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Photon decay into neutrinos in a strong magnetic field*

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The real photon decay, $\gamma \rightarrow \nu\bar{\nu}$, is calculated in strong magnetic fields ($H \sim 10^{13}$ G) using a version of Schwinger's mass operator technique, which makes use of the explicit form of the electron Green's function. The reaction is kinematically allowed by an index of refraction less than one. This latter condition may arise either from quantum electrodynamics (high frequencies) or from the effects of a material medium or plasma (low frequencies). The calculation is based on point charged- and neutral-current interactions for the leptons, which should agree with an $SU_2 \times U_1$ gauge-theory calculation as long as ω/m_W and $eH/m_W{}^2$ are both small compared to unity (where ω is the photon frequency and m_W is the mass of the W boson). Numerical results for the absorption coefficient and energy loss rate are given, since the mechanism may be important in the astrophysics of neutron stars.

I. INTRODUCTION

In recent years the importance of neutrino physics in the understanding of stellar processes has been widely recognized. On the one hand, it has been suggested that coherent neutrino pressure may be responsible for supernova explosions.[1] On the other, the very weakness of neutrino interactions means that stellar matter is virtually transparent to neutrinos, so that even if they are produced slowly within a star, they may provide a highly efficient cooling mechanism.[2]

We here wish to address ourselves to the question of neutrino production. A number of mechaisms by which neutrinos may be produced within a star have been dealt with in detail,[3] including the pair process ($e^+ + e^- \rightarrow \nu + \bar{\nu}$), the photoneutrino process ($\gamma + e \rightarrow e + \nu + \bar{\nu}$), and the plasmon process[4] ($\gamma^* \rightarrow \nu + \bar{\nu}$). These processes were first considered in the conventional weak-interaction theory, and more recently in the theoretically favored $SU_2 \times U_1$ gauge theory.[5,6] (Even the effect of heavy leptons, which may well exist,[7] has been examined.[8])

The probable existence of magnetic fields in pulsars comparable to, or even exceeding, the critical field value[9] $H_0 = m^2/e = 4.41 \times 10^{13}$ G has led to the consideration of these same processes in homogeneous magnetic fields.[10,11] Moreover, calculations have been made for processes that can only occur in the presence of a magnetic field, such as the synchrotron neutrino process[12] ($e \rightarrow e + \nu + \bar{\nu}$). Partially because of the somewhat cumbersome methods used, various unreliable approximations have proved necessary—unreliable, that is, for fields $H \gtrsim H_0$. In this paper we will exploit the far superior mass operator (or proper-time) approach,[13] which has already been used to compute astrophysically important processes, notably Compton scattering in external magnetic fields,[14] which is significant in determining the radiative opacity. We here apply this method, not to a reconsideration of the above-mentioned effects, but to a new process, one in which a real photon (not a plasmon) creates a neutrino and an antineutrino,

$$\gamma \rightarrow \nu + \bar{\nu}.$$

Like the synchrotron neutrino process, this effect can only occur in the presence of a magnetic field. Even then momentum is conserved (because all the external particles are uncharged), but as in photon splitting,[15] as long as the index of refraction, n, satisfies certain conditions (in this case, $n<1$, for strictly massless neutrinos), there is nonvanishing phase space. This index of refraction may arise from either electrodynamic[16] or plasma[17] effects. Beyond opening up the phase space, we will not here consider the other consequences of the index of refraction; we expect our results to be relevant for low plasma densities (see the Appendix). For appropriate circumstances, we find that this mechanism might provide significant energy loss for a highly magnetized neutron star, as well as a means by which highly energetic neutrinos may be produced in a pulsar magnetosphere. Of course, the plasmon mode may be more realistic in most high-density situations than the real photon decay; we hope to recompute this process in the future using mass operator methods. Such a recomputation seems necessary because the axial-vector term in the weak current, which is responsible for the entire real photon decay rate, was omitted from previous calculations.[18] A formidable difficulty is that strong-field plasma effects are not understood.

II. THE LEPTON-EXCHANGE PROCESS

We consider those processes by which a real photon, of four-momentum k^μ ($k^2=0$, $k^0=\omega$), converts itself, in the presence of a strong magnetic field, into a $\nu\bar{\nu}$ pair. The processes are shown in Fig. 1, where W and Z stand for the charged and neutral weak intermediate vector bosons in an $SU_2 \times U_1$ or U_2 gauge theory.[19,20] In such theories there are also analogous processes in which a pair of W bosons are exchanged, but those should be smaller by a factor of order m^2/m_W^2, as long as ω/m_W and eH/m_W^2 are both much less than unity. For the same reason it is possible to simplify the boson propagators, for example,

$$\Delta_W^{\mu\nu}(x,x') \to \frac{1}{m_W^2}\delta(x-x')g^{\mu\nu}. \tag{1}$$

More phenomenologically, the two terms in Fig. 1 refer to point (Fermi-type) neutral- and charged-current interactions, which are well established experimentally, unlike the gauge bosons.

We start with the neutral-current process of Fig. 1(a) by considering the Z-A mass operator, defined in terms of the vacuum persistence amplitude[21]

$$\langle 0_+ | 0_- \rangle = -i\int (dx)\,(dx')\,A_\mu(x)\mathfrak{M}_l^{\mu\lambda}(x,x')Z_\lambda(x'), \tag{2}$$

where

$$\mathfrak{M}_l^{\mu\lambda} = -\tfrac{1}{4}\, i\,\mathrm{Tr}\, eq\gamma^\mu G(x,x')\,(\lambda_1 q + \lambda_2 i\gamma_5)\,\gamma^\lambda G(x',x) + \mathrm{ct}, \tag{3}$$

where q is the charge matrix, the trace is over both Dirac and charge indices, and ct stands for a local contact term. The coupling constants, λ_1 and λ_2, would be, in Weinberg's theory,[19]

$$\begin{aligned} \lambda_1 &= \frac{1}{2}\,\frac{3g'^2 - g^2}{(g^2+g'^2)^{1/2}}, \\ \lambda_2 &= \tfrac{1}{2}(g^2+g'^2)^{1/2}, \\ g^2 &= 4\sqrt{2}\,m_W^2 G, \quad \frac{g^2+g'^2}{g^2} = \left(\frac{m_Z}{m_W}\right)^2, \\ e &= \frac{gg'}{(g^2+g'^2)^{1/2}}. \end{aligned} \tag{4}$$

The easiest way to evaluate (3) is to use the explicit form[13,22] for the electron propagation function in an external magnetic field, which is assumed to lie along the z axis, so that $F_{12} = -F_{21} = H$:

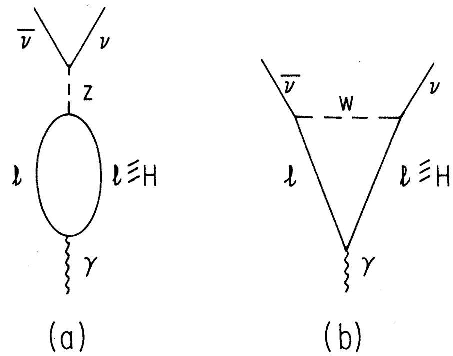

FIG. 1. Lepton-exchange processes by which a photon can decay into neutrino and antineutrino. In practice, the vector bosons are replaced by point neutral- and charged-current interactions.

$$G(x,x') = \Phi(x,x')\mathcal{G}(x-x'), \tag{5}$$

where

$$\Phi(x,x') = \exp\left[ieq\int_{x'}^{x} A^\mu(\xi)d\xi_\mu\right], \tag{6}$$

$$\mathcal{G}(x) = \int \frac{(dp)}{(2\pi)^4}\,e^{ipx}\mathcal{G}(p), \tag{7}$$

$$\mathcal{G}(p) = i\int ds\,\exp\left[-is\left(m^2 + p_\parallel^2 + \frac{\tan z}{z}\,p_\perp^2\right)\right] \times \frac{1}{\cos z}\left[(m-\gamma p_\parallel)e^{iq\sigma_3 z} - \frac{1}{\cos z}\,\gamma p_\perp\right], \tag{8}$$

making use of the notations

$$\begin{aligned} (ab)_\parallel &= -a^0b^0 + a_3b_3, \\ (ab)_\perp &= a_1b_1 + a_2b_2, \\ z &= seH. \end{aligned} \tag{9}$$

The evaluation of (3) is very similar to the vacuum polarization calculation carried out by Tsai[23] (in fact, for the parity-conserving part it is identical). We use the traces given in Eqs. (27) and (28) of Ref. 23, as well as the Dirac traces

$$\tfrac{1}{4}\,\mathrm{tr}\,e^{iq\sigma_3 z}\gamma_\mu\gamma_\nu i\gamma_5 q = i\sin z\left(\frac{{}^*F}{H}\right)_{\mu\nu} \tag{10}$$

and

$$\tfrac{1}{4}\,\mathrm{tr}\,e^{iq\sigma_3 z}\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma i\gamma_5 q = iq\cos z\,\epsilon_{\mu\nu\lambda\sigma} + i\sin z\left[-g_{\mu\nu}\left(\frac{*F}{H}\right)_{\lambda\sigma} - g_{\lambda\sigma}\left(\frac{*F}{H}\right)_{\mu\nu} + g_{\lambda\mu}\left(\frac{*F}{H}\right)_{\nu\sigma} + g_{\sigma\mu}\left(\frac{*F}{H}\right)_{\lambda\nu} + g_{\lambda\nu}\left(\frac{*F}{H}\right)_{\sigma\mu} - g_{\sigma\nu}\left(\frac{*F}{H}\right)_{\lambda\mu}\right], \tag{11}$$

in which the dual field strength appears,

$$*F^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma}F_{\lambda\sigma}.$$

We find that

$$\mathfrak{M}^{\mu\lambda}(k) = \frac{e}{(4\pi)^2}\int_0^\infty \frac{ds}{s}\int_{-1}^1 \frac{dv}{2}\,e^{-is\phi}\Big\{\lambda_1[(g^{\mu\lambda}k^2 - k^\mu k^\lambda)N_0 - (g_\parallel^{\mu\lambda}k_\parallel{}^2 - k_\parallel^\mu k_\parallel^\lambda)N_1 + (g_\perp^{\mu\lambda}k_\perp{}^2 - k_\perp^\mu k_\perp^\lambda)N_2] + \lambda_2 is\Big[\Big(2m^2 + \frac{1-v^2}{2}k_\parallel{}^2\Big)(e\,*F)^{\lambda\mu} + (1-v^2)k_\parallel^\lambda(e\,*Fk)^\mu + R\big(k_\perp^\mu(e\,*Fk)^\lambda + k_\perp^\lambda(e\,*Fk)^\mu\big)\Big]\Big\} + \mathrm{ct}, \tag{12}$$

where

$$\phi = m^2 + \frac{1-v^2}{4}k_\parallel{}^2 + \frac{\cos zv - \cos z}{2z\sin z}k_\perp{}^2, \tag{13}$$

and

$$R = \frac{1 - v\sin zv\sin z - \cos zv\cos z}{\sin^2 z}. \tag{14}$$

In Ref. 23, N_0, N_1, and N_2 are given, for which contact terms have been supplied in order to ensure gauge invariance. The last three terms, the axial-vector contribution, may be equally rendered gauge invariant by integrating by parts and omitting the surface term, since

$$i\int ds\Big(2m^2 + \frac{1-v^2}{2}k_\parallel{}^2\Big)e^{-is\phi} = 2 - i\int ds\,k_\perp{}^2 R\,e^{-is\phi}. \tag{15}$$

The result can be seen in (21) below. The net coupling between neutrino current and photon field for the neutral-current contribution is, summing over both kinds of charged leptons,

$$\langle 0_+|0_-\rangle = -i\frac{\lambda_3}{m_Z{}^2}\int\frac{(dk)}{(2\pi)^4}A_\mu(k)\times\sum_l \mathfrak{M}_l^{\mu\lambda}(k)\tfrac{1}{2}(\psi_\nu\gamma^0\gamma_\lambda i\gamma_5\psi_\nu)(-k), \tag{16}$$

where λ_3 is the coupling constant between the Z and the neutrino current and, in the Weinberg theory,[19] would be

$$\lambda_3 = \tfrac{1}{2}(g^2 + g'^2)^{1/2}. \tag{17}$$

The term in (12) proportional to λ_1 does not contribute to the $\gamma\to\nu\bar\nu$ process because the neutrino current is proportional to k_λ [see Eq. (28)], leaving the parity-violating term in the mass operator as the entire source of the effect here.

The charged-current process of Fig. 1(b) can be cast into the same form as (16) by a Fierz transformation when the approximation (1) is valid. The result can be obtained from the neutral-current expressions, (12) and (16), by deleting the summation sign in (16) and replacing

$$\frac{\lambda_2\lambda_3}{m_Z{}^2}\to -2\sqrt{2}\,G, \qquad \frac{\lambda_1\lambda_3}{m_Z{}^2}\to -2\sqrt{2}\,G, \tag{18}$$

since the charged-current normalization is fixed by the Fermi interaction.

The net coupling of the photon to two neutrinos is then (where l refers to the charged lepton corresponding to the type of neutrino emitted)

$$\langle 0_+|0_-\rangle = ie\int\frac{(dk)}{(2\pi)^4}A_\mu(k)D_l^{\mu\lambda}(k)\times\tfrac{1}{2}(\psi_\nu\gamma^0\gamma_\lambda i\gamma_5\psi_\nu)(-k), \tag{19}$$

where

$$D_l^{\mu\lambda} = 2\sqrt{2}\,G\bar{\mathfrak{M}}_l^{\mu\lambda} - \frac{\lambda_2\lambda_3}{m_Z{}^2}\sum_{l'}\bar{\mathfrak{M}}_{l'}^{\mu\lambda}, \tag{20}$$

with $\bar{\mathfrak{M}}^{\mu\lambda}$ given by

$$\bar{\mathfrak{M}}^{\mu\lambda} = \frac{i}{(4\pi)^2}\int ds\frac{dv}{2}e^{-is\phi}[(1-v^2)k_\parallel^\lambda(e\,*Fk)^\mu + R(-k_\perp{}^2e\,*F^{\lambda\mu} + k_\perp^\mu(e\,*Fk)^\lambda + k_\perp^\lambda(e\,*Fk)^\mu)]. \tag{21}$$

A *priori*, there need be no relation between the charged and neutral currents. However, in the Weinberg theory there is a simple relation between the two terms so that (20) becomes

$$D_l^{\mu\lambda} = 2\sqrt{2}\, G\left(\mathfrak{M}_l^{\mu\lambda} - \tfrac{1}{2} \sum_{l'} \mathfrak{M}_{l'}^{\mu\lambda}\right). \qquad (22)$$

Henceforth, we will make this simplifying assumption: It will certainly give us the correct order of magnitude, and it is an easy matter to go back and put in more accurate coupling constants as experimental data become available.[24]

III. ABSORPTION COEFFICIENT AND PHASE SPACE

To find the rate at which $\gamma \to \nu\bar{\nu}$, we square the amplitude we have found in terms of (19) and integrate over the neutrino phase space. The result may be expressed as an absorption coefficient, κ, the physical meaning of which is related to the decrease in intensity I of a light beam propagating a distance z:

$$I(z) = I(0)e^{-\kappa z}. \qquad (23)$$

We find, for photon polarization ϵ, and a given kind of neutrino, that

$$\kappa = \frac{1}{2\omega} e^2 \epsilon_\mu D^{\mu\alpha}(k) \mathrm{tr} \int d\omega_p d\omega_{p'} (2\pi)^4 \delta(p + p' - k) \times \gamma_\alpha \frac{1 \pm i\gamma_5}{2} \gamma p \gamma_\beta \gamma p' \times D^{\nu\beta}(k)^* \epsilon_\nu^*. \qquad (24)$$

The phase-space integral in (24) is, at best, ambiguous. The momenta of the two neutrinos must be parallel to $\vec{k}$ so that the support of the energy-momentum-conserving δ function lies on the boundary of the phase-space region. Rather than belabor the mathematical point, we note that the physics determines if the process goes or not. Because of the presence of the magnetic field, if for no other reason, the photon moves through a medium characterized by an index of refraction, n. This purely electrodynamic effect has been considered most thoroughly by Tsai and Erber.[16]

The condition for the phase space for $\gamma \to \nu\bar{\nu}$ to be nonvanishing is

$$n < 1. \qquad (25)$$

This condition assumes that the neutrino is strictly massless. If the neutrino has a small mass ν (the current limits are $\nu < 60$ eV for the electron neutrino and $\nu < 650$ keV for the muon neutrino), a stronger, frequency-dependent, inequality is required:

$$(1 - n^2)^{1/2} > \frac{2\nu}{\omega}. \qquad (26)$$

If only a magnetic field is present, the known limits[16] are

(1) $n > 1$, if $\omega/m \ll 1$, eH/m^2 arbitrary,

(2) $n < 1$, if $\omega/m \gg 1$, $eH/m^2 \ll 1$.

The deviations from unity are, of course, quite small. The second condition allows the reaction; for a more precise specification of the criterion, see Sec. IV B. The high-frequency, high-field behavior is unknown. Another possible source of an index of refraction is a plasma.[17] In the absence of a magnetic field, the classical result is (valid for low frequencies)

$$n = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}, \qquad (27)$$

where ω_p is the plasma frequency. Although weak-field corrections to this result are available,[17] it is not known how to include strong-field effects. However, (27) indicates that, for low frequencies, when plasma effects dominate over electrodynamic effects in determining the index of refraction, the process $\gamma \to \nu\bar{\nu}$ may proceed. [However, it should be noted that if the neutrino mass $\nu \neq 0$, the restriction (26) is most severe for low frequencies.]

Assuming then that we are in a region where the reaction is kinematically allowed but that $1 - n \ll 1$ (however, see the Appendix), the phase-space integral in (24) is

$$\int d\omega_p d\omega_{p'} (2\pi)^4 \delta(p + p' - k) p_\mu p'_\nu = \frac{1}{8\pi} \frac{1}{6} k_\mu k_\nu, \qquad (28)$$

so that (24) becomes

$$\kappa = \frac{\alpha}{6} \frac{1}{\omega} |\epsilon_\mu D^{\mu\nu} k_\nu|^2. \qquad (29)$$

Now making use of (22), we find the essential result,

$$\kappa = \frac{\alpha G^2}{192\pi^4} \frac{1}{\omega} |\epsilon e^* F k|^2 |M_e - M_\mu|^2, \qquad (30)$$

where, from (21),

$$M = i \int ds \frac{dv}{2} e^{-is\phi} \left(\frac{1 - v^2}{2} k_\parallel^2 + R k_\perp^2\right). \qquad (31)$$

The subscripts refer to the two types of charged leptons. The particularly simple structure of (30) reflects the $SU_2 \times U_1$ relation between coupling constants and masses. In most applications of this result, the dominant contribution will come from the electron since the muon is so much more massive.

Note that if $\vec{\epsilon}$ is perpendicular to the plane defined by $\vec{k}$ and $\vec{H}$, the amplitude for neutrino production vanishes. So suppose that $\vec{\epsilon}$ lies in the $\vec{k}$, $\vec{H}$ plane. With $\vec{H}$ lying along the z axis, and

$$\vec{k} = \omega(\sin\theta, 0, \cos\theta), \quad \vec{\epsilon} = (-\cos\theta, 0, \sin\theta), \qquad (32)$$

we have, in (30) and (31),

$$(\epsilon e\, {}^*Fk) = \omega \sin\theta\, eH\,,$$
$$k_\perp{}^2 = -k_\parallel{}^2 = \omega^2 \sin^2\theta\,. \tag{33}$$

IV. LIMITING CASES

We now consider simple limiting cases of the above general result, for the polarization given by (32). Throughout we imagine that we are in a region in which the index of refraction is less than one so that the reaction is kinematically allowed, but, otherwise, we neglect all effects due to the fact that $n \neq 1$ (thus we are not considering the plasmon process). The conditions on $1-n$ for this to be a good approximation are considered in the Appendix.

A. Low-frequency limit[16]

When $\omega \ll m$, but eH is arbitrary, we can rotate the contour in (31), $s \to -is$, and replace the exponential by its zero-frequency form:

$$e^{-s\Phi} \simeq e^{-zm^2/eH}. \tag{34}$$

Then making use of the integral

$$\int_0^\infty dz \int_{-1}^1 \frac{dv}{2}\, e^{-2zh}\, \frac{1+v\sinh z \sinh zv - \cosh z \cosh zv}{\sinh^2 z} = \int_0^\infty dz\, e^{-2zh}\left(\frac{1}{\sinh^2 z} - \frac{1}{z^2}\right) = 1 + 2h(\psi(h) - \ln h)\,, \tag{35}$$

where

$$h = \frac{m^2}{2eH} \tag{36}$$

and ψ is the digamma function, we easily find that

$$M_l \simeq \frac{\omega^2}{eH}\, \sin^2\theta\, f(h_l)\,, \tag{37}$$

where

$$f(h) = 1 + \frac{1}{6h} + 2h(\psi(h) - \ln h) \tag{38a}$$

$$\to \begin{cases} \dfrac{2}{15}\left(\dfrac{eH}{m^2}\right)^3, & \dfrac{eH}{m^2} \ll 1 \\[2ex] \dfrac{1}{3}\,\dfrac{eH}{m^2}, & \dfrac{eH}{m^2} \gg 1\,. \end{cases} \tag{38b}$$

In this limit, the absorption coefficient depends only on the electron, which reflects the physical requirement that low-frequency phenomena are dominated by low-mass states. The result is, for photon polarization in the $(\vec{k}, \vec{H})$ plane,

$$\kappa \simeq \frac{\alpha G^2}{192\pi^4}\, \omega^5 \sin^6\theta [f(h)]^2$$
$$\simeq 8.91 \times 10^{-20} \left(\frac{\hbar\omega}{mc^2}\right)^5 \sin^6\theta [f(h)]^2\ \text{cm}^{-1}\,, \tag{39}$$

for each kind of neutrino. Values of $f(h)$ for various magnetic field strengths appear in Table I.

TABLE I. The magnetic field dependence of the amplitude for $\gamma \to \nu\bar{\nu}$ for low frequencies.

h	H/H_0	$f(h)$
10^6	5×10^{-7}	1.67×10^{-20}
100	0.005	1.67×10^{-8}
20	0.025	2.08×10^{-6}
10	0.05	1.66×10^{-5}
5	0.1	1.31×10^{-4}
2	0.25	1.88×10^{-3}
1	0.5	0.0123
0.5	1	0.0630
0.2	2.5	0.362
0.1	5	1.04
0.05	10	2.58
0.01	50	15.8
0.001	500	166

B. Weak-field, high-frequency limit[16]

When $eH/m^2 \ll 1$, but with $(\omega/m)\sin\theta \gg 1$, we may evaluate the absorption coefficient in terms of Airy functions. Define

$$\lambda = \frac{3}{2}\,\frac{eH}{m^2}\,\frac{\omega}{m}\,\sin\theta\,, \tag{40}$$

which may be large. Note that if only a magnetic field is present, the condition that $n<1$, so that the reaction may proceed, is satisfied for $\lambda \gtrsim 24$ (see Ref. 16). The significant contributions to the exponent come from small z in this case,

$$z \sim \frac{eH}{m^2}\,,$$

but we must retain terms of order

$$\frac{\omega^2 \sin^2\theta}{eH}\, z^3 \sim \lambda^2\,.$$

We thus approximate

$$e^{-is\Phi} \simeq e^{-i\Theta}, \tag{41}$$

with

$$\Theta = \zeta^{3/2}(y + \tfrac{1}{3}y^3)\,, \tag{42}$$

where

$$\zeta^{3/2} = \frac{6}{\lambda}\,\frac{1}{1-v^2}\,, \tag{43}$$

$$y = \frac{1-v^2}{4}\,\frac{\omega}{m}\,\sin\theta\, z\,.$$

The leading contribution to the reduced mass operator (31) is

$$M \simeq i\,\frac{2m^2}{eH}\,\frac{m}{\omega\sin\theta}\int_0^\infty dy\int_{-1}^{1}\frac{dv}{2}\,e^{-i\Theta}\,\frac{4y^2}{1-v^2}\,, \tag{44}$$

which easily yields the absorption coefficient (for producing each kind of neutrino)

$$\kappa \cong \frac{\alpha G^2}{108\pi^4}\,\frac{m^6}{\omega}\,\lambda^2\,|J_e - J_\mu|^2\,, \tag{45}$$

where

$$J = \pi\int_0^1 dv[\mathrm{Ai}''(\xi) - i\,\mathrm{Gi}''(\xi)]\,. \tag{46}$$

Appearing here are second derivatives of the Airy functions, defined by

$$\pi\,\mathrm{Ai}(\xi) = \int_0^\infty dt\,\cos(\xi t + \tfrac{1}{3}t^3)\,, \qquad \pi\,\mathrm{Gi}(\xi) = \int_0^\infty dt\,\sin(\xi t + \tfrac{1}{3}t^3)\,, \tag{47}$$

$$\mathrm{Ai}''(\xi) = \xi\,\mathrm{Ai}(\xi)\,, \qquad \mathrm{Gi}''(\xi) = \xi\,\mathrm{Gi}(\xi) - 1/\pi\,. \tag{48}$$

We have evaluated the integrals composing J numerically, making use of the tabulated values[25,26] of Ai and Gi″; the results appear in Fig. 2.

An interesting case is the very-high-frequency limit, when $\omega\sin\theta/m$ is so large that $\lambda \gg 1$. Then $\xi \ll 1$ almost everywhere in the domain of integration, so making use of

$$\sqrt{3}\,\mathrm{Gi}(0) = \mathrm{Ai}(0) = \frac{3^{-2/3}}{\Gamma(\frac{2}{3})}\,, \tag{49}$$

we find that

$$J = i + \left(1 - \frac{i}{\sqrt{3}}\right)\frac{\pi}{(2\lambda)^{2/3}}\left[\frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}\right]^2\,. \tag{50}$$

The asymptotic formula for ImJ is accurate for $\lambda \gtrsim 50$, but for ReJ it is in error still by 50% at $\lambda = 1000$ (see Fig. 2).

Bearing in mind the above limitation on the validity of the asymptotic formulas, we can give explicit expressions for the absorption coefficient in two regions. When $\lambda_\mu \gg 1$, the muon contribution dominates, since the leading, mass-independent terms cancel in (45), so that

$$\kappa \sim \frac{\alpha G^2}{324\pi^2}\,\frac{m_\mu{}^6}{\omega}\,\lambda_\mu{}^{2/3}\,2^{2/3}\left[\frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}\right]^4 = 2.29\times10^{-8}\,\frac{m_e c^2}{\hbar\omega}\,\lambda_e{}^{2/3}\,\mathrm{cm}^{-1}\,. \tag{51}$$

In the intermediate region, $\lambda_e \gg 1 \gg \lambda_\mu$, the electron is dominant and the entire contribution comes from Gi″(0):

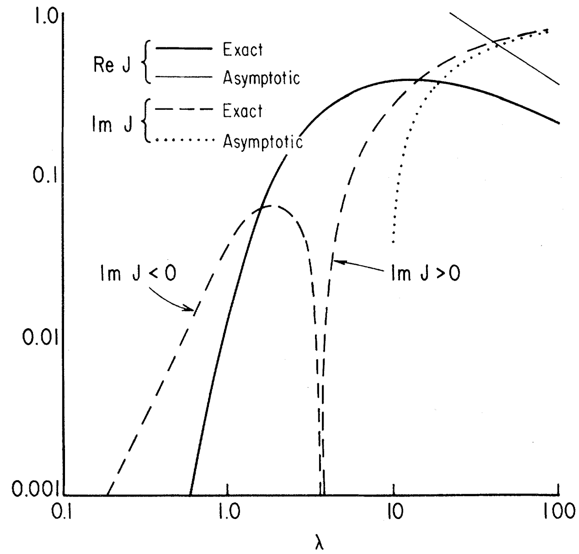

FIG. 2. The real and imaginary parts of J as a function of λ, compared with their asymptotic forms. Note that for $\lambda \lesssim 3.5$, $-\,\mathrm{Im}J$ is plotted.

$$\kappa \sim \frac{\alpha G^2}{48\pi^4}\,(eH)^2\omega\sin^2\theta = 3.56\times10^{-19}\left(\frac{\hbar\omega}{mc^2}\right)\left(\frac{H}{H_0}\right)^2\sin^2\theta\,\mathrm{cm}^{-1}\,. \tag{52}$$

These limiting values are far larger than the weak-field, low-frequency limit given earlier [see (38b)]. But in all these regimes the phenomenon is well beyond laboratory accessibility.

V. ENERGY-LOSS RATE

Suppose we consider a thermal distribution of photons in a strong magnetic field. Since κc is the probability of attenuation per unit time, and only polarizations in the $(\vec{k}, \vec{H})$ plane are effective for this energy-loss mechanism, we find the rate of energy loss per unit volume to be

$$Q = \frac{1}{(2\pi c)^2}\int_0^\infty \omega^2 d\omega\, d(\cos\theta)\,\frac{\hbar\omega\,\kappa(\omega,\theta)}{e^{\hbar\omega/kT} - 1}\,. \tag{53}$$

Particularly simple is the low-temperature limit, where

$$T \ll mc^2/k = 5.93\times10^9\,{}^\circ\mathrm{K}\,,$$

which is not at all an unrealistic limit. Then, provided the medium supplies an index of refraction less than 1, we can calculate the energy-loss rate from the low-frequency form of κ, Eq. (39). We find, summing over both kinds of neutrions, that

$$Q \simeq \frac{\alpha G^2}{420\pi^6}\,(kT)^9\,\Gamma(9)\,\zeta(9)\,[f(h)]^2\,, \tag{54a}$$

where ζ is the Riemann ζ function. Numerically, this equals

$$Q \simeq 7.11\times 10^{19}\left(\frac{kT}{mc^2}\right)^9 [f(h)]^2 \,\mathrm{erg\,cm^{-3}\,sec^{-1}}. \tag{54b}$$

For example,

$T=2.5\times 10^8\,°\mathrm{K}$: $Q=2.99\times 10^7[f(h)]^2\,\mathrm{erg\,cm^{-3}\,sec^{-1}}$,

$T=3.85\times 10^8\,°\mathrm{K}$: $Q=1.46\times 10^9[f(h)]^2\,\mathrm{erg\,cm^{-3}\,sec^{-1}}$,

$T=10^9\,°\mathrm{K}$: $Q=7.84\times 10^{12}[f(h)]^2\,\mathrm{erg\,cm^{-3}\,sec^{-1}}$.

So when $f(h)\sim 1$ (see Table I) this process may be comparable, for low densities, to the plasmon and neutrino synchrotron processes, as calculated by Canuto *et al.*[10,11] Further calculations of this type are obviously warranted, especially for the plasmon process.

VI. CONCLUSIONS

Our calculation of the real photon decay into two neutrinos is interesting on several grounds. First, the $\gamma\to\nu\bar\nu$ decay may be a significant energy-loss mechanism in highly magnetic stars (specifically in low-density regions), either for strong fields, $H \gtrsim H_0$, or high temperatures $kT \gg m_e c^2$. Moreover, in the exterior of a pulsar very-high-energy photons are believed to be present (perhaps $\hbar\omega \sim 10^{14}$ eV) and the photoabsorption due to this mechanism can be quite large. In constructing a model of such a region, it is important to understand the elementary physical processes which can arise.

Second, our calculation indicates that the more common plasmon mode may not have been accurately calculated by Canuto *et al.*,[18] since they neglect the parity-violating term [the term involving dual field strengths in (12)] which is entirely responsible for our effect, so we would expect that it should be important there as well. Furthermore, it is likely that they have not included strong-field effects accurately in the photon polarization tensor that they do calculate. For such calculations that do properly incorporate plasma effects, the kinematic restriction, $n<1$, would emerge naturally, unlike in the above, where the only function of a plasma was to open up the phase space so that the process could occur (see Appendix).

Beyond considering the effects of a plasma, we should, of course, supplement our calculation by including the processes involving W exchange, which are present in a gauge theory (as opposed to the effective phenomenological local coupling we adopted here). Certainly such terms become important when $\omega\sim m_W$, which probably does occur in pulsars as we mentioned above.

Finally, we should emphasize that our calculation is one of a very few[14-16] which really should be valid for fields up to perhaps 10^{16} G. [Beyond that point perturbation theory breaks down because $(\alpha/\pi)\,(eH/m^2)$ is now no longer small.] We may hope that such methods as employed here might find application elsewhere.

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APPENDIX: EFFECTS OF THE INDEX OF REFRACTION

In the text, the sole function of the index of refraction is to determine whether phase space is nonvanishing or not. Aside from this, we calculated the amplitude for $\gamma\to\nu\bar\nu$ in the approximation that the index of refraction is unity. We then expect our results to be a good approximation when $1-n$ is small, that is, for low-density plasmas or when the index of refraction results from the magnetic field itself.[16] We here investigate the terms linear in $1-n$ for the absorption coefficient and determine when the above is a valid approximation.

The effect arises entirely from the fact that k^2 is no longer zero:

$$-k^2=\omega^2(1-n^2)\simeq 2\omega^2(1-n). \tag{A1}$$

The corresponding alteration in the phase-space integral, (28), is

$$\int d\omega_p d\omega_{p'}(2\pi)^4\delta(p+p'-k)p_\mu p'_\nu = \frac{1}{8\pi}\left(\tfrac{1}{6}k_\mu k_\nu+\tfrac{1}{12}k^2 g_{\mu\nu}\right). \tag{A2}$$

The absorption coefficient, κ, is then given by

$$\kappa=\frac{\alpha}{6}\frac{1}{\omega}\,\epsilon_\mu D^{\mu\alpha}(k_\alpha k_\beta-k^2 g_{\alpha\beta})\epsilon^*_\nu D^{\nu\beta\,*}, \tag{A3}$$

where now we must include both vector and axial-vector contributions in $D^{\mu\alpha}$. If we ignore the muon contribution, we can write

$$D^{\mu\lambda}=\pm\frac{2\sqrt{2}G}{(4\pi)^2}\Big\{A(g^{\mu\lambda}k^2-k^\mu k^\lambda)+B(g_{\parallel}^{\mu\lambda}k_{\parallel}{}^2-k_{\parallel}^\mu k_{\parallel}^\lambda)+C(g_{\perp}^{\mu\lambda}k_{\perp}{}^2-k_{\perp}^\mu k_{\perp}^\lambda)+\frac{1}{eH}Dk_{\parallel}^\lambda(e\,{}^*Fk)^\mu$$
$$+\frac{1}{eH}E[-k_\perp{}^2 e\,{}^*F^{\lambda\mu}+k_\perp^\mu(e\,{}^*Fk)^\lambda+k_\perp^\lambda(e\,{}^*Fk)^\mu]\Big\}, \tag{A4}$$

where + (−) refer to the electron (muon) neutrino. The functions here are defined as

$$(A,B,C,D,E)=\int_0^\infty \frac{ds}{2}\int_{-1}^{1}\frac{dv}{2}e^{-is\phi}\left(\beta N_0,\,-\beta N_1,\,\beta N_2,\,i\,seH\frac{1-v^2}{2},\,iseH\frac{R}{2}\right), \tag{A5}$$

where

$$\beta=\begin{cases}1-r\,, & \text{electron neutrino}\\ r\,, & \text{muon neutrino}\end{cases} \tag{A6}$$

with, using the Weinberg relation between coupling constants,

$$2\sqrt{2}Gr=\frac{\lambda_1\lambda_3}{m_Z^{\,2}}=2\sqrt{2}G\left(\frac{3}{2}-2\frac{m_W^{\,2}}{m_Z^{\,2}}\right). \tag{A7}$$

It is now straightforward to calculate the change in the absorption coefficient, $\delta\kappa$, to linear terms in k^2. Both polarizations now contribute, that of (32), which we now call $\vec{\epsilon}_{\parallel}$, and $\vec{\epsilon}_{\perp}$,

$$\vec{\epsilon}_{\perp}=(0,1,0). \tag{A8}$$

We find that

$$\delta\kappa_{\perp}=\frac{\alpha G^2}{192\pi^4}\omega^5\sin^4\theta\frac{-k^2}{\omega^2}|C|^2, \tag{A9}$$

$$\delta\kappa_{\parallel}=\frac{\alpha G^2}{192\pi^4}\omega^5\sin^4\theta\frac{-k^2}{\omega^2}\Big[|B|^2+|D|^2(1-2\sin^2\theta)+2(-1+4\sin^2\theta)\mathrm{Re}D^*E-8|E|^2\sin^2\theta-2\frac{\omega^2\sin^2\theta}{m^2}\mathrm{Re}(-D+2E)\Delta^*\Big]. \tag{A10}$$

The new function here, Δ, is defined as

$$\Delta=m^2\int_0^\infty ds\int_{-1}^{1}\frac{dv}{2}e^{-is\phi}z\left(-\frac{1-v^2}{2}+R\right)\left(\frac{1-v^2}{4}\cos^2\theta+\frac{\cos zv-\cos z}{2z\sin z}\sin^2\theta\right). \tag{A11}$$

In all these functions, we replace

$$\phi\to m^2+\omega^2\sin^2\theta\left(-\frac{1-v^2}{4}+\frac{\cos zv-\cos z}{2z\sin z}\right). \tag{A12}$$

The general condition for the validity of our approximation is then

$$\frac{|\delta\kappa_{\perp,\parallel}|}{\kappa}\ll 1. \tag{A13}$$

As an illustration of this condition, we will consider the low-frequency, arbitrary magnetic field limit. Most of the relevant integrations can be found in Ref. 16; we will here just quote the results:

$$-\beta^{-1}B\simeq-\tfrac{1}{3}+2hK_1(h)+J_1+\tfrac{2}{3}\ln h-4K_2(h), \tag{A14}$$

$$\beta^{-1}C\simeq-\tfrac{2}{3}-6h^2J_1+2hK_1(h), \tag{A15}$$

$$D\simeq\frac{1}{6h}, \tag{A16}$$

$$E\simeq-\tfrac{3}{2}hJ_1, \tag{A17}$$

where

$$J_1=\frac{2}{3}\left[\psi(h)-\ln h+\frac{1}{2h}\right], \tag{A18}$$

$$K_1(h)=2\ln\Gamma(1+h)-(1+2h)\ln h-\ln(2\pi)+2h, \tag{A19}$$

$$K_2(h)=2\ln\Gamma_1(1+h)-2L_1-h(1+h)\ln h+\frac{h^2}{2}. \tag{A20}$$

The generalized Γ function, $\Gamma_1(x)$, is defined as

$$\ln\Gamma_1(x)=\int_0^x dt\,\ln\Gamma(t)+\tfrac{1}{2}x(x-1)-\tfrac{1}{2}x\ln(2\pi), \tag{A21}$$

and has the properties

$$\Gamma_1(1+x)=x^x\,\Gamma_1(x),\qquad \Gamma_1(0)=\Gamma_1(1)=\Gamma_1(2)=1. \tag{A22}$$

The constant L_1 is

$$L_1=\tfrac{1}{3}+\int_0^1 dx\,\ln\Gamma_1(1+x)\simeq 0.248\,754\,477. \tag{A23}$$

We will consider three cases, $H\ll H_0$, $H\gg H_0$, and $H=H_0$ and, for simplicity, take $\beta=\frac{1}{2}$ (this corresponds to taking, in the Weinberg model, $m_Z=\sqrt{2}\,m_W$).

Case I. $H \ll H_0$.

$$\begin{aligned} \|:&\ 1-n \ll \frac{1}{8}\left(\frac{H}{H_0}\right)^2 \sin^2\theta\,, \\ \perp:&\ 1-n \ll \left(\frac{H}{H_0}\right)^2 \sin^2\theta\,. \end{aligned} \tag{A24}$$

This would indicate that even for a very small $1-n$, κ would dominate only for magnetic fields not much smaller than the critical field.

Case II. $H \gg H_0$.

$$\begin{aligned} \|:&\ 1-n \ll \tfrac{1}{4}\tan^2\theta\,, \\ \perp:&\ 1-n \ll \frac{1}{2}\left(\frac{H}{H_0}\right)^2 \sin^2\theta\,. \end{aligned} \tag{A25}$$

These impose essentially no restriction so we could expect κ to dominate for rather large values of $1-n$.

Case III. $H=H_0$ (Γ_1 may be evaluated with the aid of Ref. 27).

$$\begin{aligned} \|:&\ 1-n \ll 0.057\ \frac{\sin^2\theta}{1-\frac{1}{4}\sin^2\theta}\,, \\ \perp:&\ 1-n \ll 1\,. \end{aligned} \tag{A26}$$

For the $\|$ polarization, this requires $1-n$ to be less than, say, 10^{-3} while the $\perp$ polarization is subject to essentially no restriction.

Generally, then, κ dominates for reasonable values of $1-n$ as long as the magnetic field is not much smaller than the critical field. This regime ($\omega \ll m$, $H \sim H_0$) seems relevant for pulsars. A similar argument could be carried out in the other limit, $\omega \gg m$, $H \ll H_0$, but we will not discuss this here. It is sufficient to note that the electrodynamic-induced value of $1-n$ in this region is essentially bounded by $\sim(\alpha/4\pi)\,(H/H_0)^2$.

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[1]D. Z. Freedman, Phys. Rev. D 9, 1389 (1974).

[2]B. M. Pontecorvo, Zh. Eksp. Teor. Fiz. 36, 1615 (1959) [Sov. Phys.-JETP 9, 1148 (1959)]; H.-Y. Chiu and P. Morrison, Phys. Rev. Lett. 5, 573 (1960).

[3]G. Beaudet, V. Petrosian, and E. Salpeter, Astrophys. J. 150, 979 (1967).

[4]J. B. Adams, M. A. Ruderman, and C. H. Woo, Phys. Rev. 129, 1383 (1963).

[5]D. A. Dicus, Phys. Rev. D 6, 941 (1972).

[6]A review can be found by S. Weinberg, in *Proceedings of the Seventh Texas Symposium on Relativistic Astrophysics*, edited by P. G. Bergmann *et al*. [Ann. N. Y. Acad. Sci. 262, 289 (1975)].

[7]M. L. Perl *et al*., Phys. Rev. Lett. 35, 1489 (1975).

[8]K. O. Mikaelian, Phys. Rev. Lett. 36, 1089 (1976).

[9]Here and in the following m stands for the mass of the electron, while m_l stands for either electron or muon.

[10]V. Canuto, C. Chiuderi, C. K. Chou, and L. Fassio-Canuto, Astrophys. Space Sci. 28, 145 (1974), and earlier references cited therein.

[11]For a review, see V. Canuto, in *Role of Magnetic Fields in Physics and Astrophysics*, edited by V. Canuto [Ann. N. Y. Acad. Sci. 257, 108 (1975)].

[12]V. Canuto, H.-Y. Chiu, C. K. Chou, and L. Fassio-Canuto, Phys. Rev. D 2, 281 (1970).

[13]J. Schwinger, Phys. Rev. 82, 664 (1951).

[14]L. L. DeRaad, Jr., N. D. Hari Dass, and K. A. Milton, Phys. Rev. D 9, 1041 (1974); K. A. Milton, W-y. Tsai, L. L. DeRaad, Jr., and N. D. Hari Dass, *ibid*. 10, 1299 (1974).

[15]S. L. Adler, Ann. Phys. (N.Y.) 67, 599 (1971).

[16]W-y. Tsai and T. Erber, Phys. Rev. D 12, 1132 (1975).

[17]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), p. 10; D. C. Kelly, Phys. Rev. 134, A641 (1964).

[18]V. Canuto, C. Chiuderi, and C. K. Chou, Astrophys. Space Sci. 7, 407 (1970); 9, 453 (1970).

[19]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971).

[20]J. Schwinger, Phys. Rev. D 7, 908 (1973).

[21]We use the notation of J. Schwinger, *Particles, Sources, and Fields*, Vols. I and II (Addison-Wesley, Reading, Mass., 1970 and 1973).

[22]W-y. Tsai, Phys. Rev. D 10, 1342 (1974).

[23]W-y. Tsai, Phys. Rev. D 10, 2699 (1974).

[24]The Weinberg choice is favored by the fact that then the surface term in (15) cancels between electron and muon contributions, so no contact term need be supplied here.

[25]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series, No. 55 (U. S. G. P. O., Washington, D. C., 1964), pp. 446 ff.

[26]L. N. Nosova and S. A. Tumarkin, *Tables of Generalized Airy Functions* (MacMillan, New York, 1965).

[27]British Association for the Advancement of Science, *Mathematical Tables*, Vol. I (Cambridge Univ. Press, Cambridge, 1931), Table X.