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Asymptotic field theory and non-Abelian gauge theories

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The formalism of Pugh's asymptotic field theory is applied here to the problem of quantizing non-Abelian gauge fields. The advantage of this formalism is that no ultraviolet divergences ever appear when one performs perturbation theory calculations for S-matrix elements. By fixing the form of the gauge transformation for the interpolating field, Ward identities for the S matrix can be derived. These can be used to establish that gauge invariance can be maintained to all orders in perturbation theory.

I. INTRODUCTION

In recent years, the problem of quantizing non-Abelian gauge fields[1,2] has received much attention. Feynman rules have been developed for both the Yang-Mills interaction and the gravitational interaction, and a suitable procedure, dimensional regularization, has been developed to handle the resulting divergences. It has also been established that the Yang-Mills interaction is renormalizable.

In this note, the techniques of asymptotic field theory[3-7] are used to quantize these non-Abelian gauge fields. The distinctive feature of this procedure is that no ultraviolet divergences ever appear, as only renormalized fields are introduced at the outset. All the standard results for non-Abelian gauge fields are recovered using this alternate method of quantization.

The infrared problem for non-Abelian gauge fields is not resolved in this note. The divergences that occur due to the appearance of massless gauge fields are regulated by the insertion of a mass parameter as a cutoff. This procedure is certainly not satisfactory, and indeed it may turn out that the long-range nature of the force associated with massless particles may not allow one to even consider asymptotically free fields. These problems will be ignored in this note, however, and we will use the notion of asymptotically free fields as a necessary idealization. In any case, asymptotic field theory can be used to treat the problem of spontaneously broken gauge symmetry,[8] and in this case no infrared divergences occur.

In the second section, the Yang-Mills field is quantized using asymptotic field theory. By specifying the form of the gauge transformation for the interpolating field, the Ward identities are derived in a simple fashion. A similar program is carried out in Sec. III for the gravitational interaction. The second-order propagator for the Yang-Mills interaction is calculated in Sec. IV. Here it is found that in order to maintain gauge invariance it is necessary to introduce a set of anticommuting scalars, related to the Faddeev-Popov ghost fields. In the fifth section, it is shown that to all orders in perturbation theory, gauge invariance can be maintained. This means that vertex functions consistent with the dynamics of asymptotic field theory can be introduced with the boundary condition that the Ward identities are to be satisfied. Finally, in the last section, it is shown that the contribution of the S matrix from the poles of the ghost propagator cancel against those from the longitudinal part of the vector propagator so that unitarity is maintained.

II. QUANTIZATION OF THE YANG-MILLS FIELD

We shall begin by examining the Yang-Mills field.[1] Two sets[3-8] of free particle states, $A^i_{\mu\ \mathrm{in}}(x)$ and $A^i_{\mu\,\mathrm{out}}(x)$, both spanning the same Hilbert space, are postulated. They satisfy the free-particle equations of motion

$$\Box A^i_{\mu\ \mathrm{in,out}}(x)=0 \tag{1}$$

and the commutation relations

$$\begin{aligned}[][A^i_{\mu\ \mathrm{in}}(x),A^j_{\nu\ \mathrm{in}}(y)]&=-i\delta^{ij}\delta_{\mu\nu}D(x-y),\\ [A^i_{\mu\ \mathrm{out}}(x),A^j_{\nu\ \mathrm{out}}(y)]&=-i\delta^{ij}\delta_{\mu\nu}\,D(x-y)\,.\end{aligned} \tag{2}$$

In analogy with the problem of self-interacting scalar fields discussed in Pugh's paper,[3] the S matrix is defined to be a unitary operator satisfying

$$A^i_{\mu\ \mathrm{out}}(x)=S^\dagger A^i_{\mu\ \mathrm{in}}(x)S \tag{3}$$

and

$$S\,|0\rangle=|0\rangle\,. \tag{4}$$

The interpolating field $A^i_\mu(x)$ is defined by

$$A^i_\mu(x)=S^\dagger[a^i_\mu(x)S]_+\,, \tag{5}$$

where $A^i_{\mu\ \mathrm{in}}(x)$ has been abbreviated to $a^i_\mu(x)$. If the S matrix is expanded in the form

$$S=\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}\int d^4x_1\cdots d^4x_m\omega(x_1\cdots x_m)\times :a^{a_1}_{\mu_1}(x_1)\cdots a^{a_m}_{\mu_m}(x_m):\,, \tag{6}$$

then using the Lehmann-Symanzik-Zimmermann (LSZ) reduction technique[9] it is possible to show that, on the mass shell,

$$\omega(x_1\cdots x_m)=\Box^1\cdots\Box^m\langle 0|\Phi(A^{i_1}_{\mu_1}(x_1)\cdots A^{i_m}_{\mu_m}(x_m))|0\rangle\,, \tag{7}$$

where

$$\Phi(A^{a_1}_{\mu_1}(x_1)\cdots A^{a_m}_{\mu_m}(x_m))=[A^{a_1}_{\mu_1}(x_1)\cdots A^{a_m}_{\mu_m}(x_m)]_+ + i\left\{\sum_{\text{pairs}}\delta^{a_ia_j}\delta_{\mu_i\mu_j}D_i(x_i-x_j)[A^{a_1}_{\mu_1}(x_1)\cdots\Lambda_{ij}\cdots A^{a_m}_{\mu_m}(x_m)]_+ + (i)^2\sum_{2\text{ pairs}}\cdots\right\}\,. \tag{8}$$

A complete set of physical states are created by the operators

$$A^{\alpha}_{\text{in}}(x_0)=-i\int d^3x A^i_{\mu\ \text{in}}(x)\overleftrightarrow{\partial}_0 f^{\alpha i}_{\mu}(x)\,,$$
$$A^{\alpha}_{\text{out}}(x_0)=-i\int d^3x A^i_{\mu\ \text{out}}(x)\overleftrightarrow{\partial}_0 f^{\alpha i}_{\mu}(x)\,, \tag{9}$$

where $f^{\alpha i}_{\mu}(x)$ is a positive-energy solution to Eq. (1). To ensure that the norm of a vector created by $A^{\alpha}_{\text{in}}(x_0)$ is equal to its transverse part, we adopt a device introduced into electrodynamics by Pugh.[3,5] The condition

$$\partial_\mu f^{\alpha i}_{\mu}(x)=0 \tag{10}$$

is imposed on the function $f^{\alpha i}_{\mu}(x)$. The matrix elements of S are independent of the longitudinal and scalar parts of $f^{\alpha i}_{\mu}(x)$ provided that $f^{\alpha i}_{\mu}(x)$ can be replaced by $f^{\alpha i}_{\mu}(x)-(1/g)\partial_\mu\theta^i(x)$, where

$$\Box\theta^i(x)=0\,. \tag{11}$$

Matrix elements of the operator S as expanded in Eq. (6) are of the form

$$S_{(\beta\alpha)}=(\Phi^{\beta}_{\text{in}},S\Phi^{\alpha}_{\text{in}})=(-i)^m\int d^4x_1\cdots d^4x_m\Psi^{\beta *}(x_1\cdots x_k)\omega(x_1\cdots x_m)\Psi^{\alpha}(x_{k+1}\cdots x_m), \tag{12}$$

where

$$\Psi^{\beta}(x_1\cdots x_k)=\prod_{j=1}^{k}f^{\beta_j i_j}_{\mu_j}(x_j)\,.$$

The transformation $f^{\beta i}_{\mu}(x)\to f^{\beta i}_{\mu}(x)-(1/g)\partial_\mu\theta^i(x)$ on the jth incoming wave function leads to

$$\delta S_{(\beta\alpha)}=(-i)^m\int d^4x_1\cdots d^4x_m\omega(x_1\cdots x_m)\left[-\frac{1}{g}\partial_{\mu_j}\theta^{a_j}(x_j)\right]\Psi^{\beta *}(x_1\cdots x_k)\Psi^{\alpha}(x_{k+1}\cdots\Lambda_j\cdots x_m)\,. \tag{13}$$

This is a matrix element of the operator

$$\delta S=\sum_m\frac{(-i)^{m-1}}{(m-1)!}\int d^4x_1\cdots d^4x_m\omega(x_1\cdots x_m)\left[\frac{-1}{g}\partial_{\mu_m}\theta^{a_m}(x_m)\right]:a^{a_1}_{\mu_1}(x_1)\cdots a^{a_{m-1}}_{\mu_{m-1}}(x_{m-1}):\,,$$

formed from S by having all the fields $a^a_\mu(x)$ undergo the infinitesimal change $a^a_\mu(x)\to a^a_\mu(x)-(1/g)\partial_\mu\theta^a(x)$.

The requirement that $\delta S=0$ implies that

$$\int d^4\xi\,\omega(\xi^a_\mu\cdots)\partial_\mu\theta^a(\xi)=0\,,$$

where the dots refer to variables whose Fourier transform is restricted to the mass-shell value $p^2=0$. Pugh supplies a proof[6] that

$$\int d^4\xi\,\omega(\xi^a_\mu\cdots)\partial_\mu\theta^a(\xi)=-\int d^4\xi\,\omega(\partial_\mu\xi^a_\mu\cdots)\theta^a(\xi)\,, \tag{14}$$

and hence

$$\omega(\partial_\mu \xi^a_\mu \cdots)\big|_{\text{mass shell}} = 0\,. \tag{15}$$

As a result of Eq. (15) and the definition of the interpolating field given in Eq. (5), it may be shown that

$$\partial_\mu a^a_\mu(x) = \partial_\mu A^a_\mu(x)\,. \tag{16}$$

To extend Eq. (15) off the mass shell, the form of the gauge transformation of the interpolating field is fixed as

$$A^i_\mu(x) \to A^i_\mu(x) - \frac{1}{g}\left[\partial_\mu \delta^{ij} + g c^{ipj} A^p_\mu(x)\right]\Theta^j(x) \equiv A^i_\mu(x) - \frac{1}{g}\nabla^{ij}_\mu \Theta^j\,, \tag{17}$$

where $\Theta^j(x)$ is subject to the restriction

$$\partial_\mu \nabla^{ij}_\mu \Theta^j = 0\,, \tag{18}$$

and to zeroth order in g

$$\Theta^{j(0)}(x) = \theta^j(x)\,. \tag{19}$$

Here $\theta^j(x)$ is the gauge function of Eq. (11) and c^{ijk} is the structure function for the group of which $A^i_\mu(x)$ forms a representation. A perturbative solution to Eq. (18) is of the form

$$\begin{aligned}\Theta^i(x) = \theta^i(x) &+ \int d^4\eta\, \partial^x_\mu D_c(x-\eta)[g c^{iab} A^a_\mu(\eta)]\,\theta^b(\eta)\\ &+ \int d^4\eta_1 d^4\eta_2 [\partial^x_{\mu_1} D_c(x-\eta_1)][g c^{ia_1b_1} A^{a_1}_{\mu_1}(\eta_1)][\partial^{\eta_1}_{\mu_2} D_c(\eta_1-\eta_2)][g c^{b_1a_2b_2} A^{a_2}_{\mu_2}(\eta_2)]\,\theta^{b_2}(\eta_2) + \cdots\\ &\equiv \int d^4\eta\, \Lambda^{ij}(x,\eta)\theta^j(\eta),\end{aligned} \tag{20}$$

provided we use causal boundary conditions. Fixing the gauge transformation in this fashion implies a commutation relation for the interpolating fields that in turn leads to the Ward identities for the Yang-Mills theory. If the gauge transformation $a^a_\mu(x) \to a^a_\mu(x) - (1/g)\partial_\mu \theta^a(x)$ is made, its effect on a function $F(a^a_\mu(x))$ is given by[7]

$$F \to e^\lambda F, \tag{21}$$

where

$$\lambda = \frac{-1}{g}\int d^4\xi\, \partial_\mu \theta^i(\xi)\frac{\delta F}{\delta a^a_\mu(\xi)}\,. \tag{22}$$

By the commutation relations of Eq. (2), we obtain

$$[a^a_\mu(x), F] = -i\int d^4\xi\, D(x-\xi)\frac{\delta F}{\delta a^a_\mu(\xi)}\,, \tag{23}$$

and hence

$$\int d^4\xi\, f(\xi)\frac{\delta F}{\delta a^i_\mu(\xi)} = i\int d^3\xi\, f(\xi)\overleftrightarrow{\partial}_0[a^i_\mu(\xi), F]\,. \tag{24}$$

Equation (21) implies that under a gauge transformation $A^a_\mu(x) \to e^\lambda A^a_\mu(x)$, which for infinitesimal $\theta^i(x)$ becomes $(1+\lambda)A^a_\mu(x)$. Using Eq. (24) this becomes

$$\lambda A^i_\mu(x) = \frac{-i}{g}\int d^3\xi\, \partial_\nu \theta^j(\xi)\overleftrightarrow{\partial}_0[a^j_\nu(\xi), A^i_\mu(x)]\,. \tag{25}$$

But it has been postulated that

$$\begin{aligned}\lambda A^i_\mu(x) &= \frac{-1}{g}\nabla^{ij}_\mu \Theta^j(x)\\ &\equiv -\frac{1}{g}\nabla^{il}_\mu\left\{\delta^{lj}\int d^3\xi\, D(x-\xi)\overleftrightarrow{\partial}_0\theta^j(\xi) + \int d^4\eta_1 \partial^x_\alpha D_c(x-\eta_1)[g c^{lpj} A^p_\alpha(\eta_1)]\int d^3\xi\, D(\eta_1-\xi)\overleftrightarrow{\partial}_0\theta^j(\xi) + \cdots\right\}.\end{aligned} \tag{26}$$

As $\theta^i(x)$ is an arbitrary solution to Eq. (11), these two equations together imply that

$$[\partial_\nu a_\nu^i(\xi), A_\mu^i(x)] = -i\nabla_\mu^{xil} K^{lj}(x, \xi), \tag{27}$$

where

$$K^{lj}(x,\xi) = \delta^{lj} D(x-\xi) + \int d^4\eta [\partial_\alpha^x D_c(x-\eta)][gc^{lpj} A_\alpha^p(\eta)] D(\eta - \xi) + \int d^4\eta_1 d^4\eta_2 [\partial_\alpha^x D_c(x-\eta_1)][gc^{lpb} A_\alpha^p(\eta_1)][\partial_\beta^{\eta_1} D_c(\eta_1 - \eta_2)][gc^{bmj} A_\beta^m(\eta_2)] D(\eta_2 - x) + \cdots . \tag{28}$$

Note that $\partial_\mu \nabla_\mu^{ij} K^{jl}(x,\xi) = 0$, and consequently

$$[\partial_\mu a_\mu^a(x), \partial_\nu A_\nu^a(y)] = 0. \tag{29}$$

Equation (27) implies a restriction of the S matrix. To obtain the operator form of this restriction, we use the operator form of the dynamic equation[3,4]

$$S^\dagger \frac{\delta^2 S}{\delta a_\mu^a(x) \delta a_\beta^b(y)} = (-i)^2 \Box^x \Box^y [A_\alpha^a(x) A_\beta^b(y)]_+. \tag{30}$$

Consequently, we obtain

$$\left[\partial_\mu a_\mu^j(\xi), \frac{\delta^2 S}{\delta a_\mu^a(x)\delta a_\beta^b(y)}\right] = (-i)^3 S \Box^x \Box^y \{\theta(x-y)[\nabla_\alpha^{xal} K^{lj}(x,\xi) A_\beta^b(y) + A_\alpha^a(x) \nabla_\beta^{ybl} K^{lj}(y,\xi)] + \theta(y-x)[A_\beta^b(y)\nabla_\alpha^{xal} K^{lj}(x,\xi) + \nabla_\beta^{ybl} K^{lj}(y,\xi) A_\alpha^a(x)]\}. \tag{31}$$

This is the operator form of the Ward identity.

The Ward identity for time-ordered products is also easily obtained from Eq. (27). Using Eq. (16), we obtain the following:

$$\begin{aligned}
&\langle 0| [\partial_\mu^i A_\mu(x) A_{\mu_1}^{a_1}(x_1) \cdots A_{\mu_n}^{a_n}(x_n)]_+ |0\rangle \\
&= \sum_j \langle 0| (\{\theta(x-x_j)[\partial_\mu a_\mu^{i(+)}(x), A_{\mu_j}^{a_j}(x_j)] + \theta(x_j - x)[A_{\mu_j}^{a_j}(x_j), \partial_\mu a_\mu^{i(-)}(x)]\} A_{\mu_1}^{a_1}(x_1) \cdots \Lambda_j \cdots A_{\mu_n}^{a_n}(x_n))_+ |0\rangle \\
&= -i \sum_j \langle 0| \{[\nabla_{\mu_j}^{x_j a_j l} K^{li}(x_j, x)]_c A_{\mu_1}^{a_1}(x_1) \cdots \Lambda_j \cdots A_{\mu_n}^{a_n}(x_n)\}_+ |0\rangle .
\end{aligned} \tag{32}$$

We have here defined

$$[\nabla_\mu^{ab} K^{bc}(x,y)]_c = \theta(x-y)[\partial_\mu^x \delta^{ab} + gc^{apb} A_\mu^p(x)] \left\{\delta^{bc} D^{(+)}(x-y) + \int d^4\eta [\partial_\alpha^x D_c(x-\eta)][gc^{bqc} A_\alpha^q(\eta)] D^{(+)}(\eta - y) + \cdots \right\} - \theta(y-x)[\partial_\mu^x \delta^{ab} + gc^{apb} A_\mu^p(x)] \left\{\delta^{bc} D^{(-)}(x-y) + \int d^4\eta [\partial_\alpha^x D_c(x-\eta)][gc^{bqc} A_\alpha^q(\eta)] D^{(-)}(\eta - y) + \cdots \right\}. \tag{33}$$

This method of obtaining a Ward identity can easily be extended by Eq. (29) to give

$$\langle 0| [\partial_{\mu_1} A_{\mu_1}^{a_1}(x_1) \partial_{\mu_2} A_{\mu_2}^{a_2}(x_2) \cdots \partial_{\mu_n} A_{\mu_n}^{a_n}(x_n)]_+ |0\rangle = 0. \tag{34}$$

In order to obtain the form of the Ward identity for the S-matrix elements, it is necessary to examine Eq. (7) directly and then apply the commutation relation of Eq. (27). In analogy with Eq. (32), we obtain

$$\begin{aligned}
&\omega(\partial_\mu \xi_\mu^a, x_1 \cdots x_m) \\
&= -\Box^\xi \Box^1 \cdots \Box^m \Big(\sum_j \langle 0| \{A_{\mu_1}^{a_1}(x_1) \cdots \Lambda_j [\nabla_{\mu_j}^{a_j b} K^{ba}(x_j, \xi)]_c \cdots A_{\mu_m}^{a_m}(x_m)\}_+ |0\rangle \\
&\quad + i \sum_{\text{pairs}} \sum_{j \neq k,l} \delta^{a_k a_l} \delta_{\mu_k \mu_l} D_c(x_k - x_l) \langle 0| \{A_{\mu_1}^{a_1}(x_1) \cdots \Lambda_{jkl} [\nabla_{\mu_j}^{a_j b} K^{ba}(x_j,\xi)]_c \cdots A_{\mu_m}^{a_m}(x_m)\}_+ |0\rangle + \cdots \Big).
\end{aligned} \tag{35}$$

This is the form of the Ward identity for the function $\omega(x_1 \cdots x_m)$.

III. QUANTIZATION OF THE GRAVITATIONAL FIELD

The principles that we have been using to quantize the Yang-Mills field can now be applied to the gravitational field[2] in a directly analogous fashion. Two sets of free spin-two fields, $H^{in}_{\mu}(x)$ and $H^{out}_{\mu\nu}(x)$ are postulated. They satisfy the equations of motion

$$\Box H^{in,\,out}_{\mu\nu}=0 \tag{36}$$

and the commutation relations

$$[H^{in,\,out}_{\mu\nu}(x), H^{in,\,out}_{\lambda\sigma}(y)] = iQ_{\mu\nu,\lambda\sigma}D(x-y)\,, \tag{37}$$

where

$$Q_{\mu\nu,\lambda\sigma}=\tfrac{1}{2}(\delta_{\mu\lambda}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\lambda}-\delta_{\mu\nu}\delta_{\lambda\sigma})\,. \tag{38}$$

With this form of the commutator, one might at first suspect trouble due to the fact that

$$Q_{\mu\nu,\lambda\sigma}Q_{\lambda\sigma,\rho\kappa}=I_{\mu\nu,\rho\kappa}\,, \tag{39}$$

or

$$Q^2=I\,,$$

where

$$I_{\mu\nu,\rho\kappa}=\tfrac{1}{2}(\delta_{\mu\rho}\delta_{\nu\kappa}+\delta_{\mu\kappa}\delta_{\nu\rho})\,. \tag{40}$$

This would seem to prevent the operator $B_{\mu_1\nu_1\cdots\mu_n\nu_n,\lambda_1\sigma_1\cdots\lambda_n\sigma_n}$ appearing in the dynamic equation for a spin-two field[3,4]

$$[(I_{\mu_1\nu_1\cdots,\lambda_1\sigma_1\cdots})-(B_{\mu_1\nu_1\cdots,\lambda_1\sigma_1\cdots})]\omega^{(n)}_{\lambda_1\sigma_1\cdots}=\lambda^{(n)}_{\mu_1\nu_1\cdots} \tag{41}$$

from being a projection operator.[3] However, as

$$B_{\mu_1\nu_1\cdots,\lambda_1\sigma_1\cdots}=Q_{\mu_1\nu_1,\lambda_1\sigma_1}\cdots Q_{\mu_n\nu_n,\lambda_n\sigma_n}B\,, \tag{42}$$

or, more simply,

$$=QB\,,$$

we can multiply the dynamic equation

$$(I-QB)\omega^{(n)}=\lambda^{(n)}$$

by the integral operator $(I+QB)$ to obtain the equation

$$(I-IB)\omega^{(n)}=\lambda^{(n)}\,. \tag{43}$$

Here we have used the facts[3] that $B^2=B$ and $B\lambda^{(n)}=0$. As $I^2=I$, Eq. (43) is a satisfactory form of the dynamic equation for a spin-two field.

The S operator is defined by the relations

$$\begin{aligned} &H^{out}_{\mu\nu}(x)=S^\dagger H^{in}_{\mu\nu}(x)S\,,\\ &S^\dagger S=1\,,\\ \text{and}\\ &S|0\rangle=|0\rangle\,. \end{aligned} \tag{44}$$

An interpolating field is introduced, satisfying

$$H_{\mu\nu}(x)=S^\dagger[h_{\mu\nu}(x)S]_+\,. \tag{45}$$

We have abbreviated $H^{in}_{\mu\nu}(x)$ to $h_{\mu\nu}(x)$.

An expansion for S can be made

$$S=\sum_m\frac{(-i)^m}{m!}\int d^4x_1\cdots d^4x_m\,\omega(x_1\cdots x_m)\times :h_{\mu_1\nu_1}(x_1)\cdots h_{\mu_m\nu_m}(x_m):\,, \tag{46}$$

with the LSZ[9] reduction technique giving, on the mass shell,

$$\omega(x_1\cdots x_m)=\Box^1\cdots\Box^m\Big\{\langle 0|[H_{\mu_1\nu_1}(x_1)\cdots H_{\mu_m\nu_m}(x_m)]_+|0\rangle + (-i)\sum_{\text{pairs}} I_{\mu_i\nu_i,\mu_j\nu_j}D_c(x_i-x_j)\langle 0|[H_{\mu_1\nu_1}(x_1)\cdots\Lambda_{ij}\cdots H_{\mu_m\nu_m}(x_m)]_+|0\rangle+\cdots\Big\}\,. \tag{47}$$

Physical states are created by the operator

$$H^{\alpha\ in,\,out}=-i\int d^3x\, H^{in,\,out}_{\mu\nu}(x)\overleftrightarrow{\partial}_0\phi^\alpha_{\mu\nu}(x)\,, \tag{48}$$

where $\phi^\alpha_{\mu\nu}$ is a positive-energy solution to Eq. (36). To eliminate the contribution of nonphysical degrees of freedom to S, a device similar to that used for the Yang-Mills field is used. The supplementary conditions

$$\partial_\mu\phi^\alpha_{\mu\nu}=0 \tag{49}$$

and

$$\phi^\alpha_{\mu\mu}=0 \tag{50}$$

are imposed on the function $\phi^\alpha_{\mu\nu}(x)$. The matrix elements of S are independent of the unphysical components of $h_{\mu\nu}(x)$, provided that $\phi^\alpha_{\mu\nu}$ can be replaced by $\phi^\alpha_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\nu-\delta_{\mu\nu}\partial\cdot f$, where

$$\partial\cdot f=0 \tag{51}$$

and

$$\Box f_\mu=0\,. \tag{52}$$

These restrictions imply that $h_{\mu\nu}$ satisfies the conditions

$$h^{(-)}_{\mu\mu}|\Lambda_{in}\rangle=0 \tag{53}$$

and

$$\partial_\mu h^{(-)}_{\mu\nu}|\Lambda_{in}\rangle=0\,, \tag{54}$$

which imply that the matrix elements of $\partial_\mu h_{\mu\nu}$ and $h_{\mu\mu}$ are zero.

Matrix elements of the operator S are of the form

$$S_{(\beta\alpha)}=(-i)^m\int d^4x_1\cdots d^4x_m\,\Psi^{\beta *}(x_1\cdots x_k)\times\omega(x_1\cdots x_m)\Psi^{\alpha}(x_{k+1}\cdots x_m)\,,\quad(55)$$

where

$$\Psi^{\beta}(x_1\cdots x_k)=\prod_{i=1}^{k}\phi^{\beta_i}_{\mu_i\nu_i}(x_i)\,.$$

The transformation $\phi^{\alpha}_{\mu\nu}\to\phi^{\alpha}_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\mu$ on the jth incoming wave function leads to

$$\delta S_{(\beta\alpha)}=(-i)^m\int d^4x_1\cdots d^4x_m\,\omega(x_1\cdots x_m)\times[\partial_{\mu_j}f_{\nu_j}(x_j)+\partial_{\nu_j}f_{\mu_j}(x_j)]\times\Psi^{\beta *}(x_1\cdots x_k)\times\Psi^{\alpha}(x_{k+1}\cdots\Lambda_j\cdots x_m)\,.$$

This is a matrix element of the operator

$$\delta S=\sum_m\frac{(-i)^{m-1}}{(m-1)!}\int d^4x_1\cdots d^4x_m\,\omega(x_1\cdots x_m)\times[\partial_{\mu_1}f_{\nu_1}(x_1)+\partial_{\nu_1}f_{\mu_1}(x_1)]\times :h_{\mu_2\nu_2}(x_2)\cdots h_{\mu_m\nu_m}(x_m):\quad(56)$$

The requirement that $\delta S=0$ leads to the condition

$$\omega(\partial_\mu\xi_{\mu\nu}\cdots)\big|_{\text{mass shell}}=0\,,\quad(57)$$

in the same fashion that Eq. (15) was obtained. This, in turn, implies that

$$\partial_\mu h_{\mu\nu}=\partial_\mu H_{\mu\nu}\,.\quad(58)$$

Instead of working with the in field $h_{\mu\nu}$, it is possible to work with a field $\rho_{\mu\nu}$ defined by

$$\rho_{\mu\nu}=h_{\mu\nu}-\tfrac{1}{2}\delta_{\mu\nu}h_{\lambda\lambda}\,.\quad(59)$$

If the interpolating field is defined by

$$P_{\mu\nu}=S^\dagger(\rho_{\mu\nu}S)_+\,,\quad(60)$$

then by Eq. (45) we will have

$$P_{\mu\nu}=H_{\mu\nu}-\tfrac{1}{2}\delta_{\mu\nu}H_{\lambda\lambda}\,.\quad(61)$$

The form of the field equations and the commutation relations for $\rho_{\mu\nu}$ is

$$\Box\rho_{\mu\nu}(x)=0$$

and

$$[\rho_{\mu\nu}(x),\rho_{\lambda\sigma}(y)]=iQ_{\mu\nu,\lambda\sigma}D(x-y)\,.\quad(62)$$

The gauge transformation

$$h_{\mu\nu}\to h_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\mu-\delta_{\mu\nu}\partial\cdot f\quad(63)$$

now becomes

$$\rho_{\mu\nu}\to\rho_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\mu\,.\quad(64)$$

The expansion

$$S=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int d^4x_1\cdots d^4x_n\,\tilde\omega(x_1\cdots x_n)\times :\rho_{\mu_1\nu_1}(x_1)\cdots\rho_{\mu_n\nu_n}(x_n):$$

leads to the restriction

$$\tilde\omega(\partial_\mu\xi_{\mu\nu}\cdots)\big|_{\text{mass shell}}=0,\quad(65)$$

in the same fashion that Eq. (15) was derived. From Eq. (65), we obtain

$$\partial_\mu\rho_{\mu\nu}=\partial_\mu P_{\mu\nu}\,.\quad(66)$$

By Eqs. (58), (59), (61), and (66) we see that

$$\partial_\mu h_{\nu\nu}=\partial_\mu H_{\nu\nu}\,.\quad(67)$$

To extend Eq. (57) off the mass shell, the form of the gauge transformation for the interpolating field is fixed as

$$\begin{aligned}H_{\mu\nu}\to H_{\mu\nu}&+[(\partial_\mu\delta_{\rho\nu}+\partial_\nu\delta_{\rho\mu}-\delta_{\mu\nu}\partial_\rho)\\&+K(H_{\mu\lambda}\partial_\lambda\delta_{\rho\nu}+H_{\nu\lambda}\partial_\lambda\delta_{\rho\mu}-H_{\mu\nu}\partial_\rho\\&-\partial_\rho H_{\mu\nu})]F_\rho\\&\equiv H_{\mu\nu}+\Box_{\mu\nu\rho}F_\rho\,,\end{aligned}\quad(68)$$

when the in field undergoes the transformation of Eq. (63). Also, if the trace of the in field undergoes the transformation

$$h_{\lambda\lambda}\to h_{\lambda\lambda}-2\partial\cdot f\,,\quad(69)$$

then only the diagonal elements of $H_{\mu\nu}$ undergo the following transformation:

$$H_{\mu\nu}\to H_{\mu\nu}+\tfrac{1}{4}\delta_{\mu\nu}\Box_{\lambda\lambda\rho}F_\rho\,.\quad(70)$$

The function F_μ is related to the function f_μ by the requirement that

$$\partial_\mu\Box_{\mu\nu\lambda}F_\lambda=0\,,\quad(71)$$

with, to lowest order in K,

$$F^{(0)}_\mu(x)=f_\mu(x)\,.$$

Consequently, we have

$$F_\mu(x)=\int d^4y\,\Lambda_{\mu\nu}(x,y)f_\nu(y)\,,\quad(72)$$

where

$$\Lambda_{\mu\nu}(x,y)=\delta_{\mu\nu}\delta^4(x-y)+O(K)\,.$$

The commutation relation between interpolating fields may now be developed in the same fashion as was done for the Yang-Mills field. If the gauge transformation of Eq. (63) is made, its effect on a functional $F[h_{\mu\nu}]$ is given by

$$F\to e^{\lambda}F\,,\quad(73)$$

where

$$\lambda = \int d^4y[\partial_\mu f_\nu(y) + \partial_\nu f_\mu(y) - \delta_{\mu\nu}\partial\cdot f(y)] \times \frac{\delta}{\delta h_{\mu\nu}(y)} .$$

Using the same methods as were used in deriving Eq. (25), we find that

$$\lambda H_{\alpha\beta}(x) = 2i \int d^3y\, f_\nu(y)\overleftrightarrow{\partial}_0[\partial_\mu h_{\mu\nu}(y), H_{\alpha\beta}(x)] . \tag{74}$$

But for infinitesimal λ, it has been postulated that

$$\lambda H_{\alpha\beta}(x) = \Box_{\alpha\beta\rho} F_\rho(x) . \tag{75}$$

Equations (72), (74), and (75) together imply that

$$\begin{aligned}[\partial_\mu h_{\mu\nu}(y), H_{\alpha\beta}(x)] &= \frac{1}{2i}\Box_{\alpha\beta\rho}\int d^4z\,\Lambda_{\rho\nu}(x,z)D(y-z) \\ &\equiv \frac{1}{2i}\Box_{\alpha\beta\rho}K_{\rho\nu}(x,y) .\end{aligned} \tag{76}$$

Similarly, the gauge transformations of Eqs. (69) and (70) lead to the commutation relations

$$[\partial_\nu h_{\mu\mu}(y), H_{\alpha\beta}(x)] = \frac{1}{2i}\delta_{\alpha\beta}\Box_{\lambda\lambda\rho}K_{\rho\nu}(x,y) . \tag{77}$$

Combining Eqs. (76) and (77) we see that

$$\begin{aligned}[\partial_\mu h_{\mu\nu} - \tfrac{1}{2}\partial_\nu h_{\mu\mu}(y), H_{\alpha\beta}(x)] &= \frac{1}{2i}(\Box_{\alpha\beta\rho} - \tfrac{1}{2}\delta_{\alpha\beta}\Box_{\lambda\lambda\rho})K_{\rho\nu}(x,y) \\ &\equiv \frac{1}{2i}\tilde{\Box}_{\alpha\beta\rho}K_{\rho\nu}(x,y) .\end{aligned} \tag{78}$$

Equations (76) and (78) are consistent with Eq. (37) when $K=0$.

From the commutation relations of Eqs. (76) and (78) the Ward identities can be derived. The operator form of these restrictions can be derived from the operator form of the dynamic equations,

$$S^\dagger \frac{\delta^2 S}{\delta h_{\mu\nu}(x)\delta h_{\lambda\sigma}(y)} = (i)^2\Box^x\Box^y[H_{\mu\nu}(x)H_{\lambda\sigma}(y)]_+ . \tag{79}$$

From Eqs. (76) and (79) we obtain the operator form of the Ward identity,

$$\begin{aligned}\left[\partial_\alpha h_{\alpha\beta}(z), \frac{\delta^2 S}{\delta h_{\mu\nu}(x)\delta h_{\lambda\sigma}(y)}\right] = (i)^2 S\Box^x\Box^y\{&\theta(x-y)[\Box^x_{\mu\nu\rho}K_{\rho\beta}(x,z)H_{\lambda\sigma}(y) + H_{\mu\nu}(x)\Box^y_{\lambda\sigma\rho}K_{\rho\beta}(y,z)] \\ &+\theta(y-x)[H_{\lambda\sigma}(y)\Box^x_{\mu\nu\rho}K_{\rho\beta}(x,z) + H_{\mu\nu}(x)\Box^y_{\lambda\sigma\rho}K_{\rho\beta}(y,z)]\} .\end{aligned} \tag{80}$$

Other forms of the Ward identity, derived in analogy to Eqs. (32), (34), and (35) are

$$\langle 0|[\partial_\mu H_{\mu\nu}(x)H_{\mu_1\nu_1}(x_1)\cdots H_{\mu_n\nu_n}(x_n)]_+|0\rangle = \frac{1}{2i}\sum_j \langle 0|\{[\Box^{x_j}_{\mu_j\nu_j\rho}K_{\rho\nu}(x_j,x)]_c H_{\mu_1\nu_1}(x_1)\cdots\Lambda_j\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle , \tag{81}$$

$$\langle 0|[\partial_{\mu_1}H_{\mu_1\nu_1}(x_1)\partial_{\mu_2}H_{\mu_2\nu_2}(x_2)\cdots\partial_{\mu_n}H_{\mu_n\nu_n}(x_n)]_+|0\rangle = 0 , \tag{82}$$

and

$$\begin{aligned}\omega(\partial_\mu\xi_{\mu\nu}, x_1\cdots x_n) = \frac{1}{2i}\Box^\xi\Box^1\cdots\Box^n\Big(&\sum_m \langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots\Lambda_m[\Box_{\mu_m\nu_m\rho}K_{\rho\nu}(x_m,\xi)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle \\ &+(-i)\sum_{m\neq i,j}\sum_{\text{pairs}} I_{\mu_i\nu_i,\mu_j\nu_j}D_c(x_i-x_j) \\ &\times\langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots\Lambda_{mij}[\Box_{\mu_m\nu_m\rho}K_{\rho\nu}(x_m,\xi)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle + \cdots\Big) .\end{aligned} \tag{83}$$

Similar Ward identities can be derived upon use of the commutation relation of Eq. (78). In particular, we can have

$$\begin{aligned}\langle 0|\{[\partial_\mu H_{\mu\nu}(x) - \tfrac{1}{2}\partial_\nu H_{\mu\mu}(x)]H_{\mu_1\nu_1}(x_1)\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle \\ = \frac{1}{2i}\sum_j \langle 0|\{[\tilde{\Box}_{\mu_j\nu_j\rho}K_{\rho\nu}(x_j,x)]_c H_{\mu_1\nu_1}(x_1)\cdots\Lambda_j\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle\end{aligned} \tag{84}$$

and

$$\omega(\partial_\mu \xi_{\mu\nu} - \tfrac{1}{2}\partial_\nu \xi_{\mu\mu}, x_1 \cdots x_n)$$
$$= \frac{1}{2i}\Box^{\xi}\Box^{1}\cdots\Box^{n}\Big(\sum_m \langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots\Lambda_m[\bar{\Box}_{\mu_m\nu_m\rho}K_{\rho\nu}(x_m,\xi)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle + \cdots\Big). \tag{85}$$

IV. PERTURBATION EXPANSION

The formal procedure for quantizing non-Abelian gauge fields is now complete, and the elements of the S matrix can now be calculated using perturbation theory. In accordance with the formalism for asymptotic field theory, a vertex must be postulated. For the Yang-Mills theory, we select the most general form of the three-point function consistent with the solution of $(1-B)\omega^{(1)}=0$,[3]

$$\omega^{(1)}(x,y,z) = gc^{abc}[\delta_{\alpha\beta}(\partial_y - \partial_x)_\gamma + \delta_{\beta\gamma}(\partial_z - \partial_y)_\alpha + \delta_{\alpha\gamma}(\partial_x - \partial_z)_\beta][\delta^4(x-y)\delta^4(y-z)]. \tag{86}$$

However, this must also be consistent with the Ward identity of Eq. (35). For this to be possible, a complex scalar field must be postulated.[10–12] It will later be shown that in order for unitarity to be satisfied, these fields must obey Fermi statistics. The field equations and commutation relations for these fields are

$$\Box b^a(x) = 0 \tag{87}$$

and

$$\{b^a(x), \bar{b}^b(y)\} = -i\delta^{ab}D(x-y). \tag{88}$$

If these fields do not enter the asymptotic states, the spin-statistics theorem[5] is not violated. The additional vertex function that must be postulated in order for Eq. (31) to be satisfied is shown by explicit calculations to be given by

$$S^{(1)} = (-i)^3\int d^4x\, d^4y\, d^4z[igc^{abc}\partial^y_\alpha\delta^4(x-y)\delta^4(y-z)]:a^a_\alpha(x)b^b(y)\bar{b}^c(z):. \tag{89}$$

A more intuitive approach to the introduction of these ghost particles corresponds to the way ghosts were introduced from Eqs. (A11) and (A16). With $S^{(1)}$ given by Eq. (86), the gauge transformation $a^a_\mu \to a^a_\mu - (1/g)\partial_\mu\theta^a$ leads to the change $S^{(1)} \to S^{(1)} + \delta S^{(1)}$, where

$$\delta S^{(1)} = \tfrac{1}{2}(-i)^3\int d^4x\, d^4y\, d^4z[c^{abc}(\partial^z_\beta\partial^x_\alpha - \partial^z_\alpha\partial^y_\beta)\delta^4(x-y)\delta^4(y-z)]:a^a_\alpha(x)a^b_\beta(y):\theta^c(z). \tag{90}$$

The fact that $(\partial^z + \partial^y + \partial^x)_\alpha\delta^4(x-y)\delta^4(y-z)=0$ and that $\Box a^a_\mu = 0$ have been used in deriving Eq. (90). If $\theta^a(x)$ is treated as a noninteracting, massless, spin-zero field, that is, the gauge transformation is a Bell-Treiman transformation as was used by 't Hooft and Veltman,[13] then Eq. (90) describes a vertex function whose effects are to be canceled. This is done by use of ghost fields.

As an example of a perturbation theory calculation, let us now calculate the second-order propagator, $\omega^{(2)}(x,y)$. As $\omega(x,y)$ does not appear in the expansion for S, the term $B\omega^{(2)}$ does not appear in the dynamic equation for $\omega^{(2)}$.[3,14] Consequently we have

$$\omega^{(2)}(x_1,x_2) = \Box^1\Box^2\langle 0|[A^{i_1(1)}_{\mu_1}(x_1)A^{i_2(1)}_{\mu_2}(x_2)]_+|0\rangle. \tag{91}$$

By Eq. (5),

$$A^{d(1)}_\delta(1) = \{S^\dagger[Sa^d_\delta(w)]_+\}^{(1)}, \tag{92}$$

which can be combined with Eqs. (86) and (89) to give

$$A^{d(1)}_\delta(w) = \tfrac{1}{2}g\int d^4x_1\, d^4x_2\, d^4x_3\, c^{a_1a_2a_3}[\delta_{\alpha_1\alpha_2}(\partial_{x_2} - \partial_{x_1})_{\alpha_3} + \delta_{\alpha_1\alpha_3}(\partial_{x_1} - \partial_{x_3})_{\alpha_2} + \delta_{\alpha_2\alpha_3}(\partial_{x_3} - \partial_{x_2})_{\alpha_1}]$$
$$\times[\delta^4(x_1-x_2)\delta^4(x_2-x_3)][\delta^{a_1d}\delta_{\alpha_1\delta}D_R(w-x_1):a^{a_2}_{\alpha_2}(x_2)a^{a_3}_{\alpha_3}(x_3):]$$
$$+ig\int d^4x\, d^4y\, d^4z[\partial^y_\alpha\delta^4(x-y)\delta^4(y-z)][\delta^{ad}\delta_{\alpha\delta}D_R(w-x)]:b^b(y)\bar{b}^c(z):. \tag{93}$$

Inserting this into Eq. (91), using the commutation relations, and substituting the Fourier transforms according to Appendix B gives

$$\omega^{(2)}(w,w') = \Box^{w}\Box^{w'}\frac{1}{(2\pi)^4}C_2(G)g^2\delta^{dd'} \times \int \frac{d^4p}{p^4}e^{ip\cdot(w-w')}\int_{-\infty}^{\infty}\frac{d\lambda}{\lambda-i\epsilon}[-\tfrac{1}{2}\pi\,\theta(-p^2)][e^{i\lambda(w-w')_0}\theta(p)+e^{i\lambda(w'-w)_0}\theta(-p)] \times[-\tfrac{5}{3}p_\delta p_{\delta'}+\tfrac{5}{3}p^2\delta_{\delta\delta'}]. \tag{94}$$

The constant $C_2(G)$ is defined by $\delta^{dd'}C_2(G)=c^{abd}c^{abd'}$. Multiplying this equation by $1=\int_{-\infty}^{\infty}d\kappa^2\delta(p^2+\kappa^2)$ to facilitate the integration over λ results in

$$\omega^{(2)}(w,w') = \Box^{w}\Box^{w'}\left\{\left[\frac{-ig^2\pi}{2(2\pi)^4}C_2(G)\delta^{dd'}\int d^4p\int_{\mu^2}^{\infty}\frac{d\kappa^2}{\kappa^4}\frac{1}{p^2+\kappa^2-i\epsilon}e^{ip\cdot(w-w')}\frac{5}{3}(p_\delta p_{\delta'}-p^2\delta_{\delta\delta'})\right] -\left[\frac{ig^2\pi}{2}\frac{C_2(G)\delta^{dd'}}{\mu^2}\frac{5}{3}(\delta_{\delta\delta'}-\delta_{\delta4}\delta_{\delta'4})\right]\right\}. \tag{95}$$

The constant μ^2 is an infrared cutoff whose presence is made necessary by considerations entirely unrelated to any renormalization procedure.[15]

However, this is not a unique form for $\omega^{(2)}(w,w')$. Due to the fact that the matrix elements of $\partial_\mu a_\mu^a(x)$ are zero, gauge invariance can be maintained for arbitrary M when Eq. (5) is modified to

$$A_\mu^i(x)=S^\dagger[Sa_\mu^i(x)]_+ + M\partial_\mu\partial_\nu a_\nu^i(x). \tag{96}$$

If M is taken to be second order in g, Eq. (91) becomes

$$\omega^{(2)}(w,w') = \Box^{w}\Box^{w'}\{\langle 0|[A_\delta^{d(1)}(w)A_{\delta'}^{d'(1)}(w')]_+|0\rangle + M^{(2)}\langle 0|[\partial_\delta\partial_\nu a_\nu^d(w)a_{\delta'}^{d'}(w')+a_\delta^d(w)\partial_{\delta'}\partial_\nu a_\nu^{d'}(w')]_+|0\rangle\}. \tag{97}$$

As

$$\langle 0|[\partial_\delta\partial_\nu a_\nu^d(w)a_{\delta'}^{d'}(w')+a_\delta^d(w)\partial_{\delta'}\partial_\nu a_\nu^{d'}(w')]_+|0\rangle = -2i[\partial_\delta\partial_{\delta'}D_c(w-w')-\delta_{\delta4}\delta_{\delta'4}\delta^4(w-w')]\delta^{dd'},$$

Eq. (97) becomes, for appropriate choice for $M^{(2)}$,

$$\omega^{(2)}(w,w') = \Box^{w}\Box^{w'}\left[\frac{-ig^2\pi}{4(2\pi)^4}C_2(G)\delta^{dd'}\int d^4p\,\ln\left(\frac{p^2+\mu^2}{\mu^2}\right)\left(\frac{10}{3}\right)(p_\delta p_{\delta'}-p^2\delta_{\delta\delta'})e^{ip\cdot(w-w')}\right]. \tag{98}$$

This is the same result that is obtained from evaluating the diagrams of Fig. 1 upon use of dimensional regularization,[16] in the limit $\mu^2\approx 0$.

Much the same procedure can be used to calculate elements of the S matrix in quantum gravity. Expanding the Einstein Lagrangian of Eq. (A18) to first order in K results in a term[17-19]

$$\tfrac{1}{2}h_{\mu\nu}[(\partial_\mu h_{\rho\lambda}\partial_\nu h_{\rho\lambda}-\tfrac{1}{2}\partial_\mu h_{\rho\rho}\partial_\nu h_{\lambda\lambda})+2(\partial_\lambda h_{\mu\rho}\partial_\rho h_{\nu\lambda}-\partial_\rho h_{\mu\lambda}\partial_\rho h_{\nu\lambda}+\tfrac{1}{2}\partial_\rho h_{\mu\nu}\partial_\rho h_{\lambda\lambda})]. \tag{99}$$

With this as the motivating factor, the vertex function for the gravitational interaction is set equal to

$$\omega^{(1)}(x_1,x_2,x_3)=\tfrac{1}{8}KS_{\alpha_1\beta_1}S_{\alpha_2\beta_2}S_{\alpha_3\beta_3}P_3^!(\partial_\rho^2\partial_\sigma^3) \times(\delta_{\alpha_1\rho}\delta_{\beta_1\sigma}Q_{\alpha_2\beta_2,\alpha_3\beta_3}-2\delta_{\rho\sigma}\delta_{\alpha_1\alpha_2}Q_{\beta_1\beta_2,\alpha_3\beta_3}+4\delta_{\sigma\alpha_2}\delta_{\rho\alpha_3}\delta_{\alpha_1\beta_2}\delta_{\beta_1\beta_3})\delta^4(x_1-x_2)\delta^4(x_2-x_3), \tag{100}$$

where $S_{\alpha\beta}$ stands for symmetrization between the indices α and β, and $P_3^!$ stands for summing over the 3! permutations of the indices 1, 2, and 3.[19]

Once again, in order to satisfy the Ward identities, a vertex function involving a set of ghost fields must be postulated. Upon examining Eq. (83), it is found that a set of spin-one fields, $a_\mu(x)$ and $b_\mu(x)$, must be postulated. Unitarity will imply that these ghost fields obey Fermi statistics, as was the case for the Yang-Mills interaction. The equations of motion and the commutation relations for the ghost fields are

$$\Box a_\mu(x)=\Box b_\mu(x)=0, \tag{101}$$

and

$$\{a_\mu(x),b_\nu(y)\}=-i\delta_{\mu\nu}D(x-y),$$
$$\{a_\mu(x),a_\nu(y)\}=\{b_\mu(x),b_\nu(y)\}=0.$$

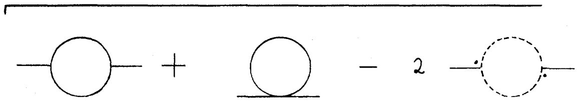

FIG. 1. Feynman diagrams for the second-order vector propagator for the Yang-Mills interaction.

The vertex function for these ghost fields as implied by Eq. (83) is

$$S^{(1)}=\tfrac{1}{2}(-i)^3K\int d^4x\,d^4y\,d^4z\big[(\partial^x_\alpha\partial^y_\mu\delta_{\lambda\beta}-\partial^z_\alpha\partial^y_\beta\delta_{\lambda\mu}-\partial^z_\beta\partial^y_\alpha\delta_{\lambda\mu}+\partial^x_\beta\partial^y_\mu\delta_{\lambda\alpha})\delta^4(x-y)\delta^4(y-z)\big]:h_{\alpha\beta}(x)a_\lambda(y)b_\mu(z):. \tag{102}$$

The second-order propagator $\omega^{(2)}(x_1,x_2)$ can be calculated in a manner directly analogous to that used for the Yang-Mills theory; the only complications are algebraic in nature. We have

$$\omega^{(2)}(w_1,w_2)=\Box^1\Box^2\langle 0|[H^{(1)}_{\mu_1\nu_1}(w_1)H^{(1)}_{\mu_2\nu_2}(w_2)]_+|0\rangle\,,$$

where

$$H^{(1)}_{\mu\nu}(x)=\{S^\dagger[h_{\mu\nu}(x)S]_+\}^{(1)}\,.$$

Upon substitution of Eqs. (100) and (102) into these equations, the resulting propagator can be determined without encountering divergent integrals.

V. GAUGE INVARIANCE TO HIGHER ORDERS

It is now possible to use the Ward identities to show that gauge invariance is obeyed up to all orders in perturbation theory for non-Abelian gauge fields. For concreteness, our attention will focused on the problem of quantum gravity, although an exactly analogous treatment holds for the Yang-Mills field.

First of all, it will be established that if the Ward identity is satisfied up to order n, then it will be satisfied, within a vertex function, up to order $n+1$. The method used to show this is very similar to that of Ref. 6.

Let us assume that the Ward identities of Eqs. (83) and (85) hold in perturbation theory up to order n. As a result, Eq. (58) is modified to become

$$\partial_\mu H_{\mu\nu}=\partial_\mu h_{\mu\nu}+\sum_{p=n}^{\infty}\partial_\mu H^{(p)}_{\mu\nu}\,. \tag{103}$$

Also for $l<n$, Eq. (76) holds, giving

$$[\partial_\mu h_{\mu\nu}(y),H^{(l)}_{\alpha\beta}(x)]=\frac{1}{2i}[\Box_{\alpha\beta\rho}K_{\rho\nu}(x,y)]^{(l)}\,. \tag{104}$$

From Eq. (103), we have

$$\begin{aligned}&\{\langle 0|[\partial_\mu H_{\mu\nu}(x)H_{\mu_1\nu_1}(x_1)\cdots H_{\mu_n\nu_n}(x_n)]_+|0\rangle\}^{(n)}\\&=\sum_i(\langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots[\partial_\mu h_{\mu\nu}(x),H_{\mu_i\nu_i}(x_i)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle)^{(n)}+\langle 0|[\partial_\mu H^{(n)}_{\mu\nu}(x)h_{\mu_1\nu_1}(x_1)\cdots h_{\mu_n\nu_n}(x_n)]_+|0\rangle\,.\end{aligned} \tag{105}$$

Insertion of Eq. (104) into Eq. (105) gives

$$\begin{aligned}&\{\langle 0|[\partial_\mu H_{\mu\nu}(x)H_{\mu_1\nu_1}(x_1)\cdots H_{\mu_n\nu_n}(x_n)]_+|0\rangle\}^{(n)}\\&=\sum_i(\langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots\Lambda_i[\Box_{\mu_i\nu_i\rho}K_{\rho\nu}(x_i,x)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle)^{(n)}\\&\quad+\langle 0|[\partial_\mu H^{(n)}_{\mu\nu}(x)h_{\mu_1\nu_1}(x_1)\cdots h_{\mu_n\nu_n}(x_n)]_+|0\rangle\\&\quad+\sum_i\langle 0|(h_{\mu_1\nu_1}(x_1)\cdots\Lambda_i\{[\partial_\mu h_{\mu\nu}(x),H^{(n)}_{\mu_i\nu_i}(x_i)]_c-(1/2i)[\Box_{\mu_i\nu_i\rho}K^{(n)}_{\rho\nu}(x_i,x)]_c\}\cdots h_{\mu_n\nu_n}(x_n))_+|0\rangle\,.\end{aligned} \tag{106}$$

This allows one to examine the corrections to the nth order form of the Ward identity. If we convert Eq. (105) into an equation for Φ products, just as Eq. (81) is converted into Eq. (83), we have

$$\omega^{(n)}(\partial_\mu x_{\mu\nu}, x_1 \cdots x_n)$$
$$= \frac{1}{2i} \Box^x \Box^1 \cdots \Box^n \Big(\sum_m \langle 0 | \{ H_{\mu_1\nu_1}(x_1) \cdots \Lambda_m [\Box_{\mu_m \nu_m \rho} K_{\rho\nu}(x_m, x)]_c \cdots H_{\mu_n\nu_n}(x_n)\}_+ | 0 \rangle$$
$$+ (-i) \sum_{m \neq i, j} \sum_{\text{pairs}} I_{\mu_i\nu_i, \mu_j\nu_j} D_c(x_i - x_j)$$
$$\times \langle 0 | \{ H_{\mu_1\nu_1}(x_1) \cdots \Lambda_{ijm} [\Box_{\mu_m\nu_m\rho} K_{\rho\nu}(x_m, x)]_c \cdots H_{\mu_n\nu_n}(x_n)\}_+ | 0 \rangle + \cdots \Big)^{(n)}$$
$$+ \Box^x \Box^1 \cdots \Box^n \langle 0 | \Phi(\partial_\mu H^{(n)}_{\mu\nu}(x) h_{\mu_1\nu_1}(x_1) \cdots h_{\mu_n\nu_n}(x_n)) | 0 \rangle$$
$$+ \Box^x \Box^1 \cdots \Box^n \sum_i \langle 0 | \Phi(h_{\mu_1\nu_1}(x_1) \cdots \Lambda_i \{ [\partial_\mu h_{\mu\nu}(x), H^{(n)}_{\mu_i\nu_i}(x_i)]_c - (1/2i)[\Box_{\mu_i\nu_i\rho} K^{(n)}_{\rho\nu}(x_i, x)]_c \} \cdots h_{\mu_n\nu_n}(x_n)) | 0 \rangle . \tag{107}$$

The commutator in Eq. (107) can now be evaluated. As

$$H_{\mu\nu}(x) = S^\dagger [S h_{\mu\nu}(x)]_+$$

we have

$$H_{\mu\nu}(x) = h_{\mu\nu}(x) + iS^\dagger \sum_n \frac{(-i)^n}{n!} \int d^4x_1 \cdots d^4x_n d^4\xi$$
$$\times D_R(x - \xi) Q_{\mu\nu, \lambda\sigma} \omega(\xi_{\lambda\sigma}, x_1 \cdots x_n) : h_{\mu_1\nu_1}(x_1) \cdots h_{\mu_n\nu_n}(x_n) : . \tag{108}$$

As a result, using Eqs. (14) and (108) we have

$$[\partial_\mu h_{\mu\nu}(x), H_{\mu_i\nu_i}(x_i)]_c = iQ_{\mu\nu, \mu_i\nu_i} \partial_\mu D_c(x - x_i)$$
$$+ iS^\dagger \sum_n \frac{(-i)^n}{(n-1)!} \int d^4x_1 \cdots d^4x_n D_R(x_i - \xi) [iQ_{\mu\nu, \mu_1\nu_1} D_{c\,x_i}(x - x_1)]$$
$$\times Q_{\mu_i\nu_i, \lambda\sigma} \omega(\xi_{\lambda\sigma}, \partial_\mu x_{\mu_1\nu_1} \cdots x_n) : h_{\mu_2\nu_2}(x_2) \cdots h_{\mu_n\nu_n}(x_n) : . \tag{109}$$

Inserting the nth-order form of the Ward identity of Eq. (85) into Eq. (109) we find that

$$[\partial_\mu h_{\mu\nu}(x), H^{(n)}_{\mu_i\nu_i}(x_i)]_c = \frac{1}{2i} [\Box_{\mu_i\nu_i\rho} K_{\rho\nu}(x_i, x)]^{(n)}_c$$
$$+ iS^\dagger \sum_n \frac{(-i)^{n-1}}{n!} \int d^4x_1 \cdots d^4x_n [D_R(x_i - \xi) D_{c\,x_i}(x - x_1) Q_{\mu_i\nu_i, \lambda\sigma} \Pi^{(n)}(x_1, \xi_{\lambda\sigma} \cdots x_n)]$$
$$\times : h_{\mu_2\nu_2}(x_2) \cdots h_{\mu_n\nu_n}(x_n) : , \tag{110}$$

where

$$\Pi(x, x_1 \cdots x_n) = \omega(\partial_\mu x_{\mu\nu} - \tfrac{1}{2} \partial_\nu x_{\mu\mu}, x_1 \cdots x_n)$$
$$- \frac{1}{2i} \Box^x \Box^1 \cdots \Box^n \sum_m (\langle 0 | \{ H_{\mu_1\nu_1}(x_1) \cdots \Lambda_m [\tilde{\Box}_{\mu_m\nu_m\rho} K_{\rho\nu}(x_m, x)]_c \cdots H_{\mu_n\nu_n}(x_n)\}_+ | 0 \rangle + \cdots) . \tag{111}$$

Substituting Eq. (110) into Eq. (107), we obtain

$$\Omega^{(n)}(x, x_1 \cdots x_n) - \int d^4\xi \, d^4\xi_1 \cdots d^4\xi_n B(x, x_1 \cdots x_n; \xi, \xi_1 \cdots \xi_n) \Pi^{(n)}(\xi, \xi_1 \cdots \xi_n) = 0 , \tag{112}$$

where

$$B(x_1 \cdots x_n; y_1 \cdots y_n) = (-1)^n \Box^1 \cdots \Box^n \Big[\sum_\lambda I_{\mu_\lambda\nu_\lambda, \rho_\lambda\sigma_\lambda} D_R(x_\lambda - y_\lambda) {\prod_{i=1}^n}' \theta(x_i - y_i) I_{\mu_i\nu_i, \rho_i\sigma_i} D_c(x_i - y_i) \Big] ,$$

and

$$\Omega^{(n)}(x,x_1\cdots x_n)=\omega(\partial_\mu x_{\mu\nu},x_1\cdots x_n) - \frac{1}{2i}\Box^x\Box^1\cdots\Box^n\sum_m(\langle 0|\{H_{\mu_1\nu_1}(x_1)\cdots\Lambda_m[\Box_{\mu_m\nu_m\rho}K_{\rho\nu}(x_m,x)]_c\cdots H_{\mu_n\nu_n}(x_n)\}_+|0\rangle+\cdots). \tag{113}$$

Similarly, the equation

$$\Pi^{(n)}(x,x_1\cdots x_n)-\int d^4\xi\, d^4\xi_1\cdots d^4\xi_n B(x,x_1\cdots x_n;\,\xi,\,\xi_1\cdots\xi_n)\Omega^{(n)}(\xi,\,\xi_1\cdots\xi_n)=0 \tag{114}$$

can be derived. Using the fact that $B^2=B$, Eqs. (112) and (114) together imply that

$$(I-B)\Omega^{(n)}=0. \tag{115}$$

This is the dynamic equation that is to be obeyed by any vertex function. Consequently gauge invariance to order n can be satisfied by the insertion of vertex functions whose form is determined by the boundary condition that the Ward identity is to be satisfied.

VI. UNITARITY

In establishing the unitarity of the S matrix, we will adhere very closely to the program of 't Hooft.[16] By use of the Ward identities, it must be shown that the contributions of the nonphysical polarizations of the gauge fields to the absorbtive part of the S matrix cancel against those contributions coming from the ghost fields.

We first must derive the Cutkosky rules[20] within the formalism of asymptotic field theory. If $S=1+T$, then $S^\dagger S=1$ implies that

$$T^\dagger+T=-TT^\dagger. \tag{116}$$

Using the expansion for T given in Eq. (6), except now including the ghost fields explicity, we obtain

$$TT^\dagger=\sum_{n,m}\sum_{n',m'}\frac{(-i)^m}{m!(n!)^2}\frac{(i)^{m'}}{m'!(n'!)^2}\int d^4x_1\cdots d^4z'_{n'}\,\omega(x_1\cdots z_n)\omega^*(x'_1\cdots z'_{n'}):a^{a_1}_{\mu_1}(x_1)\cdots\bar{b}^{c_n}(z_n): \times:a^{a'_1}_{\mu'_1}(x'_1)\cdots\bar{b}^{c'_{n'}}(z'_{n'}):. \tag{117}$$

Using the fact that the commutation relations imply that

$$:a^a_\alpha(x)a^b_\beta(y):=a^a_\alpha(x)a^b_\beta(y)+i\delta^{ab}\delta_{\alpha\beta}D^{(+)}(x-y),$$
$$:b^b(y)\bar{b}^c(z):=b^b(y)\bar{b}^c(z)+i\delta^{bc}D^{(+)}(y-z), \tag{118}$$

the product of normal-ordered products in Eq. (117) can be reduced to a single normal-ordered product. In this way, the Cutkosky rules are derived.

It has been shown that if Bogoliubov causality is postulated[21] then the dynamical axiom of Eq. (30) follows, as well as microscopic causality, which states that

$$[A^a_\alpha(x),A^b_\beta(y)]=0 \tag{119}$$

for spacelike separation of x and y. This, in turn, implies that

$$\langle 0|[\partial_\mu A^a_\mu(x)\cdots]_+|0\rangle=\partial_\mu\langle 0|[A^a_\mu(x)\cdots]_+|0\rangle.$$

The Ward identity of Eq. (34) now takes the form

$$\partial^{x_1}_{\mu_1}\cdots\partial^{x_n}_{\mu_n}\langle 0|[A^{a_1}_{\mu_1}(x_1)\cdots A^{a_n}_{\mu_n}(x_n)]_+|0\rangle=0. \tag{120}$$

This is the equation that 't Hooft uses in establishing unitarity[16]; consequently his argument can now be carried through in tact.

Unitarity for the gravitational field can be established in an analogous manner. With the Ward identities that have been derived, 't Hooft's method[16] can be used to show that the effect of the ghost fields is to cancel the contribution of the nonphysical polarizations of the graviton field to the imaginary part of the S matrix.

VII. DISCUSSION

The approach of asymptotic field theory is seen here to provide a useful vehicle for discussion problems involving non-Abelian gauge symmetry. As only renormalized quantities are introduced at the outset, there is no problem with divergent quantities, and no need for introducing a renormalization or regularization procedure. This fact allows one to avoid the Adler anomaly that occurs when a chiral gauge symmetry is present.[8]

Hopefully, the techniques introduced here may shed some light on the problem of renormalization of general relativity. It has been shown in Sec. V that higher-order vertex functions may be chosen with the boundary condition that they satisfy the Ward identity, but it remains to be seen whether these vertex functions are uniquely determined. An example of how asymptotic field theory can be used to determine the form of an interaction is given in Ref. 7, where it is shown how the Pauli interaction is to be excluded from quantum electrodynamics. Further investigations are being made in this direction.

APPENDIX A: CLASSICAL GAUGE FIELDS

Some properties of the classical gauge fields that have been analyzed up to this point are examined in this appendix.

First of all, the massless Yang-Mills field[1] is dealt with. The standard means of deriving the Yang-Mills Lagrangian is by demanding a coupling of the vector field to a spinor or scalar field that is invariant under rotations in some isospin space. However, it is useful to point out how the coupling can be derived without referring to other fields. This approach will be useful in Appendix B when the Yang-Mills formalism will be extended. It also allows for a different insight into the gravitational interaction.

First of all, consider an uncoupled vector field $A_\mu^i(x)$, where i refers to the group index and μ to the Lorentz index. The structure functions of the group are denoted by c_{ijk}. For the uncoupled Lagrangian, we take the standard Lagrangian for a spin-one particle,

$$L=-\tfrac{1}{4}f_{\mu\nu}^i f_{\mu\nu}^i, \tag{A1}$$

where

$$f_{\mu\nu}^i=\partial_\mu A_\nu^i-\partial_\nu A_\mu^i. \tag{A2}$$

The resulting equation of motion is

$$\Box A_\mu^i-\partial_\mu(\partial_\nu A_\nu^i)=0. \tag{A3}$$

Suppose we now require that the equation of motion be altered so that the vector field couples with strength g to a conserved current generated by demanding that the Lagrangian be invariant under an infinitesimal transformation

$$A_\mu^i\to A_\mu^i+c^{ijk}A_\mu^j\alpha^k. \tag{A4}$$

In general, an infinitesimal transformation

$$\phi_A(x)\to\phi_A(x)+\alpha\sum_B\lambda_{AB}\phi_B(x) \tag{A5}$$

leads to a conserved current,

$$j_\mu=-\sum_{AB}\frac{\partial}{\partial(\partial_\mu\phi_A)}\lambda_{AB}\phi_B. \tag{A6}$$

For the case of the Lagrangian of Eq. (A1), we obtain

$$j_\mu^i=c^{ijk}f_{\mu\nu}^j A_\nu^k. \tag{A7}$$

The equation of motion now becomes

$$\Box A_\mu^i-\partial_\mu(\partial_\nu A_\nu^i)=-gc^{ijk}f_{\mu\nu}^j A_\nu^k. \tag{A8}$$

This requires an alteration of the Lagrangian of Eq. (A1) by the addition of a piece

$$-gc^{ijk}\partial_\mu A_\nu^i A_\mu^j A_\nu^k. \tag{A9}$$

By Eq. (A8), such a term in the Lagrangian contributes another term to the conserved current. The process is repeated once more, and the final piece that must be added to the Lagrangian is found to be

$$-\tfrac{1}{4}g^2c^{ijk}c^{imn}A_\mu^j A_\nu^k A_\mu^m A_\nu^n. \tag{A10}$$

Upon collecting together Eqs. (A1), (A9), and (A10), we find that the full Yang-Mills Lagrangian is given by

$$L=-\tfrac{1}{4}F_{\mu\nu}^i F_{\mu\nu}^i, \tag{A11}$$

where

$$F_{\mu\nu}^i=\partial_\mu A_\nu^i-\partial_\nu A_\mu^i+gc^{ijk}A_\mu^j A_\nu^k. \tag{A12}$$

The resulting equation of motion is

$$(\partial_\mu\delta^{ij}+gc^{ipj}A_\mu^p)F_{\mu\nu}^j=0 \tag{A13}$$

or

$$\nabla_\mu^{ij}F_{\mu\nu}^j=0.$$

If L^i is a matrix representation of the group in question, and $\theta^i(x)$ an arbitrary function, then the following gauge transformation is a symmetry of the Lagrangian

$$L^iA_\rho^i(x)\to U(\theta)\left[L^iA_\rho^i(x)-\frac{i}{g}U^{-1}(\theta)\partial_\rho U(\theta)\right]U^{-1}(\theta), \tag{A14}$$

where $U(\theta)=\exp[-iL^i\theta^i(x)]$.

For infinitesimal $\theta^i(x)$, the gauge transformation takes the form

$$A_\mu^i(x)\to A_\mu^i(x)-\frac{1}{g}\nabla_\mu^{ij}\theta^j(x). \tag{A15}$$

This implies that

$$F_{\mu\nu}^i\to(\delta^{ij}+c^{ilj}\theta^l)F_{\mu\nu}^j$$

and

$$\nabla_\mu^{ij}F_{\mu\nu}^j\to(\delta^{il}+c^{ipl}\theta^p)\nabla_\mu^{lj}F_{\mu\nu}^j.$$

If one quantizes the Yang-Mills field by use of path integrals, the Lagrangian one uses is

$$L=L_{\rm inv}-\tfrac{1}{2}c^2, \tag{A16}$$

where $L_{\rm inv}$ is given by Eq. (A11) and $-\frac{1}{2}c^2$ fixes the gauge in which one works. If under the infinitesimal gauge transformation of Eq. (A15),

$$c \to c + M\theta,$$

then an extra term $\bar{\phi}M\phi$ must be added to the Lagrangian.[13] The field ϕ is a complex scalar field with Fermi statistics that appears only in closed loops and does not enter the unitary condition. For example, if $c = \partial_\mu A^i_\mu$, then $\bar{\phi}M\phi = \bar{\phi}^i(\Box\delta^{ij} + gc^{ipj}A^p_\mu\partial_\mu)\phi^j$.

Einstein's theory of gravity[2] is a theory of classical fields. Attempts to quantize it have either been those of "canonical quantization"[22,23] in which the metric field has been considered in terms of its true degrees of freedom, and of "covariant quantization"[10,11,17-19,24] in which the gravitational interaction is mediated by spin-two "gravitons" on a background of Minkowski space-time. The latter point of view was adopted in this note.

The field equations for the metric field $g_{\mu\nu}$ are, in the absence of matter,

$$R_{\mu\nu} = 0, \tag{A17}$$

where

$$R_{\eta\lambda} = \partial_\lambda \Gamma^\beta_{\beta\eta} - \partial_\beta \Gamma^\beta_{\eta\lambda} + \Gamma^\beta_{\tau\lambda}\Gamma^\tau_{\beta\eta} - \Gamma^\beta_{\tau\beta}\Gamma^\tau_{\eta\lambda}$$

and

$$\Gamma^\alpha_{\beta\gamma} = \tfrac{1}{2} g^{\alpha\kappa}(\partial_\gamma g_{\kappa\beta} + \partial_\beta g_{\kappa\gamma} - \partial_\kappa g_{\beta\gamma}).$$

These equations can be derived from the variational principle

$$\delta \int \left(\frac{2}{K^2} R\sqrt{g}\right) d^4x = 0, \tag{A18}$$

where

$$g = \det g_{\mu\nu}$$

and

$$R = R_{\mu\nu} g^{\mu\nu}.$$

Upon expanding the metric tensor in powers of the gravitational coupling constant K, we see that if

$$\sqrt{g}\, g^{\mu\nu} \equiv \bar{g}^{\mu\nu} = \delta_{\mu\nu} + K h_{\mu\nu}, \tag{A19}$$

then

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu} - K h_{\mu\nu} + K^2 h_{\mu\lambda} h_{\lambda\nu} + O(K^3).$$

Indices are now raised and lowered by use of $\delta_{\mu\nu}$, consequently they may all be taken to be lower case.

Substitution of Eq. (A19) into the Lagrangian of Eq. (A18) yields, to lowest order in K,

$$\frac{2}{K^2} R\sqrt{g} = \tfrac{1}{2}\partial_\mu h_{\nu\lambda}\partial_\mu h_{\nu\lambda} - \tfrac{1}{4}\partial_\mu h_{\nu\nu}\partial_\mu h_{\lambda\lambda} - \partial_\mu h_{\mu\lambda}\partial_\nu h_{\nu\lambda},$$

which is the Lagrangian of a spin-two field.

The gravitational equation of motion is invariant under the coordinate transformation

$$g'^{\mu\nu}(x') = \frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x'^\nu}{\partial x^\sigma} g^{\lambda\sigma}(x). \tag{A20}$$

If $x'^\mu - x^\mu = KF^\mu(x)$, then for infinitesimal F^μ Eq. (A20) takes the form

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu F_\nu + \partial_\nu F_\mu - \delta_{\mu\nu}\partial\cdot F + K(h_{\mu\lambda}\partial_\lambda F_\nu + h_{\nu\lambda}\partial_\lambda F_\mu - F_\lambda\partial_\lambda h_{\mu\nu} - h_{\mu\nu}\partial_\lambda F_\lambda). \tag{A21}$$

For $K=0$, Eq. (A21) gives the standard form for a gauge transformation for a spin-two field.

According to Duff,[25] the same approach that allowed us to arrive at Eq. (A11) from Eq. (A1) can be used to derive the Lagrangian of Eq. (A18) from the free Lagrangian L_0 for a massless spin-two field. To the free Lagrangian, an interaction term is added in which the field $h_{\mu\nu}$ is coupled with strength K to the energy-momentum tensor $T_{\mu\nu}$. To first order, $T_{\mu\nu}$ includes terms arising from the energy-momentum contributions of the free spin-two field itself. The altered Lagrangian $L_0 + KL_1$ now makes further contributions to $T_{\mu\nu}$, which adds further corrections to the Lagrangian. In the same fashion that Eq. (A11) can be derived from Eq. (A1), this process can be continued to give

$$L = L_0 + KL_1 + K^2 L_2 + \cdots.$$

If this series were summed, the hope is that the Einstein Lagrangian can be recovered upon making the identification

$$\sqrt{g}\, g^{\mu\nu} = \delta_{\mu\nu} + K h_{\mu\nu}.$$

It is of interest to examine the formalism of free spin-two fields. In the Bargmann-Wigner equations,[26] a spin-n field of mass m is represented by a symmetric spinor of $2n$ dimensions satisfying the $2n$ equations

$$\begin{aligned}&(\gamma\cdot\partial + m)_{\alpha\alpha'}\Psi_{\alpha'\beta\cdots\lambda} = 0,\\ &\cdots\\ &(\gamma\cdot\partial + m)_{\lambda\lambda'}\Psi_{\alpha\beta\cdots\lambda'} = 0.\end{aligned} \tag{A22}$$

In the representation in which

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{A23}$$

$$S_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu], \quad c = \gamma_2\gamma_4,$$

the ten symmetric matrices are $\gamma_\lambda c$ and $S_{\mu\nu} c$, and the six antisymmetric matrices are c^{-1}, $\gamma_5 c^{-1}$, and $\gamma_\lambda c^{-1}$.

For the spin-two field we take

$$\Psi_{\alpha\beta\gamma\delta} = h_{\mu\nu}(\gamma_\mu c)_{\alpha\beta}(\gamma_\nu c)_{\gamma\delta} + A_{\mu\lambda\sigma}(\gamma_\mu c)_{\alpha\beta}(S_{\lambda\sigma} c)_{\lambda\delta} + B_{\lambda\sigma\mu}(S_{\lambda\sigma} c)_{\alpha\beta}(\gamma_\mu c)_{\gamma\delta} + G_{\mu\nu\lambda\sigma}(S_{\mu\nu} c)_{\alpha\beta}(S_{\lambda\sigma} c)_{\gamma\delta}. \tag{A24}$$

In order to ensure that Ψ is symmetric, it is evident that

$$\begin{aligned} &h_{\mu\nu} = h_{\nu\mu}, \\ &A_{\mu\lambda\sigma} = B_{\lambda\sigma\mu} = -A_{\mu\sigma\lambda}, \\ &G_{\mu\nu\lambda\sigma} = G_{\lambda\sigma\mu\nu} = -G_{\nu\mu\lambda\sigma} = -G_{\mu\nu\sigma\lambda}. \end{aligned} \tag{A25}$$

Also, contraction with the antisymmetric matrices $(c^{-1})_{\beta\gamma}$, $(\gamma_5 c^{-1})_{\beta\gamma}$ and $(\gamma_\lambda c^{-1})_{\beta\gamma}$ must vanish. Contracting with $(c^{-1})_{\beta\gamma}(c^{-1})_{\delta\epsilon}$ leads to

$$\begin{aligned} &h_{\mu\nu}\gamma_\mu\gamma_\nu = 0, \\ &A_{\mu\lambda\sigma}(\gamma_\mu S_{\lambda\sigma} + S_{\lambda\sigma}\gamma_\mu) = 0, \\ &G_{\mu\nu\lambda\sigma} S_{\mu\nu} S_{\lambda\sigma} = 0. \end{aligned} \tag{A26}$$

Contracting with $(\gamma_5 c^{-1})_{\beta\gamma}(\gamma_5 c^{-1})_{\delta\epsilon}$ leads to

$$\begin{aligned} &A_{\mu\lambda\sigma}(\gamma_\mu S_{\lambda\sigma} - S_{\lambda\sigma}\gamma_\mu) = 0, \\ &G_{\mu\nu\lambda\sigma}(S_{\mu\nu} S_{\lambda\sigma} + S_{\lambda\sigma} S_{\mu\nu}) = 0. \end{aligned} \tag{A27}$$

Use of the commutation relations

$$\begin{aligned} &\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}, \\ &[\gamma_\mu, S_{\lambda\sigma}] = 2i(\delta_{\mu\sigma}\gamma_\lambda - \delta_{\mu\lambda}\gamma_\sigma), \\ &\{\gamma_\mu, S_{\lambda\sigma}\} = 2i\epsilon_{\mu\lambda\sigma\kappa}\gamma_\kappa\gamma_5, \\ &\{S_{\mu\nu}, S_{\lambda\sigma}\} = 2(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda}) - 2\epsilon_{\mu\nu\lambda\sigma}\gamma_5, \end{aligned} \tag{A28}$$

shows that Eqs. (A26) and (A27) imply that

$$\begin{aligned} &h_{\mu\mu} = 0, \\ &A_{\mu\mu\lambda} = 0, \\ &\epsilon_{\mu\nu\lambda\sigma} A_{\nu\lambda\sigma} = 0, \\ &G_{\mu\nu\mu\nu} = 0, \\ &\epsilon_{\mu\nu\lambda\sigma} G_{\mu\nu\lambda\sigma} = 0. \end{aligned} \tag{A29}$$

This is sufficient to ensure that Ψ is a symmetric function.

We can now substitute Eq. (A24) into Eq. (A22). Upon contracting the equation with $(c^{-1}\gamma_\tau)_{\beta\alpha}(c^{-1})_{\delta\epsilon}$ and with $(c^{-1}S_{\pi\tau})_{\beta\alpha}(c^{-1})_{\delta\epsilon}$ and using

$$\begin{aligned} &\mathrm{Tr}(\gamma_\nu\gamma_\mu S_{\lambda\sigma}) = 4i(\delta_{\nu\lambda}\delta_{\mu\sigma} - \delta_{\mu\lambda}\delta_{\nu\sigma}), \\ &\mathrm{Tr}(S_{\mu\nu} S_{\lambda\sigma}) = 4(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\nu\lambda}\delta_{\mu\sigma}), \\ &\mathrm{Tr}(\gamma_\mu\gamma_\nu) = 4\delta_{\mu\nu}, \\ &\mathrm{Tr}(\gamma_\mu) = \mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\lambda) = 0, \end{aligned} \tag{A30}$$

we obtain

$$\begin{aligned} &i(\partial_\rho B_{\tau\rho\nu} - \partial_\rho B_{\rho\tau\nu}) = -mh_{\tau\nu}, \\ &i(\partial_\rho G_{\mu\nu\tau\rho} - \partial_\rho G_{\mu\nu\rho\tau}) = -mA_{\tau\mu\nu}, \end{aligned} \tag{A31}$$

and

$$\begin{aligned} &i(\partial_\pi h_{\tau\nu} - \partial_\tau h_{\pi\nu}) = -m(B_{\pi\tau\nu} - B_{\tau\pi\nu}), \\ &i(\partial_\pi A_{\tau\lambda\sigma} - \partial_\tau A_{\lambda\sigma}) = -m(G_{\lambda\sigma\pi\tau} - G_{\lambda\sigma\tau\pi}). \end{aligned} \tag{A32}$$

Upon replacing $A_{\mu\nu\lambda}$ by $(2im)A_{\mu\nu\lambda}$ and $G_{\mu\nu\lambda\sigma}$ by $(2im)^2 G_{\mu\nu\lambda\sigma}$, and applying the symmetry conditions of Eqs. (A25) and (A29), Eqs. (A31) and (A32) become.

$$\begin{aligned} &\partial_\rho B_{\tau\rho\nu} = -m^2 h_{\tau\nu}, \\ &\partial_\rho G_{\mu\nu\tau\rho} = -m^2 B_{\mu\nu\tau}, \\ &\partial_\pi h_{\tau\nu} - \partial_\tau h_{\pi\nu} = B_{\pi\tau\nu}, \\ &\partial_\pi B_{\lambda\sigma\tau} - \partial_\tau B_{\lambda\sigma\pi} = G_{\lambda\sigma\pi\tau}. \end{aligned} \tag{A33}$$

From these equations it is immediately apparent that

$$\partial_\mu h_{\mu\nu} = 0 \tag{A34}$$

and

$$(\Box - m^2)h_{\mu\nu} = 0.$$

All standard features of massive spin-two fields are thus reproduced.

In the massless case, Eq. (A33) reduces to

$$\begin{aligned} &\partial_\tau B_{\tau\rho\nu} = 0, \\ &\partial_\rho G_{\mu\nu\tau\rho} = 0, \\ &\text{and} \\ &\partial_\pi h_{\tau\nu} - \partial_\tau h_{\pi\nu} = B_{\pi\tau\nu}, \\ &\partial_\pi B_{\lambda\sigma\tau} - \partial_\tau B_{\lambda\sigma\pi} = G_{\lambda\sigma\pi\tau}. \end{aligned} \tag{A35}$$

These equations cannot be derived from the Bargmann-Wigner equations.

The quantity $G_{\mu\nu\lambda\sigma}$ is invariant under the transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu f_\nu + \partial_\nu f_\mu, \tag{A36}$$

which implies $B_{\tau\pi\nu} \to B_{\tau\pi\nu} + \partial_\nu(\partial_\tau f_\pi - \partial_\pi f_\tau)$. Consequently, we take $h_{\mu\nu} = h_{\nu\mu}$, but assume that the conditions

$$\begin{aligned} &h_{\mu\mu} = 0 \\ &\text{and} \\ &\partial_\mu h_{\mu\nu} = 0 \end{aligned} \tag{A37}$$

are gauge dependent. To extend the equation of motion for $h_{\mu\nu}$ in a gauge-independent fashion, the equation

$$\partial_\rho B_{\nu\rho\tau} = 0$$

is generalized to

$$\partial_\rho B_{\nu\rho\tau} + \partial_\tau B_{\rho\nu\rho} = 0, \qquad \text{(A38)}$$

which is independent of the gauge transformation of Eq. (A36). Expressing Eq. (A38) in terms of $h_{\mu\nu}$ yields

$$\partial_\rho(\partial_\nu h_{\rho\tau} - \partial_\rho h_{\nu\tau}) + \partial_\tau(\partial_\rho h_{\nu\rho} - \partial_\nu h_{\rho\rho}) = 0, \qquad \text{(A39)}$$

as can be seen from Eq. (A35). Replacing $h_{\mu\nu}$ by $h_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu} h_{\lambda\lambda}$ gives the standard equation of motion

$$\Box h_{\nu\tau} - \tfrac{1}{2}\delta_{\nu\tau}\Box h_{\rho\rho} - \partial_{\mu\tau}{}^2 h_{\mu\nu} - \partial_{\mu\nu}{}^2 h_{\mu\tau} = 0, \qquad \text{(A40)}$$

which is invariant under

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu f_\nu + \partial_\nu f_\mu - \delta_{\mu\nu}\partial f. \qquad \text{(A41)}$$

For a spin-two field, the Lagrangian density for this equation of motion is

$$L = \tfrac{1}{2}\partial_\mu h_{\nu\lambda}\partial_\mu h_{\nu\lambda} - \tfrac{1}{4}\partial_\mu h_{\nu\nu}\partial_\mu h_{\lambda\lambda} - \partial_\mu h_{\mu\nu}\partial_\lambda h_{\lambda\nu}. \qquad \text{(A42)}$$

We shall now examine the polarization tensors associated with a spin-two field. A symmetric matrix has ten independent components. The restriction that it be traceless and transverse reduces the number of degrees of freedom to five, the number one would expect for a massive spin-two field. A symmetric matrix $h_{\mu\nu}$ with the gauge degree of freedom of Eq. (A41) will then have two degrees of freedom. The polarization tensors $\epsilon^\Lambda_{\alpha\beta}$ for the spin-two field can be constructed out of the polarization vectors e^a_α for a spin-one field.[27] If e^1_α and e^2_α denote the transverse polarization, we use the conventions that

$$\begin{aligned}
&\sum_{a=1}^{4} e^a_\alpha e^a_\beta = \delta_{\alpha\beta},\\
&e^a_\alpha e^b_\alpha = \delta^{ab},\\
&\sum_{a=1}^{3} e^a_\alpha e^a_\beta = \delta_{\alpha\beta} + k_\alpha k_\beta/m^2 \quad \text{(massive case)},\\
&\sum_{a=1}^{2} e^a_\alpha e^a_\beta = \delta_{\alpha\beta} - k_\alpha k_\beta/(k\cdot y)^2\\
&\qquad - (y_\alpha k_\beta + y_\beta k_\alpha)/(k\cdot y),
\end{aligned} \qquad \text{(A43)}$$

where y_α is defined to be $(0,0,0,i)$ in a reference frame in which $e^3_\alpha = (\vec{k}/|\vec{k}|,\ 0)$.

For the spin-two field, ten suitable polarization tensors are $e^1_\alpha e^1_\beta, \ldots, e^4_\alpha e^4_\beta$, $(e^1_\alpha e^2_\beta + e^1_\beta e^2_\alpha)/\sqrt{2}, \ldots$, $(e^3_\alpha e^4_\beta + e^3_\beta e^4_\alpha)/\sqrt{2}$. These ten tensors $\bar{\epsilon}^\Lambda_{\alpha\beta}$ obey the completeness and orthogonality conditions

$$\begin{aligned}
&\sum_\Lambda \bar{\epsilon}^\Lambda_{\alpha\beta}\bar{\epsilon}^\Lambda_{\gamma\delta} = 1 \quad \text{if } \alpha=\gamma,\ \beta=\delta \text{ and/or } \alpha=\delta,\ \beta=\gamma\\
&\qquad = 0 \quad \text{otherwise}\\
&\text{and}\\
&\bar{\epsilon}^\Lambda_{\alpha\beta}\bar{\epsilon}^{\Lambda'}_{\alpha\beta} = \delta^{\Lambda\Lambda'}.
\end{aligned} \qquad \text{(A44)}$$

This can be verified directly from Eq. (A43).

Five traceless and transverse tensors that are linear combinations of $\bar{\epsilon}^\Lambda_{\mu\nu}$ and obey Eq. (A44) are

$$\begin{aligned}
\epsilon^1_{\alpha\beta} &= \frac{1}{\sqrt{2}}(e^1_\alpha e^1_\beta - e^2_\alpha e^2_\beta),\\
\epsilon^2_{\alpha\beta} &= \frac{1}{\sqrt{2}}(e^1_\alpha e^2_\beta + e^2_\alpha e^1_\beta),\\
\epsilon^3_{\alpha\beta} &= (\tfrac{2}{3})^{1/2}(\tfrac{1}{2}e^1_\alpha e^1_\beta + \tfrac{1}{2}e^2_\alpha e^2_\beta - e^3_\alpha e^3_\beta),\\
\epsilon^4_{\alpha\beta} &= \frac{1}{\sqrt{2}}(e^1_\alpha e^3_\beta + e^3_\alpha e^1_\beta),\\
&\text{and}\\
\epsilon^5_{\alpha\beta} &= \frac{1}{\sqrt{2}}(e^2_\alpha e^3_\beta + e^3_\alpha e^2_\beta).
\end{aligned} \qquad \text{(A45)}$$

These are suitable polarization tensors for a massive spin-two field. Only $\epsilon^1_{\mu\nu}$ and $\epsilon^2_{\mu\nu}$ are polarizations for a massless spin-two field. From Eq. (A43), it may be shown that for $m \neq 0$

$$\begin{aligned}
\sum_{\Lambda=1}^{5} e^\Lambda_{\alpha\beta} e^\Lambda_{\gamma\delta}
&= \tfrac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta})\\
&\quad + \frac{1}{2m^2}(\delta_{\alpha\gamma}k_\beta k_\delta + \delta_{\beta\delta}k_\alpha k_\gamma + \delta_{\alpha\delta}k_\beta k_\gamma + \delta_{\beta\gamma}k_\alpha k_\delta)\\
&\quad + \frac{2}{3}\left(\frac{1}{2}\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{m^2}\right)\left(\frac{1}{2}\delta_{\gamma\delta} - \frac{k_\gamma k_\delta}{m^2}\right),
\end{aligned}$$

and for $m = 0$

$$\sum_{\Lambda=1}^{2} e^\Lambda_{\alpha\beta} e^\Lambda_{\gamma\delta} = \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}(e^i_\alpha e^i_\gamma e^j_\beta e^j_\delta + e^i_\alpha e^i_\delta e^j_\beta e^j_\gamma - e^i_\alpha e^i_\beta e^j_\gamma e^j_\delta),$$

which can be evaluated from Eq. (A43). Consequently, the form of the propagator for massless spin-two particles is

$$D_{\alpha\beta,\gamma\delta}(k) = \frac{i}{2}(\delta_{\alpha\delta}\delta_{\beta\gamma} + \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\beta}\delta_{\gamma\delta})\frac{1}{k^2} + (\text{gauge terms}).$$

APPENDIX B: SOME USEFUL INTEGRALS

In evaluating the second-order propagator, the following expressions are useful:

$$\theta(x) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\lambda \frac{1}{\lambda - i\epsilon} e^{i\lambda x},$$

$$\Delta^{(+)}(x) = -\Delta^{(-)}(-x) = \frac{-i}{(2\pi)^3}\int d^4p e^{ipx}\theta(p)\delta(p^2+m^2),$$

and

$$\Delta_R(x) = \frac{1}{(2\pi)^4}\int_R d^4p\, e^{ipx}\frac{1}{p^2+m^2}.$$

By going to the frame of reference to which $\vec{p}=0$, it may be shown that

$$\int d^4p'\theta(p-p')\theta(p')\delta((p-p')^2+m^2)\delta(p'^2+m^2) = \frac{-\pi}{2p^2}\theta(p)\theta(-p^2-4m^2)(p^4+4m^2p^2)^{1/2},$$

$$\int d^4p'\theta(p-p')\theta(p')\delta((p-p')^2+m^2)\delta(p'^2+m^2)p'_\delta = \frac{-\pi}{4p^2}p_\delta\theta(p)\theta(-p^2-4m^2)(p^4+4m^2p^2)^{1/2},$$

$$\int d^4p'\theta(p-p')\theta(p')\delta((p-p')^2+m^2)(p'^2+m^2)p'_\mu p'_\nu = \left[\frac{p^2+m^2}{p^2}p_\mu p_\nu - \left(m^2+\frac{p^2}{4}\right)\delta_{\mu\nu}\right] \frac{-\pi}{6p^2}\theta(p)\theta(-p^2-4m^2)(p^4+4m^2p^2)^{1/2},$$

$$\int d^4p'\theta(p-p')\theta(p')\delta((p-p')^2+m^2)\delta(p'^2+m^2)p'_\mu p'_\nu p'_\lambda = \left[\left(\frac{m^2}{2p^2}+\frac{1}{4}\right)p_\mu p_\nu p_\lambda + \frac{-p^2}{6}\left(\frac{m^2}{p^2}+\frac{1}{4}\right)(\delta_{\mu\nu}p_\lambda+\delta_{\mu\lambda}p_\nu+\delta_{\nu\lambda}p_\mu)\right]\left[\frac{-\pi}{2p^2}\theta(p)\theta(-p^2-4m^2)(p^4+4m^2p^2)^{1/2}\right],$$

$$\int d^4p'\theta(p-p')\theta(p')\delta((p-p')^2+m^2)\delta(p'^2+m^2)p'_\mu p'_\nu p'_\lambda p'_\sigma$$

$$= \left\{\left[\frac{-m^2}{3p^2}\left(\frac{m^2}{p^2}+1\right)+\frac{8}{5}\left(\frac{m^2}{p^2}+\frac{1}{4}\right)\left(\frac{m^2}{3p^2}+\frac{1}{2}\right)\right]p_\mu p_\nu p_\lambda p_\sigma + \frac{-p^2}{5}\left(\frac{m^2}{p^2}+\frac{1}{4}\right)\left(\frac{m^2}{3p^2}+\frac{1}{2}\right)(\delta_{\mu\nu}p_\lambda p_\sigma+\delta_{\lambda\sigma}p_\mu p_\nu+\delta_{\mu\lambda}p_\nu p_\sigma+\delta_{\nu\sigma}p_\mu p_\lambda+\delta_{\mu\sigma}p_\nu p_\lambda+\delta_{\nu\lambda}p_\mu p_\sigma) + \frac{p^4}{15}\left(\frac{m^2}{p^2}+\frac{1}{4}\right)^2(\delta_{\mu\nu}\delta_{\lambda\sigma}+\delta_{\mu\lambda}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\lambda})\right\}\left[\frac{-\pi}{2p^2}\theta(p)\theta(-p^2-4m^2)(p^4+4m^2p^2)^{1/2}\right].$$

[1]C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954).
[2]A. Einstein, Ann. Phys. (Leipzig) 49, 769 (1916).
[3]R. E. Pugh, Ann. Phys. (N.Y.) 23, 335 (1963).
[4]R. E. Pugh, J. Math. Phys. 6, 740 (1965).
[5]R. E. Pugh, Phys. Rev. D 4, 353 (1971).
[6]R. E. Pugh, Ann. Phys. (N.Y.) 30, 422 (1964).
[7]R. E. Pugh, J. Math. Phys. 7, 376 (1966).
[8]Gerry McKeon, University of Toronto doctoral thesis (unpublished).
[9]H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 205 (1955).
[10]R. P. Feynman, Acta Phys. Pol. 24, 697 (1963).
[11]B. S. DeWitt, Phys. Rev. 162, 1195 (1967).
[12]L. Faddeev and V. Popov, Phys. Lett. 2B, 29 (1967).
[13]G. 't Hooft and M. Veltman, CERN Report No. 73-9, 1973 (unpublished).
[14]J. G. Wray, Ph.D. thesis, University of Syracuse, 1966 (unpublished).
[15]T. D. Lee and M. Nauenberg, Phys. Rev. 133, B1549 (1964).
[16]G. 't Hooft, Nucl. Phys. B33, 173 (1971).
[17]G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré 20, 69 (1974).
[18]D. M. Capper, G. Leibbrandt, and M. Medrano, Phys. Rev. D 8, 4320 (1973).
[19]M. Brown, Nucl. Phys. B56, 194 (1973).
[20]R. Cutkosky, J. Math. Phys. 1, 429 (1960).
[21]F. Rohrlich and J. C. Stoddard, J. Math. Phys. 6, 495 (1965).
[22]B. S. DeWitt, Phys. Rev. 160, 1113 (1967).
[23]R. Arnowitt, S. Deser, and C. Misner, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962).
[24]M. Fierz and W. Pauli, Proc. R. Soc. London A173, 211 (1939).
[25]M. J. Duff, Trieste Report No. IC/73/70, 1973 (unpublished).
[26]V. Bargmann and E. Wigner, Proc. Nat. Acad. Sci. USA 34, 211 (1948).
[27]H. van Dam and M. Veltman, Nucl. Phys. B22, 397 (1970).