}[D_\mu D_{\lambda'}G_{\nu\sigma'}(x,x')+D_\nu D_{\sigma'}G_{\mu\lambda'}(x,x')-D_\nu D_{\lambda'}G_{\mu\sigma'}(x,x')-D_\mu D_{\sigma'}G_{\nu\lambda'}(x,x')]\\&=\tfrac{1}{2}\ \underset{x\to x'}{\text{symmetrical limit}}[D_\mu D_{\lambda'}G^{(1)}_{\nu\sigma'}(x,x')+D_\nu D_{\sigma'}G^{(1)}_{\mu\lambda'}(x,x')-D_\nu D_{\lambda'}G^{(1)}_{\mu\sigma'}(x,x')-D_\mu D_{\sigma'}G^{(1)}_{\nu\lambda'}(x,x')]\,.\end{aligned} \tag{16}$$

Equations (8), (13), (15), and (16) are a Green's function formulation of the photon pairing problem.[8]

I next briefly discuss what would be the ultimate aim in solving the pairing problem just formulated. The basic observation is that $\langle\mathcal{L}_{\text{lin}}\rangle$, if nonvanishing, is necessarily a scalar density function of $R_{\mu\nu\lambda\sigma}$. Hence if $\langle\mathcal{L}_{\text{lin}}\rangle$ should have the form

$$\langle \mathcal{L}_{\text{lin}} \rangle = \text{constant} \times (-g)^{1/2} R + \cdots, \quad R = R_{\mu\nu}{}^{\mu\nu}, \tag{17}$$

with the dots denoting much smaller correction terms involving more complicated scalars formed from $R_{\mu\nu\lambda\sigma}$, then $\langle \mathcal{L}_{\text{lin}} \rangle$ would behave as a gravitational kinetic Lagrangian. Since the photon field term $:\mathcal{L}_{\text{lin}}:$ involves the metric in just the usual way (and similarly for other matter field terms, such as the electron Lagrangian, which have not been indicated explicitly), varying with respect to the metric $g_{\mu\nu}$ would then give the usual Einstein theory as the classical equations of motion for the metric.

Evidently, the extended Hartree approximation equations developed above have an appearance very different from the ladder graph weak-coupling singularity arguments made in Ref. 1. A minimal test (necessary, but of course not sufficient) of whether the two approaches have the same physical content is to check that in the limits $\kappa = 0$ (no pairing interaction) or $R_{\mu\nu\lambda\sigma} = 0$ (spacetime flat through quadratic terms in the expansion of the metric about the coordinate origin) in which no effect is found in Ref. 1, the linearized pairing problem also gives no interesting effects. Consider first the case $\kappa = 0$, with the background metric still that for curved space-time. Since $\langle A_\alpha \mathfrak{F}^{\alpha\beta} A_\beta \rangle$ vanishes by the equations of motion, Eq. (13) implies that $\int d^4x \langle \mathcal{L}_{\text{lin}} \rangle = 0$ when $\kappa = 0$, or in other words, $\langle \mathcal{L}_{\text{lin}} \rangle$ in this case is a total derivative. Actually, as I shall now show, when $\kappa = 0$ the stronger statement $\langle \mathcal{L}_{\text{lin}} \rangle = 0$ also holds. To see this, note that when $\kappa = 0$ the electromagnetic Lagrangian is just the free Maxwell Lagrangian, which even in curved spacetime gives equations of motion invariant under the duality transformation[9]

$$F^{\mu\nu} \to {}^*F^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$$
$$\Rightarrow \left.\begin{matrix} \vec{E} \to -\vec{B} \\ \vec{B} \to \vec{E} \end{matrix}\right\} \text{ in inertial frames}, \tag{18a}$$

and which also implies the equal-time commutation relations

$$\left.\begin{matrix} [B^i(\vec{x},t), E^j(\vec{y},t)] = i\epsilon^{ijl} \dfrac{\partial}{\partial x^l}\delta^3(\vec{x}-\vec{y}) \\ [B^i(\vec{x},t), B^j(\vec{y},t)] = [E^i(\vec{x},t), E^j(\vec{y},t)] = 0 \end{matrix}\right\} \text{ in inertial frames}, \tag{18b}$$

which are again invariant under the duality transformation of Eq. (18a). Duality invariance implies that

$$\langle \{F_{\mu\nu}, F_{\lambda\sigma}\} \rangle = \langle \{ {}^*F_{\mu\nu}, {}^*F_{\lambda\sigma} \} \rangle, \tag{19}$$

which means that $P_{\mu\nu\lambda\sigma}$ is invariant under a double duality transformation,

$$P_{\mu\nu\lambda\sigma} = {}^*P_{\mu\nu\lambda\sigma}{}^* = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta} P^{\alpha\beta\gamma\delta} \tfrac{1}{2}\epsilon_{\lambda\sigma\gamma\delta}. \tag{20}$$

It is easy to show that Eq. (20) implies that $P_{\mu\nu\lambda\sigma}$ must have the form

$$P_{\mu\nu\lambda\sigma} = 2(M_{\mu\lambda} g_{\nu\sigma} + M_{\nu\sigma} g_{\mu\lambda} - M_{\nu\lambda} g_{\mu\sigma} - M_{\mu\sigma} g_{\nu\lambda}) - M_\rho{}^\rho (g_{\mu\lambda} g_{\nu\sigma} - g_{\nu\lambda} g_{\mu\sigma}) \tag{21}$$

with $M_{\mu\lambda}$ a symmetric tensor, from which is immediately deduced

$$\langle \mathcal{L}_{\text{lin}} \rangle = -\tfrac{1}{4}(-g)^{1/2} P = 0. \tag{22}$$

Of course, Eq. (22) can also be obtained directly by noting that $\mathcal{L}|_{\kappa=0} = -(-g)^{1/2}\mathfrak{F}$ is odd under the transformation of Eq. (18a). Using the DeWitt-Brehme[10] expressions for the Hadamard elementary function $G^{(1)}$ in an arbitrary background metric, together with a symmetric-averaging version of a covariant point-separation method due to Christensen,[11] I have verified Eq. (21) through quadratically divergent terms (that is, through terms linear in $R_{\mu\nu\lambda\sigma}$) and Eq. (22) through the logarithmically divergent term as well [where for the metric of Eq. (2a) it implies that the contributions proportional to $R_{;\rho}{}^\rho$ cancel against one another].[12] Note that when $\kappa \neq 0$ the duality argument no longer implies the vanishing of $\langle \mathcal{L}_{\text{lin}} \rangle$, since the starting Lagrangian of Eq. (1) is no longer simply transformed into minus itself by the duality transformation. Physically, the reason for this is clear: Since the electron has electric charge e but zero magnetic charge, the electromagnetic vacuum is not duality invariant.

Turning now to the other special limit, in which $\kappa \neq 0$ but $R_{\mu\nu\lambda\sigma} = 0$, I only give an incomplete argument which assumes, without proof, that the pairing amplitudes arise strictly as a response to the metric. Since when $R_{\mu\nu\lambda\sigma} = 0$ the metric of Eq. (2b) involves no externally specified tensors, under this assumption $P_{\mu\nu\lambda\sigma}$ must have the form

$$P_{\mu\nu\lambda\sigma}=C(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda}),$$
$$P_{\mu\lambda}=3Cg_{\mu\lambda}, \quad P=12C, \tag{23}$$

which on substitution into Eq. (7) gives

$$:\mathfrak{L}_{\text{lin}}:=(-g)^{1/2}(-\tfrac{1}{4}):F_{\lambda\sigma}F^{\lambda\sigma}:(1-9\kappa C),$$
$$\langle\mathfrak{L}_{\text{lin}}\rangle=-3C(1-\tfrac{9}{2}\kappa C). \tag{24}$$

Insofar as $:\mathfrak{L}_{\text{lin}}:$ is concerned, this just represents a rescaling of the $\kappa=0$ flat-space kinetic Lagrangian, and so the arguments of the preceding paragraph imply *a fortiori*

$$\langle\mathfrak{L}_{\text{lin}}\rangle=C=0. \tag{25}$$

The fact that the special cases $\kappa=0$ or $R_{\mu\nu\lambda\sigma}=0$ both give $\langle\mathfrak{L}_{\text{lin}}\rangle=0$, but that $\langle\mathfrak{L}_{\text{lin}}\rangle$ does not appear to vanish when both κ and $R_{\mu\nu\lambda\sigma}$ are nonzero, is encouraging for two reasons. First, as already noted above, it at least makes it plausible that the pairing model formulated in this paper incorporates the essential physics of the ladder graph summation calculation of Ref. 1. Second, the vanishing of $\langle\mathfrak{L}_{\text{lin}}\rangle$ in particular for $\kappa=0$ suggests that it may indeed make sense as a first approximation to treat the photon-photon interaction in effective Lagrangian approximation, eliminating the very-short-wavelength photon contributions, for which the effective strength κ_{eff} of the photon-photon interaction vanishes rapidly, by assuming a short wavelength or minimum distance cutoff of order m^{-1}.

Finally, I turn briefly to the question of how the magnitude of the Cavendish gravitational constant $G\sim10^{-45}/m^2$ might be fixed in the pairing model, focusing for definiteness on three suggestions which have been made in the literature for how such a small number might enter gravitational physics. One suggestion, made by Dirac,[13] is that the gravitational constant G is related to the Hubble constant H_0 which describes the general cosmological expansion of the universe, through a relation of the form

$$Gm^2=(\text{numerical constant})\times\frac{H_0}{m}. \tag{26}$$

Although this "law of large numbers" is appealing, there is no way of realizing Eq. (26) within the framework of the local pairing model formulated above,[14] in which the local curvature $R_{\mu\alpha\nu\beta}$ is essentially entirely determined by the local matter distribution.[15] A second suggestion, made by many authors, is that G is related to a maximum elementary particle mass $m_{\text{Planck}}\sim10^{22}m$ or a minimum distance $l_{\text{Planck}}\sim(m_{\text{Planck}})^{-1}$ by

$$G\sim(m_{\text{Planck}})^{-2}\sim(l_{\text{Planck}})^2. \tag{27}$$

A particularly interesting version of Eq. (27) is Sakharov's[16] suggestion that the vacuum Lagrangian for a quantum field in curved spacetime takes the form

$$\mathfrak{L}_{\text{vac}}(R_{\mu\nu\lambda\sigma})=(-g)^{1/2}[A(m_{\text{Planck}})^4+B(m_{\text{Planck}})^2R+\cdots], \tag{28}$$

with A and B numerical constants, and with the second term acting as the gravitational kinetic Lagrangian. In attempting to do specific calculations based on this idea, one must drop the strictly local point of view, since the vacuum loops in quantum field theory uniquely determine only the integrated vacuum action $\int d^4x\mathfrak{L}_{\text{vac}}=W_{\text{vac}}$, rather than a local Lagrangian density $\mathfrak{L}_{\text{vac}}$. This of course poses no problem in setting up a variational action principle to get the gravitational field equations. As DeWitt[17] and Christensen[18] have shown, the vacuum action functional W_{vac} can be evaluated using Schwinger's proper-time technique,[19] but here a difficulty arises: For a free electromagnetic field, which is conformally invariant, the vacuum action is found to have the form

$$W_{\text{vac}}=\int d^4x(-g)^{1/2}\left(A\int_0\frac{dt}{t^3}+B\int_0\frac{dt}{t^2}R+\cdots\right), \tag{29}$$

with t the proper-time parameter and with both A and B nonzero, whereas manifest conformal invariance would require $A=B=0$. Equation (29) does not actually contradict conformal invariance, since the divergent integrals become well defined only when truncated at a minimum proper time $t_0>0$, a process which introduces a distance scale and thereby breaks the conformal invariance of the theory. However, taking $t_0\sim l_{\text{Planck}}$ is not really a satisfactory way to realize Sakharov's idea, since it violates one's intuitive feeling that a parameter of minimum distance or maximum mass should enter into the physics by appearing in the Lagrangian for the underlying fields (i.e., in the case under discussion, in the electromagnetic Lagrangian), rather than appearing as an essentially *ad hoc* cutoff parameter.

Evidently, the pairing model introduced above is similar in spirit to Sakharov's idea, but of course quite different in detail. Attention is shifted from the vacuum sector of the field theory to the one-particle connected Green's functions, and a nonvanishing gravitational kinetic Lagrangian is obtained only when conformal invariance of the electromagnetic Lagrangian is explicitly broken by inclusion of the photon-photon scattering term. However, since the natural mass scale which appears is the electron mass m, there is no way of obtaining G in the pairing model through the mechanism of Eq. (27). This brings the discussion to a third suggestion that has frequently been made[20] for explaining the magnitude of G, the proposal that G, m, and the fine-structure constant α are related through a formula

$$Gm^2 = ae^{-b/\alpha}, \tag{30}$$

with a and b numerical constants. Since the pairing model involves α and m through κ, and m independently through the short-wavelength cutoff of the photon-photon scattering term, Eq. (30) could in principle be realized in the form

$$Gm^2 = a'e^{-b'/(m^2\kappa^{1/2})}. \tag{31}$$

Essentially singular dependence on the coupling constant is, of course, a well-known feature of superconductive models, and a square-root dependence on κ would be consistent with the fact that the weak-coupling singularities found in Ref. 1 shift from the imaginary axis to the real axis when the sign of κ is reversed. Whether anything like Eqs. (17) and (31) actually emerges will, of course, be known only after the dynamics of the pairing model which I have introduced above is analyzed in the nontrivial case with both κ and $R_{\mu\nu\lambda\sigma}$ nonzero.

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[1]S. L. Adler, J. Lieberman, Y. J. Ng, and H.-S. Tsao, preceding paper, Phys. Rev. D 14, 359 (1976).

[2]I follow in this paper the notational conventions of C. W. Misner, K. S. Thorne, and J. A. Wheeler [*Gravitation* (W. H. Freeman, San Francisco, 1973)] except that I use atomic units ($\hbar = c = 1$) rather than geometrized units.

[3]A. Z. Petrov, *Einstein Spaces* (Pergamon, New York, 1969), p. 36.

[4]Since $\epsilon^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma}=0$, no pseudoscalar quantity can be formed from the metric of Eq. (2b), and so $\langle \mathcal{G} \rangle$ can be nonvanishing only if there is spontaneous parity violation.

[5]See, for example, S. Deser and P. van Nieuwenhuizen, Phys. Rev. D 10, 401 (1974).

[6]Consistent with the neglect of covariant derivatives of $R_{\mu\nu\lambda\sigma}$ in Eq. (2b), one can write $D_\mu N^{\mu\alpha\lambda\beta}D_\lambda = N^{\mu\alpha\lambda\beta}{}_{;\mu}D_\lambda + N^{\mu\alpha\lambda\beta}D_\mu D_\lambda \approx N^{\mu\alpha\lambda\beta}D_\mu D_\lambda$.

[7]J. Hadamard, *Lectures on Cauchy's Problem in Linear Partial Differential Equations* (Yale Univ. Press, New Haven, 1923).

[8]Equations (8), (13), (15), and (16) do not completely determine the Green's functions, since a boundary condition is needed to ensure that the Green's functions for the coordinate patch being examined join on smoothly to the Green's functions for neighboring coordinate patches. I do not know whether or not this boundary condition plays a role in the photon pairing problem.

[9]For a useful discussion of duality and double duality transformations, see Misner, Thorne, and Wheeler, Ref. 2.

[10]B. S. DeWitt and R. W. Brehme, Ann. Phys. (N.Y.) 9, 220 (1960).

[11]S. M. Christensen, Phys. Rev. D (to be published).

[12]Details of this calculation will be described elsewhere.

[13]P. A. M. Dirac, Nature 139, 323 (1937); Proc. R. Soc. London A165, 199 (1938). For a recent review, see F. J. Dyson, in *Aspects of Quantum Theory*, edited by A. Salam and E. P. Wigner (Cambridge Univ. Press, Cambridge, England, 1972), p. 213.

[14]I retract a speculation to the contrary made in the preliminary version of Ref. 1.

[15]To the extent that distant matter is distributed in shells spherically symmetric about the chosen point of observation, it makes no contribution to the local spacetime curvature. I wish to thank C. G. Callan and F. J. Dyson for a discussion of this point.

[16]A. D. Sakharov, Dokl. Akad. Nauk SSSR 177, 70 (1967) [Sov. Phys. Dokl. 12, 1040 (1968)].

[17]B. S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965); Phys. Rep. 19C, 295 (1975).

[18]S. M. Christensen, Ph.D. thesis, University of Texas at Austin, 1975 (unpublished).

[19]J. Schwinger, Phys. Rev. 82, 664 (1951).

[20]The relation of Eq. (30) appears, for example, when one equates the electron mass to the electron second-order electromagnetic self-energy, calculated with the Planck mass as a momentum cutoff. See I. B. Khriplovich, Yad. Fiz. 3, 575 (1966) [Sov. J. Nucl. Phys. 3, 415 (1966)].