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Breaking of conformal invariance and the quark-counting rule for form factors

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Conformal-breaking terms are calculated by iterating the conformal-invariant vertex function of a particle belonging to the fermion-antifermion channel (e.g. the pion) by means of a massive kernel. The resulting γ_5-odd breaking terms have the effect of adding to the conformal-invariant result $(-q^2)^{1-\bar{d}}$ for the form factor ($\bar{d}$ = dimension of the particle field) a term $(-q^2)^{-1}$ for constituents of canonical dimensions. Thus the conformal-breaking terms reconstitute the agreement with the quark-counting rule of Brodsky and Farrar and Matveev, Muradyan, and Tavkhelidze, also for spin-1/2 constituents. The stability of such conformal-breaking terms is proved in a particular case.

I. INTRODUCTION

In this paper we shall examine the effect of conformal-breaking terms on the high-energy behavior of on-shell amplitudes. High-energy, large-momentum-transfer phenomena such as the form factors and fixed-angle scattering have been framed in the so-called "quark-counting rules" derived by Brodsky and Farrar and by Matveev, Muradyan, and Tavkhelidze.[1] Such rules are essentially derived by examining the high-energy behavior of lowest-order graphs connecting a cluster of elementary particles (quarks) which constitute the hadron. The main results are that the fixed-angle differential cross section $d\sigma/dt$ behaves like $s^{2-\Sigma n}$, where $\sum n$ is the sum of the number of elementary constituents of the four particles appearing in the initial state and in the final state, and that the form factor of a hadron composed of n quarks behaves like $(-q^2)^{1-n}$. One should like to prove such rules from field theory. However, owing to the fact that these phenomena are not, strictly speaking, short-distance phenomena, the problem is far from trivial. The problem is further complicated by the fact that, as the external particles are composite, and thus described by fields of high dimensions, a jump occurs when one goes from off-shell to on-shell amplitudes,[2] and thus one cannot work with off-shell amplitudes. Thus perturbation and renormalization-group techniques are not directly applicable, and one needs a rather detailed knowledge of the vertex functions of the composite objects.[3] A further difficulty appears in gauge theories owing to the gauge dependence of the vertex functions.[4,5] In theories with anomalous dimensions, where conformal invariance[6] is realized on the light cone,[7] a careful computation of the form factor[8,9] using the conformal wave function gives an asymptotic behavior $(-q^2)^{1-\bar{d}}$ for an object described by a field of minimal dimension $\bar{d}$ connected to the quark-antiquark channel (e.g. the pion). The asymptotic behavior $(-q^2)^{1-\bar{d}}$ is due to the absence of infrared singularities in the form-factor graph computed using the conformal wave function. Such a result may be embarrassing because if we admit that dimensional anomalies in nature are small and thus attribute to the pion $\bar{d}=3$ we would obtain $F(q^2)\sim(-q^2)^{-2}$ for the pion form factor which goes against the "quark-counting rule".[10] For "scalar quarks," on the other hand, one would obtain $\bar{d}=2$ and $F(q^2)\sim(-q^2)^{-1}$. This is the main motivation why we investigate in this paper the conformal-breaking effects.

There is, as far as we know, no general treatment of conformal breaking. The reason appears to be that Schroer[11] proved that the trace of the energy-momentum tensor is soft only at the Gell-Mann–Low point $g=g_\infty$. Such a result is clearly enough, by examining directly the asymptotic behavior of vertex functions, to prove that at short distances conformal invariance is realized also for $g\neq g_\infty$, but in doing so one avoids understanding the nature of the breaking terms. One has to resort directly to the structure of field theory. In a rather artificial model with only scalar particles in six dimensions we have shown[12] that such conformal-breaking terms are actually unimportant as far as the high-energy behavior of on-shell amplitudes is concerned. On the other hand, if we deal with fermion constituents the problem is more complex. In fact the Dirac γ matrices can produce polynomials in the numerators of Feynman integrals resulting in more singular behavior of the vertex function when one squared momentum goes to infinity.

The breaking scheme we shall adopt in the present paper is the following: Starting from the γ_5-even conformal vertex function[13] of the composite object, we shall iterate it by means of a kernel given by the simplest skeleton graph, i.e., the triangle skeleton graph, where the massless propagators and in particular the fermion propagators have been replaced with massive propagators. Thus the main idea is to start from the exact vertex function on the light cone as given by the conformal invariance and to move inside the light cone by means of a massive iteration. Such an iteration obviously produces breaking terms and in particular γ_5-odd-breaking terms. A rather simple argument (see Sec. II) gives for such γ_5-odd terms a behavior, when only one squared momentum goes to ∞, $(q^2)^{-1}$ which, when properly inserted in the form-factor graph, adds to the above-reported $(-q^2)^{1-\bar{d}}$ behavior the contribution $(-q^2)^{-1}$. With regard to the contribution due to massive kernels of the crossed type, one expects a contribution to the limit when one squared momentum goes to ∞ not higher than the one given by the triangle skeleton graph. The reason is that the large-one-momentum-squared contribution is, as appears clear from the treatment in Sec. II, an infrared effect. As argued in Ref. 5, such infrared effects should be dominant in the triangle graph which is of the disconnected type.

Some care has to be exercised in dealing with such γ_5-odd-breaking terms. In fact, the same power-counting argument would also give a $(q^2)^{-1}$ behavior for the terms containing two Dirac γ matrices in the vertex function; however, we know from a general argument[14] that such $(q^2)^{-1}$ terms cancel out exactly. Thus we must make sure that the coefficients of the γ_5-odd terms behaving like $(q^2)^{-1}$ do not sum up to zero. Such a job is undertaken in Sec. III, where it is shown that for a particular choice of dimensions of the fields (and thus in general), such cancellation in the γ_5-odd-breaking terms does not occur. In the same section we examine also the problem of the stability of the breaking, i.e., whether further iterations of such breaking terms with the same massive kernel (which qualitatively speaking correspond to moving further inside the light cone) break the above-reported result. The outcome, again for a special choice of dimensions, is that the second iteration modifies in an unessential way the breaking term, and that after the second iteration the breaking term becomes stable.

Finally in Sec. IV we examine the role played by these breaking terms in determining the form factor. One has to examine both the interference of the breaking terms with the unperturbed wave function and the convolution of the breaking terms with themselves. As already reported above the result is to add to the dimensional contribution $(-q^2)^{1-\bar{d}}$ the contribution $(-q^2)^{-1}$ (which obviously dominates for $\bar{d}>2$), thus reaching agreement with the quark-counting rules[1] also for spin-$\frac{1}{2}$ constituents. Such an additional $(-q^2)^{-1}$ contribution is a conformal-breaking effect.

II. BREAKING OF THE CONFORMAL VERTEX FUNCTION

As we described in the Introduction, we shall compute conformal- (and dilatation-) breaking terms by iterating the conformal-invariant wave function by means of a massive kernel. Such a kernel, in the simplest instance, is given by the skeleton graph of Fig. 1 in which the massless (conformal) fermion propagators have been replaced by massive ones. Such a procedure introduces breaking terms both γ_5-even and γ_5-odd. The dimensionally leading breaking terms are the γ_5-odd terms and have the form $\not{q}_1 v_1(q_1^2, q_2^2)$ or $\not{q}_2 v_2(q_1^2, q_2^2)$. We shall consider fermions of almost canonical dimension, and thus we set $d'=\frac{3}{2}$. Then the change in the propagator will be given by the replacement $(\not{q})^{-1} \to (\not{q}-m)^{-1}$.

The conformal on-shell and off-shell vertex functions in Fig. 1 are each composed of two parts: a part s which multiplies the identity Dirac matrix and a second part t which multiplies $\not{q}_1\not{q}_2$, where q_2 and q_1 are in the incoming and outgoing fermion momenta. Let us consider at first, for the sake of illustration, the simplest case where three s-type terms are convoluted together, and only the internal fermion leg on the left has been replaced by a massive propagator. In the unperturbed graph the lower wave function (fermion propagators included) is given by[8,9]

$$\Phi(q_1,q_2)=\not{q}_1\not{q}_2\int_0^1 \mathfrak{F}^{-3/2-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz + \frac{1/2-\bar{d}/2}{1/2+\bar{d}/2}\int_0^1 \mathfrak{F}^{-1/2-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz$$

$$=\not{q}_1\not{q}_2 T + S\ , \tag{2.1}$$

where

$$\mathfrak{F}=(-q_1^2+m^2)z+(-q_2^2+m^2)(1-z)\ , \tag{2.2}$$

while the left upper vertex with boson propagator

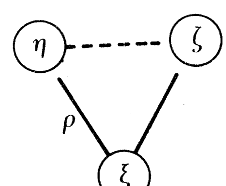

FIG. 1. Iteration of the on-shell vertex function.

included is given by[9]

$$\not q_1 \not q_2 \int_0^1 \mathcal{G}^{d/2-5/2}\delta(1-\Sigma)(\alpha\beta)^{3/2-d/2}d\alpha\, d\beta\, \gamma^{d/2-1/2}d\gamma + \frac{d/2+1/2}{d/2-3/2}\int_0^1 \mathcal{G}^{d/2-3/2}\delta(1-\Sigma)(\alpha\beta)^{3/2-d/2}d\alpha\, d\beta\, \gamma^{d/2-3/2}d\gamma = \not q_1 \not q_2 \bar{t} + \bar{s}\,, \tag{2.3}$$

where

$$\Sigma = \alpha + \beta + \gamma \tag{2.4}$$

and

$$\mathcal{G} = (-q_1^2 + m^2)\beta\gamma + (-q_2^2 + m^2)\alpha\gamma + (-p^2 + \mu^2)\alpha\beta\,, \tag{2.5}$$

with $p = q_2 - q_1$.

The right upper vertex, with boson and fermion propagators removed, is given by

$$\not q_1 \not q_2 \int_0^1 \mathcal{G}^{-1/2-d/2}\delta(1-\Sigma')(\alpha'\beta')^{d/2-1/2}d\alpha'\, d\beta'\, \gamma'^{3/2-d/2}d\gamma' + \frac{5/2-d/2}{1/2-d/2}\int_0^1 \mathcal{G}^{1/2-d/2}\delta(1-\Sigma')(\alpha'\beta')^{d/2-1/2}d\alpha'\, d\beta'\, \gamma'^{1/2-d/2}d\gamma' = \not q_1 \not q_2 t + s\,. \tag{2.6}$$

As discussed in Ref. 9, nonzero threshold masses have been introduced in the functions $\mathcal{F}$ and $\mathcal{G}$. Such a procedure is necessary because in examining nonleading terms one must consider the mass of the composite particle to be strictly positive; the masses introduced in $\mathcal{F}$ and $\mathcal{G}$ are then necessary to satisfy the stability condition.[15]

When we replace a massless fermion propagator with $(\not q + m)/(-q^2 + m^2)$, the first part $\not q(-q^2+m^2)$ is going to reproduce asymptotically, after the convolution, a part of the conformal on-shell vertex function, while the second part $m/(-q^2+m^2)$ is going to produce γ_5-odd conformal-breaking terms.

The convolution is performed along the lines explained in Ref. 9 to get

$$m\int [\not q_1(1-A) - \not q_2 B]C^{\bar{d}/2-3/2}\mathfrak{D}^{-1/2-\bar{d}/2} \times d\xi^{\bar{d}/2+1/2}d\rho\, d\eta^{3/2-d/2}d\zeta^{d/2-1/2}\delta(1-\xi-\rho-\eta-\zeta)\,, \tag{2.7}$$

where

$$\begin{aligned} \mathfrak{D} &= -q_1^2(AD + C\eta\beta\gamma) - q_2^2(BD + C\zeta\alpha'\gamma') + MC\Delta\,,\\ A &= \xi z + \eta\alpha\gamma + \rho\,, \quad B = \xi(1-z) + \zeta\beta'\gamma'\,,\\ D &= \eta\alpha\beta + \zeta\alpha'\beta'\,,\\ C &= A + B + D\,, \quad M = C + \eta\beta\gamma + \zeta\gamma'\alpha'\,. \end{aligned} \tag{2.8}$$

Δ is a quantity which, owing to the stability condition of the external particle, is strictly positive.

Equation (2.7) has to be integrated further over the parameters $z, \alpha\beta\gamma, \alpha'\beta'\gamma'$ with the weights appearing in Eqs. (2.1), (2.3), and (2.6).

It is easily seen from (2.7) that such an integral cannot decrease for $-q_1^2 \to \infty$ faster than $(-q_1^2)^{-1}$. The reason is that we can get a lower bound by replacing the coefficient of $-q_1^2$ with $\eta + \zeta$ as $AD + C\eta\beta\gamma \leq \text{const} \times (\eta + \zeta)$. The sum of the two exponents in the differentials $d\eta, d\zeta$ in (2.7) is 1, and thus we conclude that the asymptotic behavior cannot be lower than $\not q_1(-q_1^2)^{-1}$. From a more qualitative point of view the situation is as follows. The kernel of Fig. 2, for $-k^2 \to \infty$, cannot decrease faster than $(-k^2)^{-1}$, as can be seen by summing the nominal powers of the two s terms in (2.3) and (2.6). As, for $-q_1^2 \to \infty$, the momentum can also flow along the line k, we cannot expect a decrease of the loop integral faster than $(-q_1^2)^{-1}$.

At first sight such an argument appears contradictory with the behavior of the γ_5-even conformal vertex function which (external fermion propaga-

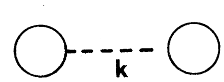

FIG. 2. Conformal kernel with external fermion propagators removed.

tors removed) is[9]

$$\Gamma(q_1,q_2)=\not q_1\not q_2\int_0^1[-q_1^2z-q_2^2(1-z)]^{-1/2-\bar d/2}\times[z(1-z)]^{\bar d/2-1/2}dz+\frac{1/2+\bar d/2}{1/2-\bar d/2}\int_0^1[-q_1^2z-q_2^2(1-z)]^{1/2-\bar d/2}\times[z(1-z)]^{\bar d/2-1/2}dz=\not q_1\not q_2 t_B+s_B. \tag{2.9}$$

Equation (2.9) has a t part, multiplying $\not q_1\not q_2$, which for large q_1^2 behaves like $(-q_1^2)^{-1/2-\bar d/2}\times\ln(-q_1^2)$, i.e., is more rapidly decreasing than $(-q_1^2)^{-1}$. The reason is that for such a term a special cancellation occurs in the convolution of Fig. 1, which eliminates all such $(-q^2)^{-1}$ contributions (see Sec. III). Thus we have to make sure that similar cancellations do not occur also for the γ_5-odd term.

To complete the analysis now we have to perform two checks. First we must make sure that the γ_5-odd contributions arising from the various terms do not exceed such $(-q^2)^{-1}$ behavior. Such a program will be accomplished in this section. The second point is to prove that the various γ_5-odd contributions do not cancel out such a leading $(-q^2)^{-1}$ behavior. This will be done in Sec. III, where also the stability of the perturbation, in the special case of canonical dimensions ($d'=\frac{3}{2}$ and $d=1$), will be proved.

In calculating the asymptotic behavior of the described mass insertions, one cannot rely on simple power-counting arguments. The reason is that, as pointed out in Refs. 8 and 9, when internal spinor lines are present as a rule one finds that the final result is the outcome of rather subtle cancellations in the polynomials in the Feynman numerator. Thus the technique is that of computing the convolution exactly by using the Feynman parameters and then of looking at the asymptotic behavior. Instead of going through the examination of all eight combinations of s and t vertex parts in Fig. 1, we shall examine in detail only the case of the convolution of the three t parts where the role of such cancellations is particularly apparent. The γ_5-odd-breaking term for the convolution of type $\bar tTt$ is given by

$$m\int\not q_1\bar t\frac{\not q_1+\not k}{-(q_1+k)^2+m^2}(q_1+k)^2T(q_2+k)^2t\not q_2d^4k. \tag{2.10}$$

In performing such a loop integration with the Feynman parametrization of Fig. 1, one has to make the shift

$$k=k'-\frac{A}{C}q_1-\frac{B}{C}q_2, \tag{2.11}$$

which symmetrizes the denominator, and by straightforward computation one finds that

$$\not q_1(\not q_1+\not k)(q_1+k)^2(q_2+k)^2\not q_2 \tag{2.12}$$

goes over to a linear combination of the following terms:

$$\begin{aligned}&(k'^2)^2(\not q_1q_2^2-q_1^2\not q_2),\\&k'^2(q_1^2-q_2^2)\frac{D}{C}\left(\not q_1q_2^2\frac{B}{C}-q_1^2\not q_2\right),\\&k'^2\left(q_1^2-q_2^2\frac{B}{C}\right)\frac{D}{C}\left(\not q_1q_2^2-q_1^2\frac{A}{C}\not q_2\right),\\&\left(\frac{D}{C}\right)^2\left(q_1^2-q_2^2\frac{B}{C}\right)\left(q_1^2\frac{A}{C}-q_2^2\right)\left(\not q_1q_2^2\frac{B}{C}-q_1^2\not q_2\right).\end{aligned} \tag{2.13}$$

In deriving (2.13) one exploits several times the relation[16] $A+B+D=C$ [cf. Eq. (2.8)]. In performing the integration over d^4k one has to keep in mind that

$$(q_1^2)^a(q_2^2)^b\int d^4k'(k'^2)^N(-Ck'^2+\mathfrak{D}C^{-1})^{-a-b-N-5/2-\bar d/2}=\text{const}\times(q_1^2)^a(q_2^2)^b\mathfrak{D}^{-a-b-\bar d/2-1/2}C^{\bar d/2+a+b-N-3/2}. \tag{2.14}$$

Let us consider the behavior of such γ_5-odd terms as e.g. $-q_1^2\to\infty$. As we already know from the argument shown above that we cannot expect a decrease of the separate contributions, for $-q_1^2\to\infty$, faster than $(-q_1^2)^{-1}$, we perform straightaway in (2.14) the majorization

$$\begin{aligned}&\mathfrak{D}^{-a-b-\bar d/2-1/2}C^{\bar d/2+a+b-N-3/2}\\&\quad\le\text{const}\times\mathfrak{D}^{-a-1+\epsilon}C^{a-N-1-\epsilon}M^{-b-\bar d/2+1/2-\epsilon}\\&\quad\le\text{const}\times(-q_1^2AD)^{-a-1+\epsilon}C^{a-N-1-\epsilon}M^{-b-\bar d/2+1/2-\epsilon}.\end{aligned} \tag{2.15}$$

Thus if we consider the contribution proportional to $\not q_1$ in the first term of (2.13) we have to prove the convergence of the integral

$$\int\mathcal{T}\,dP, \tag{2.16}$$

where

$$\mathcal{T}=(AD)^{-1+\epsilon}C^{-3-\epsilon}M^{-1/2-\bar d/2-\epsilon} \tag{2.17}$$

and the differential dP is given by [cf. Eqs. (2.1)–(2.6)]

$$dP = d\xi^{\bar d/2+3/2}[z(1-z)]^{\bar d/2-1/2}dz\, d\eta^{5/2-d/2}d\alpha^{5/2-d/2}d\beta^{5/2-d/2}d\gamma^{d/2+1/2}\delta(1-\Sigma)d\zeta^{d/2+1/2}d\alpha'^{\,/2+1/2}d\beta'^{\,d/2+1/2}d\gamma'^{\,5/2-d/2} \times \delta(1-\Sigma')d\rho\, \delta(1-\xi-\eta-\zeta-\rho)\,. \tag{2.18}$$

The convergence of such an integral for arbitrary small ϵ is straightforward (see Appendix) if one takes into account that, owing to (2.8),

$$C \geq \xi + \rho + \eta\alpha\beta + \zeta\alpha'\beta' \tag{2.19}$$

and

$$M \geq \tfrac{1}{2}[\xi + \rho + \gamma\gamma' \min(\alpha + \beta\,;\ \alpha' + \beta')]\,. \tag{2.20}$$

Similarly one deals with the other terms appearing in (2.13). The final result is that in these γ_5-odd first-order-breaking terms which are of the form $\not{q}_1 v_1(q_1^2, q_2^2) + \not{q}_2 v_2(q_1^2, q_2^2)$, the two functions v_1 and v_2 behave, except possibly for logarithms,[17] like $(-q^2)^{-1}$ when one of the two arguments goes to infinity and the other is kept fixed.

III. ASYMPTOTIC BEHAVIOR AND STABILITY OF THE BREAKING TERM

In Sec. II we have proved that each of the γ_5-odd terms arising as a result of a mass insertion in the skeleton graph of Fig. 1 behaves, when one squared momentum becomes large, as $(-q^2)^{-1}$. As the various s and t parts appear in the vertex functions with well-defined weights, one should make sure first that such leading contributions do not cancel out. This is most easily performed by taking a particular case, e.g., the case where both d' and d assume their canonical value. In such a case one can keep track of all the coefficients and such a cancellation does not occur. In fact, for $d=1$, ($d'=\frac{3}{2}$) the upper vertices in Fig. 1 become pointlike vertices and one obtains for the γ_5-odd-breaking terms proportional to $\not{q}_1$

$$m\pi^2 i \int (1-\xi z-\rho)[z(1-z)]^{\bar d/2-1/2}dz\, \xi^{\bar d/2-1/2} \times \left(\frac{1-\bar d}{1+\bar d} - \frac{2}{1+\bar d}\xi\right)\mathfrak{D}^{-1/2-\bar d/2}d\xi\, d\rho\, d\gamma \times \delta(1-\xi-\rho-\gamma)\,, \tag{3.1}$$

where in (3.1)

$$\mathfrak{D} = -q_1^2(\xi z+\rho)\gamma - q_2^2\xi(1-z)\gamma + m^2 - m_B^2(1-z)\xi(\xi z+\rho)\,, \tag{3.2}$$

and γ is the Feynman parameter associated with the boson propagator. One easily checks that the large $-q_1^2$ behavior of (3.1) is in fact $(-q_1^2)^{-1}$ and also that the large $-q_2^2$ behavior of (3.1) is $(-q_2^2)^{-1}$. Similarly one deals with the coefficients of $\not{q}_2$ obtaining the same asymptotic behavior.[18]

A more subtle question is that of the stability of the perturbation which generates such γ_5-odd terms. In other words, one should make sure that by iterating such γ_5-odd terms by means of a massive kernel one does not generate γ_5-even terms higher (in the limit of one large momentum squared) than the original conformal wave function, and at the same time the γ_5-odd breaking term stabilizes itself. Such a result is proved in two steps. First we examine the result of the double mass insertion in the fermion propagators in Fig. 1 on the functions s_B and t_B in Eq. (2.9). Clearly the two propagators $(\not{q}_1+\not{k})[-(q_1+k)^2+m^2]^{-1}$ and $(\not{q}_2+\not{k})[-(q_2+k)^2+m^2]^{-1}$, owing to conformal invariance, reproduce asymptotically after the convolution the conformal γ_5-even vertex function. On the other hand, one easily proves by using the familiar Feynman parameter technique that the contribution of $m[-(q_1+k)^2+m^2]^{-1}$, $m[-(q_2+k)^2+m^2]^{-1}$ to the convolution is not higher than the conformal one. Next we have to examine the iteration of the γ_5-odd-breaking term found in Sec. II. One has to compute the asymptotic behavior, for $-q_1^2\to\infty$ or $-q_2^2\to\infty$, of the integral

$$\int \frac{\not{q}_1+\not{k}+m}{-(q_1+k)^2+m^2}(\not{q}_1+\not{k})[-(q_1+k)^2 z-(q_2+k)^2(1-z)+m^2]^{-\bar d/2-1/2}dz\frac{\not{q}_2+\not{k}+m}{-(q_2+k)^2+m^2}\,\frac{1}{-k^2+m^2}d^4k\,. \tag{3.3}$$

The terms of asymptotic dimension $-\bar d$, which are obviously γ_5-odd, are of the form

$$\int [\not{q}_1(\xi z+\rho)+\text{const}\times\not{q}_2][q_1^2(1-\xi z-\rho)\gamma - q_2^2(\xi(1-z)+\sigma)]\gamma\mathfrak{D}^{-3/2-\bar d/2}d\xi^{\bar d/2+1/2}d\rho\, d\sigma\, d\gamma\, dz\, \delta(1-\Sigma) \tag{3.4}$$

and

$$\int (\not q_1 + \text{const} \times \not q_2) \mathfrak{D}^{-1/2-\bar{d}/2} d\xi^{\bar{d}/2+1/2} d\rho\, d\sigma\, d\gamma\, dz\, \delta(1-\Sigma)\,, \tag{3.5}$$

where now

$$\mathfrak{D} = -q_1^2(\xi z + \rho)\gamma - q_2^2[\xi(1-z)+\sigma]\gamma + m^2 - m_B^2(\xi z+\rho)[\xi(1-z)+\sigma] \tag{3.6}$$

and

$$\Sigma = \xi + \gamma + \rho + \sigma. \tag{3.7}$$

The asymptotic behavior of (3.4) and (3.5) for q_1^2 (or q_2^2) going to infinity is performed with the standard technique of introducing the new integration variable $\gamma' = -q_1^2\gamma(\xi z + \rho)$ (or $\gamma' = -q_2^2\gamma \times [\xi(1-z)+\sigma]$), and for $-q_1^2 \to \infty$ we have the behavior

$$\not q_1(-q_1^2)^{-1} + \not q_2(-q_1^2)^{-1}\ln(-q_1^2). \tag{3.8}$$

This result shows that the γ_5-odd-breaking term gets modified after the first iteration and assumes the form

$$\int \{\not q_1[a - \ln(1-z)] + \not q_2[a - \ln z]\} \times [-q_1^2 z - q_2^2(1-z) + m^2]^{-\bar{d}/2-1/2} dz. \tag{3.9}$$

However, it is very important to remark that the introduction of (3.9) into (3.3) gives again for $-q_1^2 \to \infty$ the asymptotic behavior (3.8) and for $-q_2^2 \to \infty$ the asymptotic behavior (3.8) with q_1 exchanged with q_2. Thus the outcome is that the breaking term (3.9) is stable under the iteration of Fig. 1 with $d=1$. Similarly one examines the terms proportional to m (which are γ_5-even) and those proportional to m^2 (γ_5-odd). One finds that such contributions are not higher respectively than the conformal γ_5-even terms and the γ_5-odd term (3.9).

Summing up we have proved that in the case of $d=1$ the breaking term due to mass insertion of Fig. 1, in the critical region when only one squared momentum goes to infinity, is actually of the form (3.9), and that such a breaking term is stable under further iteration with massive propagators.

IV. EFFECT OF BREAKING TERMS ON THE FORM FACTORS

We work out now the consequences of the results of Sec. III on the asymptotic behavior of the elastic form factor. It was shown in Refs. 8 and 9 that the conformal γ_5-even contribution to the wave functions gives rise to the behavior $(-q^2)^{1-\bar{d}}$. Such a result is due in part to the structure of the γ_5-even leading conformal contribution, and in part, as explained in detail in Ref. 9, to the special ordering in which the γ matrices occur in computing the trace inside the loop integral of Fig. 3. With the same technique as used in Ref. 9, one also proves that the introduction of masses in the propagators connecting the vertices building up the triangle graph of the form factor does not alter the above-mentioned result. More important instead, as we shall immediately see, is the role played by the γ_5-odd breaking term treated in Secs. II and III. We have to discuss the convolution of this breaking term with itself and with the old unbroken γ_5-even part. In both cases the vertices are connected by massive propagators (Fig. 3).

With regard to the convolution of the two γ_5-odd terms, one gets the trace

$$\mathrm{Tr}\{\gamma_\mu(\not p_1 + \not k - m)^{-1}[(\not p_1+\not k)(a - \ln(1-z)) + \not k(a-\ln z)]\,(\not k - m)^{-1}[\not k(a - \ln z') + (\not p_2 + \not k)(a - \ln(1-z'))](\not p_2 + \not k - m)^{-1}\}. \tag{4.1}$$

One has to combine (4.1) with the two powers $[-(p_1+k)^2 z - k^2(1-z) + m^2]^{-\bar{d}/2-1/2}$ and $[-k^2(1-z') - (p_1+k)^2 z']^{-\bar{d}/2-1/2}$ with Feynman parameters ξ and ζ, respectively, then perform the shift

$$k = k' - p_1 A - p_2 B$$
$$= k' - p_1(\xi z + \alpha) - p_2(\zeta z' + \beta), \tag{4.2}$$

and integrate over d^4k'. The result is a sum of terms majorized by

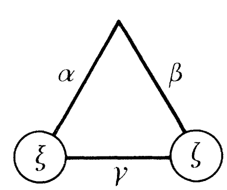

FIG. 3. Feynman parametrization for the form-factor graph.

$$p_\mu \mathfrak{D}^{-\bar{d}} \ln z \ln z', \tag{4.3a}$$

$$p_\mu (-q^2) B \mathfrak{D}^{-\bar{d}-1} \ln z'. \tag{4.3b}$$

$\mathfrak{D}$ is given by

$$\mathfrak{D} = -q^2 AB + \text{m.t.}, \tag{4.4}$$

and m.t. is a strictly positive mass term.

Consider (4.3b). The contribution to the form factor is

$$-q^2 \int (-q^2 AB + \text{m.t.})^{-1-\bar{d}} B \ln z' d\xi^{\bar{d}/2+1/2} \times d\zeta^{\bar{d}/2+1/2} dz\, dz' d\alpha\, d\beta\, d\gamma\, \delta(1-\xi-\zeta-\alpha-\beta-\gamma). \tag{4.5}$$

Setting

$$\bar{\alpha} = -q^2 \alpha, \quad \bar{z} = -q^2 z, \tag{4.6}$$

we get for (4.5) in the large-q^2 limit after integration over $\bar{\alpha}$ from 0 to ∞

$$\frac{1}{q^2} \int [\xi \bar{z}(\zeta z' + \beta) + \text{m.t.}]^{-\bar{d}} d\bar{z}\, d\xi^{\bar{d}/2+1/2} d\zeta^{\bar{d}/2+1/2} d\beta \times d\gamma \ln z' dz'\, \delta(1-\xi-\zeta-\beta-\gamma), \tag{4.7}$$

where also $\bar{z}$ is integrated from 0 to ∞.

The integral in (4.7), as $1 < \bar{d}$, is convergent. The term originating from (4.3a) goes to zero faster than $(-q^2)^{-1}$. Essentially what happens is that in (4.1) the propagator $(\not{p}_1 + \not{k} - m)^{-1}$ eliminates the kinematical factor $\not{p}_1 + \not{k}$ of the γ_5-odd wave function. As such a wave function decreases at infinity like $[(p_1+k)^2]^{-1}$ we have, when the momentum flows through the $p_1 + k$ line, a form factor behaving like $(-q^2)^{-1}$.

The terms originating from the interference between the γ_5-odd-breaking term and the γ_5-even conformal contribution, connected by massive fermion propagators, result in terms with the same nominal power as (4.3) because only those terms containing an even number of γ matrices contribute to the trace. However, the t part [see Eq. (2.9)] gives rise to terms such as (4.3), where $\ln z$ and $\ln(1-z)$ are replaced by $[z(1-z)]^{\bar{d}/2-1/2}$, thus giving rise to a more convergent integration in dz as $\bar{d} > 1$. The integration in ξ retains the same weight $d\xi^{\bar{d}/2+1/2}$. Thus we end up with the same contribution $(-q^2)^{-1}$ to the form factor.

Finally with regard to the s term [see Eq. (2.9)] we get contributions majorized by

$$p_\mu \mathfrak{D}^{-\bar{d}} \ln z', \tag{4.8a}$$

$$p_\mu (-q^2) A \mathfrak{D}^{-\bar{d}-1}. \tag{4.8b}$$

Such terms, keeping in mind that the integration in ξ has now the weight $d\xi^{\bar{d}/2-1/2}$ and that dz retains the factor $[z(1-z)]^{\bar{d}/2-1/2}$ [see Eq. (2.9)], give again the result $(-q^2)^{-1}$. Summing up, if we take into account the conformal contribution worked out in Refs. 8 and 9 we have for the asymptotic behavior $(-q^2)^{\max(-1, 1-\bar{d})}$. The contribution $(-q^2)^{-1}$ is an effect of the conformal breaking.

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APPENDIX

We give here the method for proving the convergence of the integrals appearing in Sec. II. Consider e.g. the integral

$$\int \mathcal{T} dP \tag{A1}$$

with $\mathcal{T}$ given by (2.17). Using the majorization (2.19) and (2.20) we can replace $\mathcal{T}$, for $\alpha + \beta < \alpha' + \beta'$, by

$$[(\xi z + \eta\alpha\gamma + \rho)(\eta\alpha\beta + \zeta\alpha'\beta')]^{-1+\epsilon}(\xi + \rho + \eta\alpha\beta + \zeta\alpha'\beta')^{-3-\epsilon}[\xi + \rho + \gamma\gamma'(\alpha+\beta)]^{-1/2-\bar{d}/2-\epsilon}$$
$$\leq \rho^{-1+\epsilon}(\eta\alpha\beta)^{d/2-3/2+\epsilon-\epsilon/4}(\zeta\alpha'\beta')^{-d/2+1/2+\epsilon/4}\xi^{-1+\epsilon/8}\rho^{-\epsilon+\epsilon/8}(\eta\alpha\beta)^{-1-\epsilon/8}(\zeta\alpha'\beta')^{-1-\epsilon/8}\xi^{-1/2-\bar{d}/2}(\gamma\gamma')^{-\epsilon}\alpha^{-\epsilon/2}\beta^{-\epsilon/2}. \tag{A2}$$

Comparing (A2) with dP as given by (2.18) one sees that convergence is ensured. Similarly one deals with the region $\alpha + \beta > \alpha' + \beta'$.

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[13]As discussed in Ref. 5, one has to choose between the wave function which on the light cone is γ_5-even or γ_5-odd. The γ_5-even solution (i.e., with an even number of Dirac γ matrices) appears to be favored by perturbation theory (see also G. Mack and I. T. Todorov, Ref. 6), and thus we shall bind ourselves to this case.

[14]This can be seen by keeping in mind that the iteration with massive kernel has to reproduce exactly the same light cone of the composite particle vertex function from which we start. See Sec. III.

[15]For notational simplicity we shall put in the following $\mu = m$. The stability condition is $m_B < 2m$, where m_B is the mass of the composite external particle.

[16]The coefficients A/C, B/C, D/C are the results of cancellations in the γ-matrix algebra and play an important role in determining the asymptotic behavior of the breaking terms.

[17]In the treatment of this section an additional power of a logarithm cannot be excluded. Such a power of a logarithm is actually absent in this first iteration for $d=1$; a simple logarithm appears in an unimportant corner in the second iteration and then stabilizes itself. See Sec. III.

[18]In the convolution of the unperturbed T term with two γ matrices, the Feynman parameters in the numerator sum up exactly to γ, which lowers the asymptotic behavior as $q_1{}^2$ or $q_2{}^2$ goes to infinity.