Spontaneous breakdown of fermion number conservation and supersymmetry

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(Received 13 April 1976)

We have examined the spontaneous breakdown of fermion-number-conserving supersymmetry. In the tree approximation the vacuum expectation values of the difermion fields play a passive role, the values of the potential at the minima being entirely determined by the vacuum expectation values of the usual boson fields. One-loop calculations for three different models show that the vacuum expectation values of the difermion fields should be zero at the potential minima. Our general conjecture is that spontaneous breakdown of fermion number cannot accompany the spontaneous breakdown of supersymmetry.

I. INTRODUCTION

For physical applications of supersymmetric theories[1] it is necessary that supersymmetry be broken to accommodate different masses for the bosons and the fermions belonging to the same supermultiplet. To preserve the renormalizability properties of the unbroken theory it is required that the breaking be spontaneous. It has been established that spontaneous breakdown is possible with[2] or without[3,4] gauge invariance. In the latter case some stringent conditions must be satisfied for the breaking.[4]

Salam and Strathdee[5] and Fayet[6] have introduced fermion number in supersymmetry through γ_5 transformations. A consequence of this is that the usual spin-zero bosons are accompanied by difermion spin-zero particles. If fermion number is to be conserved, then the Lagrangian is restricted to have certain specific forms which can make it favorable for spontaneous breakdown.[3,4]

In this paper, we examine the most general fermion-number-conserving Lagrangian for an arbitrary number of bosons and difermion fields. We find that when the number of difermion fields exceeds the number of boson fields spontaneous breakdown of supersymmetry can occur quite easily. A very interesting feature is that the values of the potential at the minima (both supersymmetric and supersymmetry breaking), in the tree approximation, are entirely determined by the vacuum expectation values of the boson fields. The vacuum expectation values of the difermion fields are required to satisfy certain constraints, but they play no role in fixing the values of the potential at the minima. These constraints may leave one or more of the vacuum expectation values of the difermion fields undetermined. Hence, in tree approximations spontaneous breakdown of fermion-number conservation can take place arbitrarily.

To find out whether the above situation persists in higher orders we have to calculate the one-loop corrections to the classical potential. The general features can be brought out by dealing with a few specific models. We discuss three different models, O'Raifeartaigh's discrete model,[4] Fayet's $SU(2)\times U(1)$ model,[3] and an $SU(3)$ model cooked up by using the general conditions of this paper. In all three cases we find that spontaneous breakdown of supersymmetry may not be accompanied by a spontaneous breakdown of fermion-number conservation; this is imposed by one-loop corrections to the effective potential. However, for supersymmetric minima the one-loop corrections are absent and the features of the tree approximation allowing an unspecified amount of fermion-number breaking are retained.

II. FERMION-NUMBER-CONSERVING LAGRANGIANS

We shall follow the conventions of Ref. 5 and introduce N positive chiral fields $\Phi_+^i(x,\theta)$, $i=1,2,\ldots,N$ and M negative chiral fields $\Phi_-^\alpha(x,\theta)$, $\alpha=1,2,\ldots,M$ with the following γ_5 transformations:

$$\Phi_+^i(x,\theta)\to\Phi_+^i(x,e^{-\eta\gamma_5}\theta)\,, \tag{2.1}$$

$$\Phi_-^\alpha(x,\theta)\to e^{2i\eta}\Phi_-^\alpha(x,e^{-\eta\gamma_5}\theta)\,. \tag{2.2}$$

The fields $\Phi_\pm$ have the usual expansions

$$\Phi_\pm(x,\theta)=\exp(\mp\tfrac{1}{4}\bar\theta\partial\!\!\!/\gamma_5\theta)\times[A_\pm(x)+\bar\theta\,\psi_\pm(x)+\tfrac{1}{4}\bar\theta(1\pm i\gamma_5)\theta F_\pm(x)]\,. \tag{2.3}$$

For the fermion fields

$$\psi_+^i=\tfrac{1}{2}(1+i\gamma_5)\psi^i\,, \tag{2.4}$$

$$\psi_-^\alpha=\tfrac{1}{2}(1-i\gamma_5)\chi^\alpha\,. \tag{2.5}$$

Since we are not going to introduce parity transformations, ψ^i's and χ^α's are not related.

The negative chirality fields A_-^α are difermion fields with fermion number $F=2$. The positive chirality fields A_+^i are the usual bosons with $F=0$.

The fermions $\psi_\pm$ carry $F=1$. The dummy fields F_-^α and F_+^i carry $F=0$ and $F=2$, respectively.

The most general fermion-number-conserving and renormalizable Lagrangian for the above fields is

$$\mathcal{L}=\tfrac{1}{8}(\bar{D}D)^2(\Phi_+^{\dagger i}\Phi_+^i+\Phi_-^{\dagger\alpha}\Phi_-^\alpha) -\tfrac{1}{2}\bar{D}D[(\lambda_\alpha\Phi_-^{\dagger\alpha}+m_{\alpha i}\Phi_-^{\dagger\alpha}\Phi_+^i +g_{\alpha,ij}\Phi_-^{\dagger\alpha}\Phi_+^i\Phi_+^j)+\mathrm{H.c.}]\,, \tag{2.6}$$

where the $m_{\alpha i}$ form an $M\times N$ array without any restrictions, and the $g_{\alpha,ij}$ have the symmetry

$$g_{\alpha,ij}=g_{\alpha,ji}\,. \tag{2.7}$$

The dummy fields F_+^i and F_-^α satisfy the equations of motion

$$-F_+^i=m_{\alpha i}^*A_-^\alpha+2g_{\alpha,ij}^*A_-^\alpha A_+^{*j}\,, \tag{2.8}$$

$$-F_-^\alpha=\lambda_\alpha+m_{\alpha i}A_+^i+g_{\alpha,ij}A_+^iA_+^j\,. \tag{2.9}$$

These may be used to eliminate F_+^i and F_-^α from the Lagrangian. We can write

$$\mathcal{L}=\mathcal{L}_{\mathrm{KE}}+\mathcal{L}_F-V\,, \tag{2.10}$$

where we have

$$\mathcal{L}_{\mathrm{KE}}=\partial_\mu A_+^{*i}\partial^\mu A_+^i+\bar{\psi}_+^i i\partial\!\!\!/\psi_+^i+\partial_\mu A_-^{*\alpha}\partial^\mu A_-^\alpha+\bar{\psi}_-^\alpha i\partial\!\!\!/\psi_-^\alpha \tag{2.11}$$

for the kinetic part,

$$\mathcal{L}_F=-m_{\alpha i}\bar{\psi}_-^\alpha\psi_+^i-g_{\alpha,ij}A_-^{*\alpha}\bar{\psi}_+^{ci}\psi_+^j-2g_{\alpha,ij}\bar{\psi}_-^\alpha\psi_+^iA_+^j -m_{\alpha i}^*\bar{\psi}_+^i\psi_-^\alpha-g_{\alpha,ij}^*A_-^\alpha\bar{\psi}_+^i\psi_+^{cj}-2g_{\alpha,ij}^*\bar{\psi}_+^i\psi_-^\alpha A_+^{*j} \tag{2.12}$$

for the fermion part, and

$$\begin{aligned}V&=F_+^{*i}F_+^i+F_-^{*\alpha}F_-^\alpha\\&=(m_{\alpha i}^*A_-^\alpha+2g_{\alpha,ij}^*A_-^\alpha A_+^{*j})(m_{\beta i}A_-^{*\beta}+2g_{\beta,ik}A_-^{*\beta}A_+^k)\\&\quad+(\lambda_\alpha+m_{\alpha i}A_+^i+g_{\alpha,ij}A_+^iA_+^j)(\lambda_\alpha^*+m_{\alpha k}^*A_+^k+g_{\alpha,kl}^*A_+^kA_+^l)\end{aligned} \tag{2.13}$$

for the potential.

The potential is positive-definite and $V=0$ corresponds to F_+^i and F_-^α equal to zero and gives absolute minima which are supersymmetric. So to break supersymmetry it is essential that $V\neq 0$ always.

III. POTENTIAL MINIMA IN THE TREE APPROXIMATION

For the sake of compactness we shall introduce the following generalized notations:

We shall take indices like (a,b) to run from 1 to $N+M$, indices like (i,j) to run from 1 to N, and indices like (α,β) to run from 1 to M. Then we define

$$\Phi_a\equiv\begin{cases}\Phi_+^i & \text{for } a=i\\ \Phi_-^{\dagger\alpha} & \text{for } a=\alpha+N\,.\end{cases} \tag{3.1}$$

In particular, for the fermion component

$$\psi_a\equiv\begin{cases}\psi_+^i & \text{for } a=i\\ \psi_-^{c\alpha} & \text{for } a=\alpha+N\,.\end{cases} \tag{3.2}$$

We introduce the mass matrix M by

$$M=\begin{array}{c}\begin{matrix}N\times N & N\times M\end{matrix}\\ \begin{pmatrix}0 & m^T\\ m & 0\end{pmatrix}\\ \begin{matrix}M\times N & M\times M\end{matrix}\end{array}\,, \tag{3.3}$$

which is symmetric. We can define a totally symmetric coupling constant by

$$g_{abc}=g_{a,bc}+g_{b,ac}+g_{c,ab}\,, \tag{3.4}$$

with

$$g_{a,bc}\equiv\begin{cases}g_{\alpha,ij} & \text{for } a=\alpha+N\,,\ b=i,\ c=j\\ 0 & \text{otherwise}\,.\end{cases} \tag{3.5}$$

Lastly, we define

$$\lambda_a\equiv\begin{cases}0 & \text{for } a=i\\ \lambda_\alpha & \text{for } a=\alpha+N\,.\end{cases} \tag{3.6}$$

With these generalized notations the Lagrangian looks more symmetric:

$$\mathcal{L}=\tfrac{1}{8}(\bar{D}D)^2(\Phi_a^\dagger\Phi_a) -\tfrac{1}{2}\bar{D}D(\lambda_a\Phi_a+\tfrac{1}{2}M_{ab}\Phi_a\Phi_b+\tfrac{1}{3}g_{abc}\Phi_a\Phi_b\Phi_c+\mathrm{H.c.})\,. \tag{3.7}$$

Equations (2.8) and (2.9) are combined into one:

$$-F_a^*=\lambda_a+M_{ab}A_b+g_{abc}A_bA_c\,, \tag{3.8}$$

and the potential takes the form

$$V=F_a^*F_a =\sum_a|\lambda_a+M_{ab}A_b+g_{abc}A_bA_c|^2\,. \tag{3.9}$$

We shift the fields by

$$A_a\to A_a+z_a\,, \tag{3.10}$$

where

$$z_a\equiv\begin{cases}\kappa_i & \text{for } a=i\\ y_\alpha^* & \text{for } a=\alpha+N\,.\end{cases} \tag{3.11}$$

This does not alter the form of Eq. (3.8). However, we have to make the following replacements:

$$\lambda_a\to\lambda_a(z)=\lambda_a+M_{ab}z_b+g_{abc}z_bz_c\,, \tag{3.12}$$

$$M_{ab}\to M_{ab}(z)=M_{ab}+2g_{abc}z_c\,. \tag{3.13}$$

O'Raifeartaigh[4] has discussed in detail the conditions for minima for potentials of the form of Eq. (3.9). We simply quote that for minima it is necessary that

$$M_{ab}(z)\lambda_b^*(z)=0\,. \tag{3.14}$$

This may be decomposed into the two conditions

$$2g_{\alpha,ij}y_\alpha^* m_{\beta j}^*(\kappa)y_\beta+m_{\alpha i}(\kappa)\lambda_\alpha^*(\kappa)=0\,, \tag{3.15}$$

$$m_{\alpha j}(\kappa)m_{\beta j}^*(\kappa)y_\beta=0\,, \tag{3.16}$$

where

$$m_{\alpha i}(\kappa)=m_{\alpha i}+2g_{\alpha,ij}\kappa_j\,, \tag{3.17}$$

$$\lambda_\alpha(\kappa)=\lambda_\alpha+m_{\alpha i}\kappa_i+g_{\alpha,ij}\kappa_i\kappa_j\,, \tag{3.18}$$

leading to

$$m_{\alpha i}(\kappa)\lambda_\alpha^*(\kappa)=0\,, \tag{3.19}$$

$$m_{\alpha i}(\kappa)y_\alpha^*=0\,. \tag{3.20}$$

For minima we also require the positivity of the boson (mass)^2 matrix

$$\mathfrak{M}^2(z)=\begin{pmatrix} M^\dagger(z)M(z) & S^\dagger(z)\\ S(z) & M(z)M^\dagger(z)\end{pmatrix}, \tag{3.21}$$

where

$$S_{ab}(z)=2g_{abc}\lambda_c^*(z)\,. \tag{3.22}$$

Using Eqs. (3.12) and (3.20) we find

$$\lambda_a(z)=\begin{cases}0 & \text{for } a=i\\ \lambda_\alpha(\kappa) & \text{for } a=\alpha+N\,.\end{cases} \tag{3.23}$$

Our claim that the potential at the minima, $V_{\min}$, does not depend on y_α is obvious from Eq. (3.23),

$$V_{\min}=\sum_a |\lambda_a(z)|^2 = \sum_\alpha |\lambda_\alpha(\kappa)|^2\,. \tag{3.24}$$

Note that this is true for both supersymmetric and supersymmetry-breaking minima. Of course, y_α are constrained to satisfy Eq. (3.20). Depending on the relative values of M and N and the structure of $m_{\alpha i}$, some of the y_α may be left totally arbitrary.

IV. ONE-LOOP CORRECTIONS

In the last section we have seen that the difermion field vacuum expectation values y_α play a passive role in determining the potential minima. In this section, we calculate the effects of one-loop corrections to the tree-approximation results of the last section.

The masses of the bosons and the fermions would depend on the values of κ_i and y_α satisfying Eqs. (3.19) and (3.20). We shall treat the y_α left arbitrary by the latter equation as the independent variables on which the masses depend. The tree-approximation potential at the minima has no dependence on these variables. We diagonalize the mass matrices and confine our attentions to the masses that are nonzero.

For our purpose the effective potential in the following form[7] is all that is needed:

$$V_{\text{eff}}(y)=V_{\min}+\frac{\hbar}{64\pi^2}\left[\sum_{i=1}^{N_B}M_i^4(y)\ln\frac{M_i^2(y)}{M_0^2}-\sum_{i=1}^{N_F}N_i^4(y)\ln\frac{N_i^2(y)}{M_0^2}\right], \tag{4.1}$$

where y stands for the independent y_α. See the Appendix for a brief derivation of the above equation. N_B is the number of boson masses counting fields A_a as well as the conjugate fields [these (mass)^2's are denoted by M_i^2], while N_F is the number of fermion masses counting the right-handed fields, ψ_a, and their left-handed conjugates [the fermion (mass)^2's are denoted by N_i^2].

For supersymmetric minima $N_B=N_F$ and $M_i^2=N_i^2$, so we find from Eq. (4.1) that there are no one-loop corrections to the tree-approximation results in this case. While for supersymmetry breaking N_B may not be equal to N_F and the masses M_i^2 are different from N_i^2, the effective potential would depend on y_α. We then search for minima of V_{eff} in these variables.

Now we discuss three different models, two with $N=M$ and one with $M>N$.

(i) O'Raifeartaigh's discrete model.[4] This is an example with $M=2$ and $N=1$, so that $m_{\alpha i}\to m_\alpha$, $g_{\alpha,ij}\to g_\alpha$. Supersymmetry is broken in this case unless λ_α, m_α, and g_α are related in some special ways, such as $m_1/g_1=m_2/g_2$; $\lambda_1/g_1=\lambda_2/g_2$. Instead of discussing the general case, we take the model discussed by O'Raifeartaigh. We set $m_1=0$, $m_2=m$, $g_1=g$, $g_2=0$, $\lambda_1=\lambda$, and $\lambda_2=0$ by imposing the discrete symmetry, $\Phi_-^1\to\Phi_-^1$, $\Phi_-^2\to-\Phi_-^2$, $\Phi_+\to-\Phi_+$. Fields are shifted by $A_+\to A_++\kappa$, $A_-^\alpha\to A_-^\alpha+y_\alpha$. We have

$$m_1(\kappa)=2g\kappa\,,\quad \lambda_1(\kappa)=\lambda+g\kappa^2\,,$$

$$m_2(\kappa)=m\,,\quad \lambda_2(\kappa)=m\kappa\,.$$

It is clear that both $\lambda_1(\kappa)$ and $\lambda_2(\kappa)$ cannot vanish, so supersymmetry is broken. Without loss of generality we take m, g, and λ to be real. With $-2g\lambda>m^2$ we have the deepest minima for

$$\kappa^2=\frac{-2g\lambda-m^2}{2g^2}\,. \tag{4.3}$$

The mass matrix $M(z)$ is 3×3,

$$M(z)\begin{pmatrix} u & 2g\kappa & m \\ 2g\kappa & 0 & 0 \\ m & 0 & 0 \end{pmatrix}, \tag{4.4}$$

where $u=2gy_1^*$. Equation (3.20) takes the form

$$my_2+2g\kappa y_1=0\,. \tag{4.5}$$

We shall treat u as the independent variable. Using Eq. (4.3) we have $2g(\lambda+g\kappa^2)=-m^2$. Hence the matrix $S(z)$ is

$$S(z)=\begin{pmatrix} -m^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4.6}$$

The fermion fields ψ_α have the (mass)^2 matrix $M^\dagger(z)M(z)$. While the boson (mass)^2 matrix is given by Eq. (3.21). Diagonalizing these we find four massive bosons with masses

$$\begin{aligned} M_1^2&=m_1^2-\tfrac{1}{2}m^2+\tfrac{1}{2}|u|^2 \\ &\quad+\tfrac{1}{2}[m^4+|u|^2(4m_1^2-2m^2+|u|^2)]^{1/2}\,, \\ M_2^2&=m_1^2-\tfrac{1}{2}m^2+\tfrac{1}{2}|u|^2 \\ &\quad-\tfrac{1}{2}[m^4+|u|^2(4m_1^2-2m^2+|u|^2)]^{1/2}\,, \\ M_3^2&=m_1^2+\tfrac{1}{2}m^2+\tfrac{1}{2}|u|^2 \\ &\quad+\tfrac{1}{2}[m^4+|u|^2(4m_1^2+2m^2+|u|^2)]^{1/2}\,, \\ M_4^2&=m_1^2+\tfrac{1}{2}m^2+\tfrac{1}{2}|u|^2 \\ &\quad-\tfrac{1}{2}[m^4+|u|^2(4m_1^2+2m^2+|u|^2)]^{1/2}\,, \end{aligned} \tag{4.7}$$

where

$$\begin{aligned} m_1^2&=4g^2\kappa^2+m^2 \\ &=-4g\lambda-m^2>m^2\,. \end{aligned} \tag{4.8}$$

The two left-handed and the two right-handed fermions have the masses

$$\begin{aligned} N_1^2&=m_1^2+\tfrac{1}{2}|u|^2 \\ &\quad+\tfrac{1}{2}[|u|^2(4m_1^2+|u|^2)]^{1/2}\,, \\ N_2^2&=m_1^2+\tfrac{1}{2}|u|^2 \\ &\quad-\tfrac{1}{2}[|u|^2(4m_1^2+|u|^2)]^{1/2}\,, \end{aligned} \tag{4.9}$$

each mass occurring once for left-handed and once for right-handed fields.

The effective potential in one-loop approximation is

$$V_{\rm eff}(u)=V_{\rm min}+\frac{\hbar}{64\pi^2}\left[\sum_{i=1}^4 M_i^4(\kappa)\ln\frac{M_i^2(u)}{M_0^2} -2\sum_{i=1,2}N_i^4(u)\ln\frac{N_i^2(u)}{M_0^2}\right]. \tag{4.10}$$

Differentiating we obtain

$$\begin{aligned} \frac{d}{d|u|}V_{\rm eff}(u)=\frac{\hbar}{16\pi^2}|u|\Bigg[&\frac{1}{[m^4+|u|^2(4m_1^2-2m^2+|u|^2)]^{1/2}}\left(M_1^4\ln\frac{M_1^2}{M_0^2}-M_2^4\ln\frac{M_2^2}{M_0^2}\right) \\ &+\frac{1}{[m^4+|u|^2(4m_1^2+2m^2+|u|^2)]^{1/2}}\left(M_3^4\ln\frac{M_3^2}{M_0^2}-M_4^4\ln\frac{M_4^2}{M_0^2}\right) \\ &-\frac{2}{[|u|^2(4m_1^2+|u|^2)]^{1/2}}\left(N_1^4\ln\frac{N_1^2}{M_0^2}-N_2^4\ln\frac{N_2^2}{M_0^2}\right)\Bigg]\,, \end{aligned} \tag{4.11}$$

which vanishes at $|u|=0$, and

$$\frac{d^2}{d|u|^2}V_{\rm eff}(u)\bigg|_{u=0}=\frac{(m_1^2+m^2)^2}{m^2}\ln\left(1+\frac{m^2}{m_1^2}\right)-\frac{(m_1^2-m^2)^2}{m^2}\ln\left(1-\frac{m^2}{m_1^2}\right)-2m_1^2>0\,. \tag{4.12}$$

Thus the point $u=0$ is a minimum.[8] As $V_{\rm eff}(u)$ is a monotonic function of $|u|$ increasing logarithmically, this is the only minimum.

(ii) Fayet's SU(2)×U(1) model.[3] This is an example with $M=N=4$. Isospin I and hypercharge Y for the chiral fields are chosen as

$$\begin{aligned} &\Phi_{0-}^{(0)}\sim(0,0)\,,\quad \vec{\Phi}_{1-}\sim(1,0)\,, \\ &\Phi_{1+}\sim(\tfrac{1}{2},1)\,,\quad \Phi_{2+}\sim(\tfrac{1}{2},-1)\,, \end{aligned} \tag{4.13}$$

where the use of the parentheses indicates (I,Y) contents. We have

$$\begin{aligned} &\vec{\Phi}_{1-}=\begin{pmatrix} \Phi_{1-}^{(0)} & \sqrt{2}\,\Phi_{1-}^{(+)} \\ \sqrt{2}\,\Phi_{1-}^{(-)} & -\Phi_{1-}^{(0)} \end{pmatrix}, \\ &\Phi_{1+}=\begin{pmatrix} \Phi_{1+}^{(+)} \\ \Phi_{1+}^{(0)} \end{pmatrix},\quad \Phi_{2+}=\begin{pmatrix} \Phi_{2+}^{(0)} \\ \Phi_{2+}^{(-)} \end{pmatrix}. \end{aligned} \tag{4.14}$$

The superscripts indicate charges. In our notations the fields are put into 4-dimensional columns with the following index assignments:

$$\Phi_-^\alpha = \begin{pmatrix} \Phi_{1-}^{(+)} \\ \Phi_{1-}^{(-)} \\ \Phi_{1-}^{(0)} \\ \bar{\Phi}_{0-}^{(0)} \end{pmatrix}, \quad \Phi_+^i = \begin{pmatrix} \Phi_{1+}^{(+)} \\ \Phi_{2+}^{(-)} \\ \Phi_{1+}^{(0)} \\ \Phi_{2+}^{(0)} \end{pmatrix}. \tag{4.15}$$

Thus $\alpha, i = 1, 2$ correspond to the charged sector and $\alpha, i = 3, 4$ to the neutral sector.

The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{2}\bar{D}D(\lambda\Phi_{0-}^{\dagger(0)} + ig\Phi_{1+}^T\tau_2\vec{\Phi}_{1-}^\dagger\Phi_{2+} + ih\Phi_{0-}^{\dagger(0)}\Phi_{1+}^T\tau_2\Phi_{2+} + \text{H.c.}) + \text{kinetic energy part}. \tag{4.16}$$

We are looking for charge-conserving minima and hence we put κ_2, κ_2, y_1, and y_2 equal to zero. Then the mass matrix $m_{\alpha i}(\kappa)$ is

$$m(\kappa) = \begin{pmatrix} \sqrt{2}g\kappa_4 & 0 & 0 & 0 \\ 0 & -\sqrt{2}g\kappa_3 & 0 & 0 \\ 0 & 0 & -g\kappa_4 & -g\kappa_3 \\ 0 & 0 & -h\kappa_4 & -h\kappa_3 \end{pmatrix}, \tag{4.17}$$

while

$$\lambda(\kappa) = \begin{pmatrix} 0 \\ 0 \\ -g\kappa_3\kappa_4 \\ \lambda - h\kappa_3\kappa_4 \end{pmatrix}. \tag{4.18}$$

The deepest minima is given by

$$\kappa_3\kappa_4 = \frac{\lambda h}{g^2 + h^2}. \tag{4.19}$$

y_3 and y_4 are constrained by

$$gy_3 + hy_4 = 0. \tag{4.20}$$

We shall take as the independent variable

$$u = -gy_3^* + hy_4^* = 2hy_4^*. \tag{4.21}$$

Because of charge conservation we can treat the charged and the neutral sectors separately. There is no supersymmetry breaking in the neutral sector, so we shall ignore it. To simplify the diagonalization of the mass matrices (the boson mass matrix is 8×8) we shall put $\kappa_3 = \kappa_4$. By doing this we shall not lose any general features of the problem.

Out of eight bosons in the charged sector two are massless (this is true even for $\kappa_3 \neq \kappa_4$), and six have the following masses:

$$\begin{aligned} M_1^2 &= |u|^2 + a^2, \\ M_2^2 &= \tfrac{1}{2}[|u|^2 + 3a^2 + (|u|^4 + 6a^2|u|^2 + a^4)^{1/2}], \\ M_3^2 &= \tfrac{1}{2}[|u|^2 + 3a^2 - (|u|^4 + 6a^2|u|^2 + a^4)^{1/2}], \\ a &= |\sqrt{2}g\kappa_3| = |\sqrt{2}g\kappa_4| \end{aligned} \tag{4.22}$$

each mass occurring twice.

There are two left-handed and two right-handed fermions for each of the masses

$$\begin{aligned} N_1^2 &= \tfrac{1}{2}\{|u|^2 + 2a^2 + [|u|^2(4a^2 + |u|^2)]^{1/2}\}, \\ N_2^2 &= \tfrac{1}{2}\{|u|^2 + 2a^2 - [|u|^2(4a^2 + |u|^2)]^{1/2}\}. \end{aligned} \tag{4.23}$$

The effective potential in the one-loop approximation is

$$V_{\text{eff}}(u) = V_{\min} + \frac{\hbar}{64\pi^2}\left[2\sum_{i=1}^{3} M_i^4(u)\ln\frac{M_i^2(u)}{M_0^2} - 4\sum_{i=1,2} N_i^4(u)\ln\frac{N_i^2(u)}{M_0^2}\right], \tag{4.24}$$

and

$$\frac{d}{d|u|}V_{\text{eff}}(u) = \frac{\hbar}{16\pi^2}|u|\left[2(|u|^2 + a^2)\ln\frac{|u|^2 + a^2}{M_0^2} + \frac{2}{(|u|^4 + 6a^2|u|^2 + a^4)^{1/2}}\left(M_2^4\ln\frac{M_2^2}{M_0^2} - M_3^4\ln\frac{M_3^2}{M_0^2}\right) - \frac{4}{[|u|^2(4a^2 + |u|^2)]^{1/2}}\left(N_1^4\ln\frac{N_1^2}{M_0^2} - N_2^4\ln\frac{N_2^2}{M_0^2}\right)\right]. \tag{4.25}$$

We have

$$\begin{aligned} \frac{d}{d|u|}V_{\text{eff}}\bigg|_{u=0} &= 0, \\ \frac{d^2}{d|u|^2}V_{\text{eff}}\bigg|_{u=0} &= \frac{\hbar}{16\pi^2}(8a^2\ln 2 - 4a^2) > 0. \end{aligned} \tag{4.26}$$

Thus the point $u = 0$ is the minimum.

(iii) Global SU(3) model. This is an example with $M = 9$ and $N = 8$. We put the negative chirality fields in an SU(3) octet and a singlet, Φ_-^α, $\alpha = 1, \ldots, 8$, Φ_-^9. The eight positive chirality fields are put in an octet, Φ_+^i, $i = 1, \ldots, 8$. The Lagrangian is

$$\mathcal{L} = \tfrac{1}{8}(\bar{D}D)^2(\Phi_-^{\dagger\alpha}\Phi_-^\alpha + \Phi_-^{\dagger 9}\Phi_-^9 + \Phi_+^{\dagger i}\Phi_+^i) - \tfrac{1}{2}\bar{D}D(\lambda\Phi_-^{\dagger 9} + m\Phi_-^{\dagger\alpha}\Phi_+^i\delta_{\alpha i} + g_0\Phi_-^{\dagger 9}\Phi_+^i\Phi_+^i + gd_{\alpha ij}\Phi_-^{\dagger\alpha}\Phi_+^i\Phi_+^j + \text{H.c.}), \tag{4.27}$$

where the sums over α and i run from 1 to 8 and $d_{\alpha ij}$ are the symmetric structure constants of SU(3).

We shall look for isospin- and hypercharge-conserving minima. So only $\kappa_8 \equiv \kappa$ and y_α for $\alpha = 8, 9$ are nonzero. Then the mass matrix $m_{\alpha i}(\kappa)$, $\alpha = 1, \ldots, 9$, $i = 1, \ldots, 8$, is

$$m(\kappa) = \begin{bmatrix} m_1(\kappa) I_3 & 0 & 0 \\ 0 & m_2(\kappa) I_4 & 0 \\ 0 & 0 & m_3(\kappa) \\ 0 & 0 & m_4(\kappa) \end{bmatrix}, \tag{4.28}$$

where I_3 and I_4 are 3×3 and 4×4 unit matrices respectively, and

$$\begin{aligned} m_1(\kappa) &= m + \frac{2g}{\sqrt{3}}\kappa, \\ m_2(\kappa) &= m - \frac{g}{\sqrt{3}}\kappa, \\ m_3(\kappa) &= m - \frac{2g}{\sqrt{3}}\kappa, \\ m_4(\kappa) &= 2g_0\kappa. \end{aligned} \tag{4.29}$$

We have

$$\lambda(\kappa) = \begin{bmatrix} 0 \\ 0 \\ \left(m - \frac{g}{\sqrt{3}}\kappa\right)\kappa \\ \lambda + g_0\kappa^2 \end{bmatrix}, \tag{4.30}$$

where the first zero stands for a 3-dimensional null column and the second zero for a 4-dimensional null column.

There is only one condition arising from Eq. (3.19),

$$2g_0\lambda\kappa^* + m^2\kappa - \frac{mg}{\sqrt{3}}(\kappa^2 + 2|\kappa|^2) + (2g_0^2 + \tfrac{2}{3}g^2)|\kappa|^2\kappa = 0, \tag{4.31}$$

with real solutions

$$\kappa = \frac{\sqrt{3}\, mg \pm [3m^2g^2 - 4(2g_0^2 + \frac{2}{3}g^2)(2g_0\lambda + m^2)]^{1/2}}{2(2g_0^2 + \frac{2}{3}g^2)}, \tag{4.32}$$

where $-2g_0\lambda > m^2$. If we take $mg > 0$, the upper solution would be deeper; we shall use this.

y_8 and y_9 are constrained by

$$2g_0 y_9\kappa + \left(m - \frac{2g}{\sqrt{3}}\kappa\right) y_8 = 0. \tag{4.33}$$

Owing to conservation of isospin and hypercharge the three sectors ($\alpha, i = 1, 2, 3$), ($\alpha, i = 4, 5, 6, 7$), and ($\alpha = 8, 9$; $i = 8$) may be treated separately.

($\alpha, i = 1, 2, 3$) sector. In this sector there are three bosons for each of the following masses:

$$\begin{aligned} M_1^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 + v_1 + X_{1+}), \\ M_2^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 + v_1 - X_{1+}), \\ M_3^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 - v_1 + X_{1-}), \\ M_4^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 - v_1 - X_{1-}), \end{aligned} \tag{4.34}$$

where $m_1 \equiv m_1(\kappa)$ and

$$\begin{aligned} X_{1\pm} &= [|u_1|^4 + 2|u_1|^2(2m_1^2 \pm v_1) + v_1^2]^{1/2}, \\ u_1 &= \left(-\frac{m}{\kappa} + \frac{4g}{\sqrt{3}}\right) y_8^*, \\ v_1 &= 2g_0(\lambda + g_0\kappa^2) + \frac{2g}{\sqrt{3}}\left(m - \frac{g}{\sqrt{3}}\kappa\right)\kappa. \end{aligned} \tag{4.35}$$

The condition for no tachyon for any values of $|u_1|$, $2m_1^2 > |v_1|$, is satisfied.

There are three left-handed and three right-handed fermions for each of the following masses:

$$\begin{aligned} N_1^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 + Y_1), \\ N_2^2 &= \tfrac{1}{2}(|u_1|^2 + 2m_1^2 - Y_1), \end{aligned} \tag{4.36}$$

with

$$Y_1 = [|u_1|^2(4m_1^2 + |u_1|^2)]^{1/2}. \tag{4.37}$$

($\alpha, i = 4, 5, 6, 7$) sector. Here we have four bosons for each of the following masses:

$$\begin{aligned} M_5^2 &= |u_2|^2 + m_2^2, \\ M_6^2 &= \tfrac{1}{2}(|u_2|^2 + 3m_2^2 + X_2), \\ M_7^2 &= \tfrac{1}{2}(|u_2|^2 + 3m_2^2 - X_2), \end{aligned} \tag{4.38}$$

where

$$\begin{aligned} u_2 &= \left(-\frac{m}{\kappa} + \frac{g}{\sqrt{3}}\right) y_8^*, \\ X_2 &= (|u_2|^4 + 6m_2^2|u_2|^2 + m_2^4)^{1/2}, \quad m_2 \equiv m_2(\kappa). \end{aligned} \tag{4.39}$$

In this sector there are four massless bosons.

We have four left-handed and four right-handed fermions for each of the following masses:

$$\begin{aligned} N_3^2 &= \tfrac{1}{2}(|u_2|^2 + 2m_2^2 + Y_2), \\ N_4^2 &= \tfrac{1}{2}(|u_2|^2 + 2m_2^2 - Y_2), \end{aligned} \tag{4.40}$$

where

$$Y_2 = [|u_2|^2(4m_2^2 + |u_2|^2)]^{1/2}. \tag{4.41}$$

($\alpha = 8, 9$, $i = 8$) sector. The four boson masses are

$$M_8^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}+v_3+X_{3+})\,,\qquad M_9^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}+v_3-X_{3+})\,,$$
$$M_{10}^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}-v_3+X_{3-})\,,\qquad M_{11}^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}-v_3-X_{3-})\,, \tag{4.42}$$

where

$$u_3=-\frac{m}{\kappa}\,y_8^*\,,\qquad v_3=2g_0(\lambda+g_0\kappa^2)-\frac{2g}{\sqrt{3}}\left(m-\frac{g}{\sqrt{3}}\,\kappa\right)\kappa\,, \tag{4.43}$$

$$X_{3\pm}=[|u_3|^4+2|u_3|^2(2m_3^{\,2}+2m_4^{\,2}\pm v_3)+v_3^{\,2}]^{1/2}\,,\qquad m_3\equiv m_3(\kappa),\qquad m_4\equiv m_4(\kappa)\,.$$

We have one left-handed and one right-handed fermion for each of the following masses:

$$N_5^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}+Y_3)\,,\qquad N_6^{\,2}=\tfrac{1}{2}(|u_3|^2+2m_3^{\,2}+2m_4^{\,2}-Y_3)\,, \tag{4.44}$$

with

$$Y_3=[|u_3|^2(4m_3^{\,2}+4m_4^{\,2}+|u_3|^2)]^{1/2}\,. \tag{4.45}$$

The effective potential is

$$\begin{aligned} V_{\text{eff}}=V_{\min}+\frac{\hbar}{64\pi^2}\Bigg\{&3\left(\sum_{i=1}^{4}M_i^{\,4}\ln\frac{M_i^{\,2}}{M_0^{\,2}}-2\sum_{i=1,2}N_i^{\,4}\ln\frac{N_i^{\,2}}{M_0^{\,2}}\right)\\ &+4\left[(|u_2|^2+m_2^{\,2})\ln\frac{|u_2|^2+m_2^{\,2}}{M_0^{\,2}}+\sum_{i=6,7}M_i^{\,4}\ln\frac{M_1^{\,2}}{M_0^{\,2}}-2\sum_{i=3,4}N_i^{\,4}\ln\frac{N_i^{\,2}}{M_0^{\,2}}\right]\\ &+\left(\sum_{i=8}^{11}M_i^{\,4}\ln\frac{M_i^{\,2}}{M_0^{\,2}}-2\sum_{i=5,6}N_i^{\,4}\ln\frac{N_i^{\,2}}{M_0^{\,2}}\right)\Bigg\}\,. \end{aligned} \tag{4.46}$$

Then

$$\begin{aligned} \frac{d}{d|y_8|}V_{\text{eff}}=\frac{\hbar}{16\pi^2}|y_8|\Bigg\{&3\left(-\frac{m}{\kappa}+\frac{4g}{\sqrt{3}}\right)^2\left[\frac{1}{X_{1+}}\left(M_1^{\,4}\ln\frac{M_1^{\,2}}{M_0^{\,2}}-M_2^{\,4}\ln\frac{M_2^{\,2}}{M_0^{\,2}}\right)+\frac{1}{X_{1-}}\left(M_3^{\,4}\ln\frac{M_3^{\,2}}{M_0^{\,2}}-M_4^{\,4}\ln\frac{M_4^{\,2}}{M_0^{\,2}}\right)\right.\\ &\left.\qquad-\frac{2}{Y_1}\left(N_1^{\,4}\ln\frac{N_1^{\,2}}{M_0^{\,2}}-N_2^{\,4}\ln\frac{N_1^{\,2}}{M_0^{\,2}}\right)\right]\\ &+4\left(-\frac{m}{\kappa}+\frac{g}{\sqrt{3}}\right)^2\left[(|u_2|^2+m_2^{\,2})\ln\frac{|u_2|^2+m_2^{\,2}}{M_0^{\,2}}+\frac{1}{X_2}\left(M_6^{\,4}\ln\frac{M_6^{\,2}}{M_0^{\,2}}-M_7^{\,4}\ln\frac{M_7^{\,2}}{M_0^{\,2}}\right)\right.\\ &\left.\qquad-\frac{2}{Y_2}\left(N_3^{\,4}\ln\frac{N_3^{\,2}}{M_0^{\,2}}-N_4^{\,4}\ln\frac{N_4^{\,2}}{M_0^{\,2}}\right)\right]\\ &+\left(\frac{m}{\kappa}\right)^2\left[\frac{1}{X_{3+}}\left(M_8^{\,4}\ln\frac{M_8^{\,2}}{M_0^{\,2}}-M_9^{\,4}\ln\frac{M_9^{\,2}}{M_0^{\,2}}\right)+\frac{1}{X_{3-}}\left(M_{10}^{\,4}\ln\frac{M_{10}^{\,2}}{M_0^{\,2}}-M_{11}^{\,4}\ln\frac{M_{11}^{\,2}}{M_0^{\,2}}\right)\right.\\ &\left.\qquad-\frac{2}{Y_3}\left(N_5^{\,4}\ln\frac{N_5^{\,2}}{M_0^{\,2}}-N_6^{\,4}\ln\frac{N_6^{\,2}}{M_0^{\,2}}\right)\right]\Bigg\}\,. \end{aligned} \tag{4.47}$$

The terms in the first and the third sets of square brackets of the above two equations are exactly the same type as Eq. (4.10) and Eq. (4.11), while the terms in the second set of square brackets are the same as Eq. (4.24) and Eq. (4.25). Hence we conclude that $(d/d|y_8|)V_{\text{eff}}=0$ at $y_8=0$ and $(d^2/d|y_8|^2)V_{\text{eff}}|_{y_8=0}>0$. Thus the only minimum is at $y_8=y_9=0$.

The above three models show that the arbitrariness of tree approximations for the vacuum expectation values of the difermion fields is removed by the one-loop corrections in the case of supersymmetry breaking. The one-loop corrections require that the potential minima occur for zero vacuum expectation values of the difermion fields.

The above models have all the general features. Hence we make the conjecture that when supersymmetry is broken spontaneously it cannot be accompanied by spontaneous breakdown of fermion-number conservation. For supersymmetric minima there are no one-loop corrections to the effective potential and so the results of tree approximations remain unchanged.

ACKNOWLEDGMENTS

The author would like to thank Professor A. Salam, The International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He is grateful to Professor Salam for suggesting the investigation of the problem and to Dr. Strathdee for many helpful suggestions.

APPENDIX

If we do not eliminate F_a's, then the potential V is

$$V=-F_a^* F_a + [F_a(\lambda_a + M_{ab}A_b + g_{abc}A_bA_c) + \mathrm{H.c.}]\,. \tag{A1}$$

Now let us shift the fields by

$$A_a \to A_a + z_a, \quad F_a \to F_a + \omega_a\,. \tag{A2}$$

In tree approximation condition Eq. (3.14) is replaced by

$$M_{ab}(z)\omega_b = 0\,, \tag{A3}$$

$$\omega_a^* = -\lambda_a(z)\,. \tag{A4}$$

Using Eq. (A4) in Eq. (A3) we recover Eq. (3.14).

The quadratic part of the Lagrangian may be written as

$$\mathcal{L}_{\text{quad}} = \tfrac{1}{2}(F_a^* F_a A_a^* A_a)\begin{bmatrix} \delta_{ab} & 0 & 0 & M_{ab}^*(z) \\ 0 & \delta_{ab} & M_{ab}(z) & 0 \\ 0 & M_{ab}^*(z) & -\Box\,\delta_{ab} & f_{ab}^* \\ M_{ab}(z) & 0 & f_{ab} & -\Box\,\delta_{ab} \end{bmatrix}\begin{bmatrix} F_b \\ F_b^* \\ A_b \\ A_b^* \end{bmatrix}$$

$$+\tfrac{1}{2}(\psi_{ai}\,\psi_{ai}^c)\begin{pmatrix} -i(\sigma_2)_{ij}M_{ab}(z) & i(\sigma_2\sigma_\mu\hat{k}^\mu)_{ij}\delta_{ab} \\ -i(\sigma_2\sigma_\mu k^\mu)_{ij}\delta_{ab} & i(\sigma_2)_{ij}M_{ab}^*(z)\end{pmatrix}\begin{pmatrix} \psi_{bj} \\ \psi_{bj}^c \end{pmatrix}, \tag{A5}$$

where $\hat{k}=(k_0-\vec{\mathrm{k}})$, and we have introduced two component notations for the right-handed and left-handed fields ψ_a and ψ_a and ψ_a^c. We have

$$f_{ab} = 2g_{abc}\,\omega_c\,. \tag{A6}$$

One-loop contributions to the effective potential are[9]

$$V_1 = -\frac{i}{2}\hbar\int\frac{d^2k}{(2\pi)^4}\left[\ln\begin{vmatrix} 1 & 0 & 0 & M^\dagger(z) \\ 0 & 1 & M(z) & 0 \\ 0 & M^\dagger(z) & k^2 & f^\dagger \\ M & 0 & f & k^2\end{vmatrix} - \frac{1}{2}\left(\ln\begin{vmatrix} -i\sigma_2 M(z) & i\sigma_2\sigma_\mu\hat{k}^\mu \\ -i\sigma_2\sigma_\mu k^\mu & i\sigma_2 M^\dagger(z)\end{vmatrix} + \ln\begin{vmatrix} i\sigma_2 M^\dagger(z) & -i\sigma_2\sigma_\mu\hat{k}^\mu \\ i\sigma_2\sigma_\mu k^\mu & -i\sigma_2 M(z)\end{vmatrix}\right)\right]$$

$$= \frac{\hbar}{64\pi^2}\left(\mathrm{Tr}X^4\ln\frac{X^2}{M_0^{\,2}} - \mathrm{Tr}Y^4\ln\frac{Y^2}{M_0^{\,2}}\right) + \mathrm{Tr}(f^\dagger f + ff^\dagger)\left\{\frac{i}{4}\,\frac{\hbar}{64\pi^2}\left[\int\frac{d^4k}{k^4} + \frac{i}{2}\pi^2\ln(0+\tfrac{3}{2})\right] + \frac{\hbar}{64\pi^2}\ln M_0^{\,2}\right\}$$

($M_0^{\,2}$ is the normalization point), where

$$X^2 = \begin{pmatrix} M^\dagger M & -f^\dagger \\ -f & MM^\dagger \end{pmatrix}, \quad Y^2 = \begin{pmatrix} M^\dagger M & 0 \\ 0 & MM^\dagger\end{pmatrix} \tag{A8}$$

and

$$\mathrm{Tr}(X^4 - Y^4) = \mathrm{Tr}(f^\dagger f + ff^\dagger)\,. \tag{A9}$$

Observe that only the term proportional to $\mathrm{Tr}(f^\dagger f + ff^\dagger)$ is infinite and is taken care of by wave-function renormalization of the unshifted theory (hence it is supersymmetric).[10] Since we are interested in retaining only terms up to order of $\hbar$ we may use the tree-approximation result, $f=-S$, to find that X^2 and Y^2 are the (mass)^2 matrices in the tree approximation for the bosons and the fermions, respectively. Diagonalization leads to Eq. (4.1).

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