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Lifetime and decay of "excited vacuum" states of a field theory associated with nonabsolute minima of its effective potential

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We discuss the decay of the excited vacuum states of a field theory associated with the nonabsolute minima of its effective potential. Using the equivalence of the sine-Gordon theory in (1+1) dimensions to Thirring fermion theory we demonstrate explicitly the nature of one such decay and compute its lifetime. The fact that ordinary perturbation theory cannot be used to describe the decay is also examined.

I. INTRODUCTION

In order to study the ground states of a quantum field theory it is usual to consider the effective potential[1,2] of the theory, whose minima are candidates for such states.

In perturbation theory it often seems possible to have more than one minimum and these putative vacua have either the same energy or an infinite energy gap. In the later case the vacuum state should be unstable, as has been argued on general grounds by Lee and Wick.[3] One does not see this instability in perturbation theory, however.[4] This paper is devoted to a discussion of this point and to a calculation of the decay rate in a two-dimensional model where, for one value of the coupling constant, one can get an exact result.

The first section is a general discussion of vacuum instability. The second reviews the main ideas of the (1+1)-dimensional equivalence theorems and the third and fourth sections discuss the model and its generalizations.

II. GENERAL DISCUSSION

When the effective potential $V(\varphi)$ of a quantum field theory has more than one minimum it is normally supposed that only states built on the absolute minima are stable. "Vacua" built on other minima are energetically capable of decay by some kind of tunneling mechanism and in the absence of conservation laws should actually do so. Superficially one might object that the full quantum effective potential contains in itself all the quantum effects and that if a minimum in the bare potential leads to an unstable state then radiative corrections should fill it in. This is not necessarily true, however, because $V(\varphi)$ has a physical interpretation[2]: $-E(J)=V(\varphi)+J\varphi=$ ground-state energy density for system with

$$\text{Hamiltonian}=\underline{H}+J\underline{\varphi}\,, \tag{2.1}$$

where $\varphi=\langle 0|\underline{\varphi}|0\rangle_J$ in this ground state. If we vary J so that $|0\rangle_J$ is no longer the lowest-energy state it may still be possible to analytically continue $E(J)$, and, further, E may develop an imaginary part which will give the decay rate of this state. This continuation process is exactly analogous to the way that one may continue the Gibbs function for a thermodynamic system past a phase transition into a superheated or supercooled phase.

In rather more detail let us use

$$\langle 0_{\text{out}}|0_{\text{in}}\rangle=e^{iW(J)}\,, \tag{2.2}$$

where

$$W(J)=\int dx\,E(J)=E(J)(VT) \tag{2.3}$$

and VT is the volume of space-time. The insertion of a complete set of out states $|0_{\text{out}}\rangle$, $|X\rangle$ into $\langle 0_{\text{in}}|0_{\text{in}}\rangle=1$ yields

$$1-\exp[-2\,\text{Im}E(J)(VT)]=\sum_X|\langle X|0_{\text{in}}\rangle|^2\,, \tag{2.4}$$

so that $\exp[-2\,\text{Im}E(J)(VT)]$ is the probability of no decay in volume VT. If we divide space into a number of independent regions, then this is the product of the probability of no decay in each of the regions separately. If these regions are so small that much less than one event is expected, then

$$e^{-2\text{Im}E(J)\times\text{Volume}}\cong 1-2\,\text{Im}[E(J)\times\text{Volume}]\,, \tag{2.5}$$

and $2\,\text{Im}E(J)$ is the probability per unit volume per unit time of an event. This assumes that the volumes are independent, and they must therefore be larger than the scale of quantum fluctuations. This is related to the interpretation of $W(J)$ as the generating functional for *connected* Green's functions. We need to take our volumes such that $\mathcal{G}_{\text{con}}(x_1\in V_1,\ x_2\in V_2,\ldots)$ is negligible. This is consistent with the idea that the imaginary part of a connected diagram with one point fixed describes the creation of particles within the correlation length of that point,[5] where this length is the reciprocal of a typical mass in the propagator.

In perturbation theory $V(\varphi)$, and hence $E(J)$, is calculated by the following prescription.[6]

Write the Lagrangian $\mathcal{L}(\varphi)$ as $\mathcal{L}(\varphi'+\varphi)$. Use φ' as the quantum field to specify propagators and vertices and sum all the one-particle-irreducible (1PI) vacuum-vacuum graphs to obtain a function of φ.

Unfortunately, when one uses the prescription to low orders in perturbation theory the vacuum states seem to remain stable for much longer than one would expect. Consider for example $\lambda\varphi^4$ theory with negative m^2:

$$V(\varphi)=\frac{\lambda\varphi^4}{24}-\frac{m^2\varphi^2}{2}+\frac{\hbar}{64\pi^2}\left(\frac{\lambda\varphi^2}{2}-m^2\right)^2\ln\left[\frac{\lambda\varphi^2/2-(m^2+i\epsilon)}{\mu^2}\right]$$
$$+\,(\text{finite polynomials in } \varphi \text{ depending on renormalization scheme})+O(\hbar^2). \tag{2.6}$$

The $-i\epsilon$ ensure that $\operatorname{Im}V$ is negative. The branch cut appears when

$$\tfrac{1}{2}\lambda\varphi^2-m^2+O(\hbar)=0\,,$$

i.e., at

$$\partial^2V/\partial\varphi^2=0+O(\hbar)\,. \tag{2.7}$$

To this order the state becomes unstable only at the point of inflection on the graph of V. Since the addition of a linear term $J\varphi$ to V cannot alter the curvature, the minimum of $V+J\varphi$ and the point of inflection coincide when the second minimum of $V+J\varphi$ vanishes (Fig. 1).

To this order the vacuum remains stable until the state is *classically* unstable—*no tunneling*.

It is not easy to see the persistence of this phenomena, in formal perturbation theory, to higher orders because the point of inflection shifts. To find the new position of the branch cut even to next order involves summing all the terms due to mass insertions. A self-consistent Hartree-Fock scheme in which graphs are resummed to 2PI graphs with complete propagators[6,7] will show it, but a much clearer argument comes from perturbative unitarity and energy conservation.

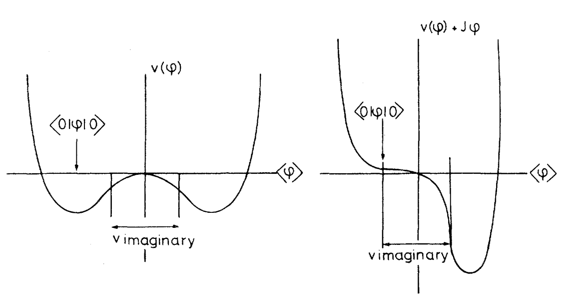

FIG. 1. $V(\varphi)+J\varphi$ for typical system showing regions of reality: (a) $J=0$; (b) J such that $V(\varphi)+J\varphi$ becomes complex in perturbation theory.

The usual unitarity rules[8] which are summarized in Fig. 2 give us an expression for the imaginary part of the connected vacuum-vacuum amplitude. In this figure the lines represent states on the mass shell and energy flows from + to −. To be nonzero the right-hand side needs an on-mass-shell particle with zero energy. The inverse propagator at zero momentum must vanish, and so

$$\partial^2V/\partial\varphi^2=0\,. \tag{2.8}$$

Thus in general one expects perturbation theory for V to indicate no tunneling. Nevertheless the states can decay.

Consider for simplicity a two-dimensional version of our $\lambda\varphi^4$ theory, that is, a "string" with more than one equilibrium point. Assume that a $J\varphi$ addition makes one minimum higher than the other. Vacuum fluctuations cannot make the whole string move as a block out of the upper trough because it has infinite mass, but small sections of it can tunnel to the lower region. If the length in the lower trough is large enough, the energy difference will be enough to "pay back" the energy borrowed in order to tunnel. The final state will be two solitons moving away from each other, the space between them being in the lower state (Fig. 3).

We will not see this event in perturbation theory unless we have a scheme embodying the off-mass-shell propagator for the solitons.

The (1 + 1)-dimensional Bose-Fermi correspondences do just this if we identify the sine-Gordon

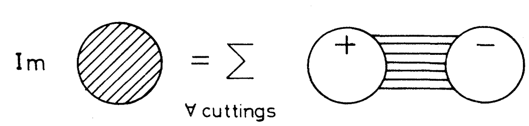

FIG. 2. Optical theorem for vacuum-vacuum unitarity.

(SG) soliton with the Thirring fermion.[9,10] We will find, just as we expect, that the Fermi translation of the SG equation shows excited vacua decaying to solitons. For one value of the SG coupling one can calculate the rate exactly.

III. EQUIVALENCE THEOREMS IN (1+1) DIMENSIONS

The work of a number of authors[9-11] has enabled us to build a dictionary between two-dimensional Bose and Fermi theories.

The basic relations are expressed by the following *free* theory equations:

$$i\bar\psi\partial\!\!\!/\psi \longleftrightarrow \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\,, \tag{3.1a}$$

$$\bar\psi\gamma^\mu\psi \longleftrightarrow \frac{1}{\sqrt{\pi}}\,\epsilon^{\mu\nu}\,\partial_\nu\varphi\,, \tag{3.1b}$$

$$\sigma_\pm = \tfrac{1}{2}\bar\psi(1\pm\gamma_5)\psi \longleftrightarrow \frac{C}{2\pi}mN_m[\exp(\pm 2i\sqrt{\pi}\varphi)]\,, \tag{3.1c}$$

where

$$N_m[\exp(\pm 2i\sqrt{\pi}\varphi)] = \exp[-2\pi i\Delta(0,m)]\exp(\pm 2i\sqrt{\pi}\varphi)\,.$$

The sense of correspondence is as follows. Suppose we calculate

$$\langle 0\,|\,T\sigma_-(x)\sigma_+(0)\,|\,0\rangle$$

with the propagator corresponding to $i\bar\psi\partial\!\!\!/\psi$. We obtain

$$\mathrm{Tr}\left\{\tfrac{1}{2}(1+\gamma_5)\frac{x\!\!\!/}{x^2}\tfrac{1}{2}(1-\gamma_5)\frac{x\!\!\!/}{x^2}\right\}\frac{1}{(2\pi)^2} = \frac{1}{(2\pi)^2}\frac{1}{x^2}\,, \tag{3.2}$$

while if

$$(2\pi)^2c^2m^2\langle 0\,|\,TN_m[\exp(-2i\sqrt{\pi}\varphi(x))] \times N_m[\exp(+2i\sqrt{\pi}\varphi(0))]\,|\,0\rangle \tag{3.3}$$

is also evaluated using the propagator obtained from the boson Lagrangian in (3.1a), we obtain the same result, where we have used

$$\Delta_F(x,y,\mu) = \frac{i}{4\pi}\ln[c^2\mu^2(x-y)^2]\,,\quad (x-y)^2\mu^2 \ll 1\,. \tag{3.4}$$

A similar result is obtained for any Green's function evaluated in a similar way–modulo infrared problems due to the strict nonexistence of massless scalar fields in two dimensions. These are discussed in Ref. 9.

We need the extension of the correspondences to the Heisenberg fields of interacting theories. That they remain true is seen by expanding any Green's function for the interacting fields by use of the Gell-Mann–Low formula and using (3.1) term by term on the resulting free-field expressions in the perturbation series.

For example, the Thirring interaction

$$i\bar\psi\partial\!\!\!/\psi - \tfrac{1}{2}g\,\bar\psi\gamma^\mu\psi\bar\psi\gamma_\mu\psi \tag{3.5}$$

corresponds to

$$\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{g}{2}\frac{1}{\pi}\epsilon^{\mu\nu}\partial_\nu\varphi\epsilon_{\mu\sigma}\partial^\sigma\varphi = (1+g/\pi)\partial_\mu\varphi\partial^\mu\varphi = \tfrac{1}{2}\partial_\mu\tilde\varphi\partial^\mu\tilde\varphi\,, \tag{3.6}$$

where

$$\tilde\varphi = (1+g/\pi)^{1/2}\varphi\,. \tag{3.7}$$

Thus the effect of the Thirring interaction is just a finite rescaling, (3.7), which identifies $\tilde\varphi$ as the canonical Bose field.

In terms of $\tilde\varphi$ the correspondences involve the pairs

$$\begin{aligned} j^\mu = \bar\psi\gamma^\mu\psi &\longleftrightarrow \frac{\beta}{2\pi}\epsilon^{\mu\nu}\partial_\nu\tilde\varphi\,,\\ \sigma_\pm &\longleftrightarrow \frac{cm}{2\pi}N_m[\exp(i\beta\tilde\varphi)]\,, \end{aligned} \tag{3.8}$$

where

$$\beta^2/4\pi = (1+g/\pi)^{-1}\,.$$

An even more interesting result[11] is the extension to include the (Schwinger) electromagnetic interaction

$$i\bar\psi\partial\!\!\!/\psi + e\bar\psi\gamma^\mu\psi A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3.9}$$

into the fermion Lagrangian. As far as fermion Green's functions are concerned an equivalent addition to (3.9) is the nonlocal term

$$\tfrac{1}{2}e^2\int d^2x\,d^2y\,j^\mu(x)G_{\mu\nu}(|x-y|)j^\nu(y)\,, \tag{3.10}$$

where $G_{\mu\nu}$ denotes the photon propagator in some gauge. To see this one may use a functional integral representation of the generating functional. The electromagnetic terms are quadratic in the A_μ field and may be integrated out to yield (2.10).

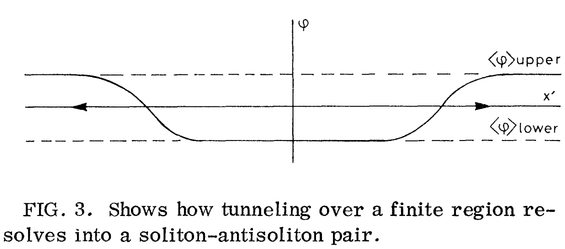

FIG. 3. Shows how tunneling over a finite region resolves into a soliton-antisoliton pair.

If we use the gauge in which $A_1=0$ so that

$$G_{\mu\nu}=\tfrac{1}{2}\delta_{\mu 0}\delta_{\nu 0}\delta(x^0)\,|x^1|\,, \tag{3.11}$$

then we can simplify the interaction for the boson theory equivalent to (3.10). Using (3.1) we get

$$\frac{e^2}{2\pi}\int dx^1dy^1dt\frac{\partial\varphi(x)}{\partial x^1}|x^1-y^1|\frac{\partial\varphi(y)}{\partial y^1} = \frac{e^2}{2\pi}\int d^2x[\varphi(x)-\varphi(\infty)]^2\,. \tag{3.12}$$

In (3.12) we have integrated by parts. The value $\varphi(\infty)$ of $\varphi(x)$ for $x\to\pm\infty$ is important as it allows the Bose theory to be chiral invariant despite the fact that it possesses what looks like a mass term.

IV. SINE-GORDON MODEL

This (1+1)-dimensional theory has the Lagrangian[9]

$$\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\frac{\alpha_0}{\beta^2}\cos\beta\varphi\,. \tag{4.1}$$

The bare potential is periodic with discretely degenerate minima, and since the theory is invariant under $\varphi\to\varphi+2\pi\beta^{-1}$ the discreteness of the vacuum should survive quantization.

The one-loop contribution to $V(\varphi)$ is

$$V(\varphi)=-\frac{\alpha}{\beta^2}\cos\beta\varphi\left[1+\frac{\beta^2\hbar}{8\pi}\ln\left(\frac{\alpha\cos\beta\varphi+i\epsilon}{\mu^2}\right)\right] + O(\hbar^2)\,, \tag{4.2}$$

where α is a renormalized coupling constant and μ a renormalization mass.

This low-order potential has the reality properties we expect: real for positive $\partial^2V/\partial\varphi^2$ and a negative imaginary part where $\alpha\cos\beta\varphi<0$. We will see in fact that $E(J)$, and hence V, is in fact imaginary everywhere, but that for small J the imaginary part is small, so the vacua are approximately stable.

Translating (4.1) plus a $J\varphi$ term into Fermi language:

$$\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\frac{\alpha}{\beta^2}N_\mu(\cos\beta\varphi)+J(x)(\varphi-\varphi_\infty)\leftrightarrow i\bar\psi\partial\!\!\!/\psi-\tfrac{1}{2}gj^\mu j_\mu+m\bar\psi\psi+eA^0j^0(x)\,, \tag{4.3}$$

where $\beta^2/4\pi=(1+g/\pi)^{-1}$ and m is proportional to α. When $g=0$, m should be the physical mass of the soliton, $\partial_1A^0=J$, and $e=2\pi/\beta$.

The equivalent Fermi theory is just the massive Thirring model in an external electric field. So as not to involve φ_∞ we will assume that the electric field is constant over most of space, but of compact support.

The J term breaks the degeneracy between the ground states, and we expect decay. We can calculate the decay to fermions in the Fermi theory to the one-loop level (the details are given in the Appendix)

$$E(J)=-\frac{1}{4\pi}\int_0^\infty\frac{ds}{s}Je\cot(Jes)e^{-m^2s}\,. \tag{4.4}$$

Since $\cot x=(1/x)(1-\frac{1}{3}x^2+\cdots)$, the only divergence in (4.4) is independent of J and can be absorbed into a trivial additive constant to the ground-state energy. $\cot x$ has poles at $x=n\pi$, and we take our contour above them.[12]

Then

$$\begin{aligned}\mathrm{Im}E(J)&=-\frac{1}{4\pi}\int_0^\infty\frac{ds}{s}\sum_1^\infty\pi\delta(s-n\pi/eJ)e^{-m^2s}\\&=-\frac{1}{4\pi}eJ\sum\frac{1}{n}e^{-m^2n\pi/eJ}\\&=-\frac{eJ}{4\pi}\ln[(1-e^{-m^2\pi/eJ})^{-1}]\,.\end{aligned}$$

This calculation is not in conflict with our previous discussion on energy conservation because the electric field carries energy, and this energy is enough to support a pair of real fermions in an intermediate state.

For $\beta^2=4\pi$ this one-loop estimate is exact. If $\beta^2>4\pi$, the Thirring force is repulsive but short range and should increase the decay rate. For $\beta^2<4\pi$, the force is attractive and will reduce the rate to zero by the time $\beta=0$. There is no sign in this calculation of a threshold at $\cos\beta\varphi=0$. This would have been due to massless mesons, but they occur as bound states of the fermions, which are only possible if $\beta^2<4\pi$. For small β and large J there should be such a threshold.

V. GENERALIZATIONS

In the above case the fermions are not themselves sources of the field. The decay products do not neutralize the electric field, so decay continues with $\langle\varphi\rangle$ cascading down to $-\infty$. If we add a mass term to the SG equation so that

$$V_0=-\frac{\alpha}{\beta^2}\cos\beta\varphi+\frac{1}{2}\left(\frac{e\beta}{2\pi}\right)^2(\varphi-\varphi_{\rm ext}-\varphi_\infty)^2\,, \tag{5.1}$$

where we have included $J\varphi$ by putting

$$J=\left(\frac{e\beta}{2\pi}\right)^2\varphi_{\rm ext}\,, \tag{5.2}$$

then the ground state remains stable until $\varphi_{\text{ext}} = \pi/\beta$ when the second minimum has the same energy as the original lowest minimum. This corresponds to the electric field produced by charges $\frac{1}{2}e$ near ∞. This field is just enough to create a pair. If the original field is produced by charges $\pm(\frac{1}{2}e+\epsilon)$, then after pair creation the net field between the pair is due to charges $\pm(\frac{1}{2}e-\epsilon)$, and no further creation can take place.

The $(3+1)$-dimensional analog of this decay will be similar to the way that superheated water boils–the vapor phase spreading out from a nucleus rather than boiling throughout the liquid.

Just as the two-dimensional decay rate is described by the number $m^2/eJ = m^2/(\text{energy gap})$, so the $(3+1)$-dimensional case will be characterized by the ratio of the "break-even" radius of the bubble $R_0 = 3\times(\text{surface energy/energy gap})$ to $\partial^2 V/\partial\varphi^2$ in the upper state.

CONCLUSION

Guided by the identification of the Thirring fermion with the SG soliton and our intuition as to the nature of the decay, we have explicitly demonstrated how excited vacuum states can decay as soon as it is energetically possible. The decay rate per unit volume is finite, making it possible for the excited vacuum states to exist for a short time.

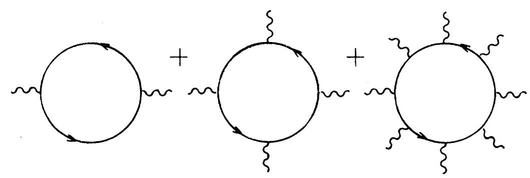

FIG. 4. The graphs for pair production in an external field. The external lines represent a classical field $A^{\mu} = -\frac{1}{2}F^{\mu\nu}\alpha_{\nu}$.

APPENDIX

The one-loop contribution to $E(J)$ is given by the graphs of Fig. 4. The combinatorics of summing such one-loop graphs is well known. We shall essentially follow the method of Brown and Duff[13,14] applied in two dimensions.

We need

$$G = \ln\det[\gamma^{\mu}(-i\partial_{\mu}+eA_{\mu})+m]; \tag{A1}$$

this equals[15] (put $x\to -x$)

$$G = \ln\det[-\gamma^{\mu}(-i\partial_{\mu}+eA_{\mu})+m]. \tag{A2}$$

So, using $\det AB = \det A \det B$, we get

$$G = \tfrac{1}{2}\ln\det[-(-\partial^2 - \tfrac{1}{4}e^2F^2_{\sigma\tau}x^{\sigma}x^{\tau} + iF^{\mu}{}_{\sigma}x^{\sigma}\partial_{\mu} - \tfrac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}) + m^2], \tag{A3}$$

here

$$\sigma^{\mu\nu} = \tfrac{1}{2}i[\gamma^{\mu},\gamma^{\nu}],\quad A_{\mu} = -\tfrac{1}{2}F_{\mu\nu}x^{\nu},\quad F^2_{\sigma\tau} \equiv F_{\sigma\mu}F^{\mu}{}_{\tau}. \tag{A4}$$

All terms without explicit σ's have 2×2 identity matrices understood. In terms of p's

$$G = \tfrac{1}{2}\ln\det(-p^2 + m^2 - \tfrac{1}{4}e^2F^2_{\sigma\tau}\partial^{\sigma\tau} + iF^{\mu}{}_{\sigma}p_{\mu}\partial^{\sigma} + \tfrac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}). \tag{A5}$$

We now use $(d/dm)\ln\det(X+m^2) = \text{Tr}(X+m^2)^{-1}$, so we need the inverse to (A3), i.e., $G(p)$, such that $(-p^2+m^2-\frac{1}{4}e^2F^2_{\sigma\tau}\partial^{\sigma\tau} + eF^{\mu}{}_{\sigma}p_{\mu}\partial^{\sigma} + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu})G(p) = 1$. We try the ansatz $G(p) = \int_0^{\infty} ds\, e^{pAp+C}$, where $A_{\mu\nu}$ and C are functions of s. We find[13]

$$A_{\mu\nu} = [(eF)^{-1}\tan eFs]_{\mu\nu}, \tag{A6}$$

$$C = (m^2 + \tfrac{1}{2}\sigma^{\mu\nu}F_{\mu\nu})s - \tfrac{1}{2}\text{Tr}\ln\cos(eFs)$$

So

$$\text{Tr}(\ )^{-1} = \int\frac{d^2p}{(2\pi)^2}\text{Tr}G(p)$$

$$= \frac{1}{(2\pi)^2 i}(\sqrt{\pi})^2\,\text{Tr}\int_0^{\infty} ds\,\det\left(\frac{\tan eFs}{eF}\right)\exp[-(m^2+\tfrac{1}{2}\sigma\cdot F)s - \tfrac{1}{2}\text{Tr}\ln\cos Fs]$$

$$= \frac{\text{Tr}}{i4\pi}\int_0^{\infty}\frac{ds}{s}\exp\left[-(m^2+\tfrac{1}{2}e\sigma\cdot F)s - \tfrac{1}{2}\text{Tr}\ln\left(\frac{\sin eFs}{eFs}\right)\right]. \tag{A7}$$

Now all we need are the traces over spinor indices. To evaluate these we use a representation of the ∂'s such that

$$\gamma^0=\sigma_1,\quad \gamma^1=i\sigma_2 \Rightarrow \sigma^{01}=-i\sigma_3\,, \tag{A8}$$

and

$$\sigma\cdot F=\sigma^{01}F_{01}+\sigma^{10}F_{10} \tag{A9}$$

together with the fact that

$$F_\mu{}^\nu=\begin{bmatrix}0 & E\\ E & 0\end{bmatrix} \tag{A10}$$

has eigenvalues $\pm E$. So

$$\exp\left[-\tfrac{1}{2}\mathrm{Tr}\ln\left(\frac{\sin eFs}{eFs}\right)\right]=\frac{Ees}{\sin eEs}, \tag{A11}$$

$$\mathrm{Tr}e^{-(1/2)e\sigma\cdot F}=2\cos eFs\,, \tag{A12}$$

where the last expression takes case of the implicit 1 factor. Integrating with respect to m^2 and using the boundary condition that $G=0$ when $m=\infty$ yields finally

$$G=\frac{1}{i4\pi}\int\frac{ds}{s}Ee\cot eEse^{-m^2s}\,. \tag{A13}$$

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[1]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973); E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).

[2]S. Coleman, in *Laws of Hadronic Matter,* Proceedings of the International 1973 Summer School "Ettore Majorana", Erice, Italy, edited by A. Zichichi (Academic, New York, 1975).

[3]T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974).

[4]A familiar example of this is the Stark effect for the hydrogen atom. Perturbation theory reveals the shift in energy levels but fails to show that the eigenstates have in fact disappeared because the electrons can now tunnel out of the $1/r$ potential.

[5]$m\neq 0$ for the decays we want to study. In thermodynamic language we are not at the critical point.

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[11]J. Kogut and L. Susskind, Phys. Rev. D 10, 3468 (1974), and references therein.

[12]This choice of contour is equivalent to the $i\epsilon$ prescription. To get analyticity in the lower $\frac{1}{2}m^2$ plane the contour should go along the imaginary axis. One rotates through $\frac{1}{2}\pi$ to get the present contour.

[13]J. Schwinger, Phys. Rev. 82, 664 (1951).

[14]M. R. Brown and M. J. Duff, Phys. Rev. D 11, 2124 (1975).

[15]There is a δ function implicit in all the differential operator matrices. This yields the $\delta(0)=VT$ factor in G.