Comments on the neutron charge radius and the quark-parton model*

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It is argued that for the neutron mean square charge radius to be negative in the quark-parton model, the quark-parton distribution functions in the region $x < 0.3$ have to be drastically different from any of the specific forms hitherto proposed.

The mean-square charge radius $\langle r_n^2\rangle$ of the neutron has been determined experimentally to be negative.[1] Sehgal[2] investigated the negative sign by means of the quark-parton model. Recently, using three specific kinds of quark-parton distribution functions (QPDF's), Parashar and Kaushal[3] showed that all gave a positive sign for $\langle r_n^2\rangle$ in Sehgal's framework. In this paper we show quite generally (without assuming any specific form for the QPDF's) that $\langle r_n^2\rangle > 0$ in the quark-parton model unless the QPDF has some unexpected behavior in the small-x region ($x \lesssim 0.3$). Our discussion will also clarify why the first two models[4] employed by Parashar and Kaushal give $\langle r_n^2\rangle > 0$.

Following Sehgal, we have

$$\langle r_n^2\rangle = \tfrac{3}{2}\int_0^1 dx \sum_i e_i f_i(x) g_i(x) \ , \tag{1}$$

where

$$g_i(x) \equiv \langle b^2\rangle_{i,x} = \frac{\int d^2b\, b^2 h_i(x,b)}{\int d^2b\, h_i(x,b)} \equiv [f_i(x)]^{-1}\int d^2b\, b^2 h_i(x,b) \ .$$

In the above, i denotes the kind of quark partons inside a neutron, and $h_i(x,b)$ is the distribution function of quark parton of type i in the transverse plane, with a fraction x of the total longitudinal momentum. $f_i(x)$ and e_i are, respectively, the usual QPDF and the charge of the quark parton of type i.

Throughout the following discussion, we will neglect[5] the net contribution from the strange quark and strange antiquark partons. The assumptions made by Sehgal are (i) $g_i(x) = g(x)$ and (ii) $g(x)$ is a monotonically decreasing (and of course positive) function of x. These assumptions are consistent[2] with experimental evidence[6,7] and (ii) is also supported by some theoretical models.[8] We then have

$$\langle r_n^2\rangle = \int_0^1 dx\, q(x) g(x) \tag{2}$$

and

$$q(x) \equiv d(x) - \bar{d}(x) - \tfrac{1}{2}[u(x) - \bar{u}(x)] \ , \tag{3}$$

where $d(x)$, $\bar{d}(x)$, $u(x)$, and $\bar{u}(x)$ are, respectively, the d, $\bar{d}$, u, and $\bar{u}$ QPDF inside a *proton*.

We shall now deduce some general properties of $q(x)$. First we recall that the quark-parton-model sum rule requires[9]

$$\int_0^1 q(x)dx = 0. \tag{4}$$

This equation represents charge neutrality of a neutron. Next we note that one of Nachtmann's inequalities,[10] derived from the positivity of the QPDF's, reads

$$\bar{u}(x) \le 2\bar{d}(x) \ . \tag{5}$$

Equation (5) together with the experimental data,[11]

$$\tfrac{1}{4} \le F_2^{en}(x)/F_2^{ep}(x) < \tfrac{2}{3} \text{ for } 1 > x > x_0 \simeq 0.3 \ ,$$

gives

$$u(x) > 2d(x) \ , \quad x_0 < x < 1 \ . \tag{6}$$

Equations (3), (5) and (6) then lead to

$$q(x) < 0 \ , \quad x_0 < x < 1 \ . \tag{7}$$

In view of Eq. (4), this means $q(x)$ must become positive at some $x = x_c < x_0 \simeq 0.3$. Finally, if the quark-parton model and Regge-pole theory are compatible, then for small x, say $x < x_R$, $q(x) > 0$ is favored. This may be seen as follows. For electroproduction processes (we are not concerned with ν-N processes here, see Ref. 12), the non-diffractive parts $f_2^{ep}(x)$ and $f_2^{en}(x)$ of $F_2^{ep}(x)$ and $F_2^{en}(x)$, respectively, can be described by the leading Regge-pole terms, i.e., the f and A_2 terms.[13] In the quark-parton model, $f_2^{ep}(x)$ and $f_2^{en}(x)$ are usually expressed in terms of $[u(x) - \bar{u}(x)]$ and $[d(x) - \bar{d}(x)]$. Thus we get

$$\begin{aligned} f_2^{ep}(x) &= \tfrac{4}{9}[u(x) - \bar{u}(x)] + \tfrac{1}{9}[d(x) - \bar{d}(x)] \\ &= \tfrac{5}{6}R_f x^{-\alpha_f(0)} + \tfrac{1}{6}R_{A_2} x^{-\alpha_{A2}(0)} \ , \\ f_2^{en}(x) &= \tfrac{1}{9}[u(x) - \bar{u}(x)] + \tfrac{4}{9}[d(x) - \bar{d}(x)] \\ &= \tfrac{5}{6}R_f x^{-\alpha_f(0)} - \tfrac{1}{6}R_{A_2} x^{-\alpha_{A2}(0)} \ , \end{aligned} \tag{8}$$

for $x < x_R$, where the R's are constants. From the above formula, we get

$$u(x) - \bar{u}(x) = \tfrac{1}{2}(3R_f x^{-\alpha_f(0)} + R_{A_2} x^{-\alpha_{A_2}(0)}) \,,$$
$$d(x) - \bar{d}(x) = \tfrac{1}{2}(3R_f x^{-\alpha_f(0)} - R_{A_2} x^{-\alpha_{A_2}(0)}) \,, \qquad (9)$$

for $x < x_R$. Following Chaichian *et al.*,[14] we use the positivities of the functions $f_2^{ep}(x)$ and $f_2^{en}(x)$ to get $\alpha_f(0) \geq \alpha_{A_2}(0)$ and $R_f > 0$. Obviously, the requirement of the valence-sea version[13] of the quark-parton model that the left-hand sides of Eq. (9) be positive leads to the same conditions. Experimentally, $\alpha_f(0) > \alpha_{A_2}(0)$ seems to hold.[15] Hence for small x, Eqs. (3) and (9) give $q(x) > 0$ for $0 < x < x_R$.

We cannot say anything about $q(x)$ for $x_R < x < x_c$. It might seem reasonable to expect that $q(x)$ remains positive in this region. In fact, to the best of our knowledge this is a common feature of all the models[16] proposed so far in which the explicit form of the QPDF's is given. The function $q(x)$ is sketched in Fig. 1. Since $g(x)$ is a monotonically decreasing (positive) function of x [assumption (ii)], Eqs. (2) and (4) immediately give

$$\langle r_n^2 \rangle > 0 \,.$$

It is clear that in order for the quark-parton model to yield $\langle r_n^2 \rangle < 0$, the quark-parton distribution functions in the small-x region, $x < x_c < 0.3$, must be reexamined carefully.

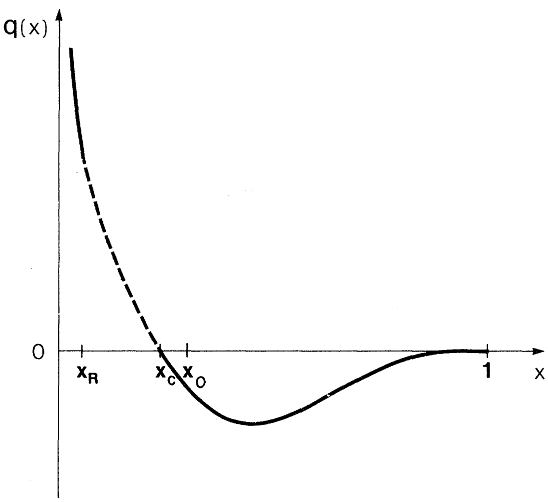

FIG. 1. Schematic sketch of the function $q(x)$. The definitions of x_0 ($\simeq 0.3$), x_c, and x_R are given in the text.

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[3]D. Parashar and R. S. Kaushal, Phys. Rev. D 13, 2684 (1976). While Sehgal used a two-parameter form for his $A(x)$, $A(x) = \alpha - \beta x$ with $0 < \beta < \alpha$, Parashar and Kaushal use only one free parameter in $A(x) = 1 - \beta x$, resulting in $\beta > 1$. This makes their $h_i(x, b)$ [their Eq. (2)] divergent for large x when $b \to \infty$. Their conclusion that $\langle r_n^2 \rangle > 0$, however, remains unchanged, even if the Sehgal form for $A(x)$ is used.

[4]The third model employed in Ref. 3 does not satisfy the quark-parton-model sum rules, so Eq. (4), below, does not hold.

[5]In the valence-sea version of the quark-parton model (see Ref. 13), $s(x) = \bar{s}(x)$, where $s(x)$ and $\bar{s}(x)$ are, respectively, the s and $\bar{s}$ QPDF inside a proton. It is also expected that $g_s(x) = g_{\bar{s}}(x)$ holds in this type of model.

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[7]The experimental data, Table VI and Fig. 5 of Ref. 6, suggest that $g_d(x)$ is not exactly equal to $g_u(x)$ but is slightly larger. Here d and u represent, respectively, the d- and u-quark parton inside a *proton*. We can show in the valence-sea version of the quark-parton model (see Ref. 13) that this in fact will strengthen our conclusion.

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