PHYSICAL REVIEW D VOLUME 14, NUMBER 11 1 DECEMBER 1976

# Forbidden coupling and inhibited decay: A study of disconnected quark diagrams*

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(Received 12 April 1976; revised manuscript received 14 June 1976)

An attempt is made to correlate deviation from ideal-mixing mass formula, high-energy scattering behavior, and violation of the Okubo-Zweig-Iizuka rule. Pole or cut nature of the intermediary is discussed. Timelike and spacelike forbidden communications are compared. Off-shell effects are examined. Some problems for phenomenology are revealed.

## I. INTRODUCTION

In a recent series of papers[1-3] we have attempted to develop a consistent phenomenological scheme, with underpinnings in dual resonance models[4] and the recent topological-expansion[5] development, for correlating various decay and production processes which are forbidden by ideal mixing and the Okubo-Zweig-Iizuka[6] (OZI) rule. We would like now, in this longer and more discursive paper, to review some of the successes of the model, report some of its shortcomings, relax some of our initial, highly specific assumptions, and compare and contrast our picture with other efforts, such as the charmonium–asymptotic-freedom approach,[7] the "asymptotic planarity" approach of Chew and Rosenzweig,[8] the dual unitarization approach,[9] and the classic wave-function–mass-matrix–mixing model of several groups.[10] Beyond the different language and pictures which dress many of these calculations, we find some significant similarities and differences which we feel will be useful to point out.

The *specific* model which is our starting point is that of Freund and Nambu[4] (also Refs. 11 and 12) in which the OZI-violating transition is associated with an $s$-channel (timelike) Pomeron effect which is pole-dominated by "particles"—"$O$" mesons or closed strings—identified with Pomeron or Pomeron-daughter trajectories.

Our general approach is the conventional one in which the new states are bound systems of a new degree or degrees of freedom—the charm picture or suitable generalizations—and the dynamics are simply those of strong-interaction physics—Regge behavior, duality, unitarity, etc.—modified by the new mass scale involved. Thus $\psi\to 3\pi$ is inhibited, for the same reason that $\phi\to 3\pi$ is inhibited, to a greater degree only because nearby states mix more strongly than distant states and hence receive larger corrections to the OZI rule.

The basic problem, in this approach, is to give meaning to the dual diagrams which, in some Born-term sense the theory must eventually specify, represent "forbidden" quark communication. For example, if Regge exchange is represented by Fig. 1(a), then in the Harari-Freund[13,14] picture the Pomeron is represented as one of the topologically equivalent diagrams of Fig. 1(b), in which the quarks make a forbidden "U turn" or in which two intermediate exchanges are twisted, or as a cylinder correction in terms of the topological expansion. (In terms of the topological expansion, planar diagrams have the structure of a sphere with one hole; singly disconnected diagrams are cylinders with the structure of a sphere with two holes; doubly disconnected, a sphere with three holes, and so on.) The double-twist diagram can be cut into two single-twist diagrams [see Fig. 1(c)], each of which is a line reversal of an allowed diagram, representing "$s$-channel" background (relatively real) which, iterated via unitarity, contributes to the Pomeron.

Similarly, the Pomeron link appears in an OZI-violating decay, as in Fig. 1(d), which again illustrates the role that unitarity plays in *requiring* OZI violation, e.g., $\psi\to D\bar{D}^*$ as well as $D\bar{D}^*\to\rho\pi$ are both OZI allowed, whereas their iteration, as illustrated in the middle diagram of Fig. 1(d), is forbidden.

At this stage, even if values can be assigned to the forbidden links, or cylinder corrections, one is thinking in a perturbative framework in which the real particles are represented, approximately, by quark lines of a single species (bare states) with transitions represented as overlaps with other bare states of different species (not continuously connected) with the OZI-violating connection "small." This may simply be a bad approximation when multiplets are involved which appreciably deviate from ideal mixing (e.g., $0^-$ and $0^+$) where some sector of the perturbation theory must be summed. Additionally, one can hope to explain the deviation from the ideal-mixing mass formula simultaneously with the deviations from ideal mixing in the wave functions; a propagator-mass-

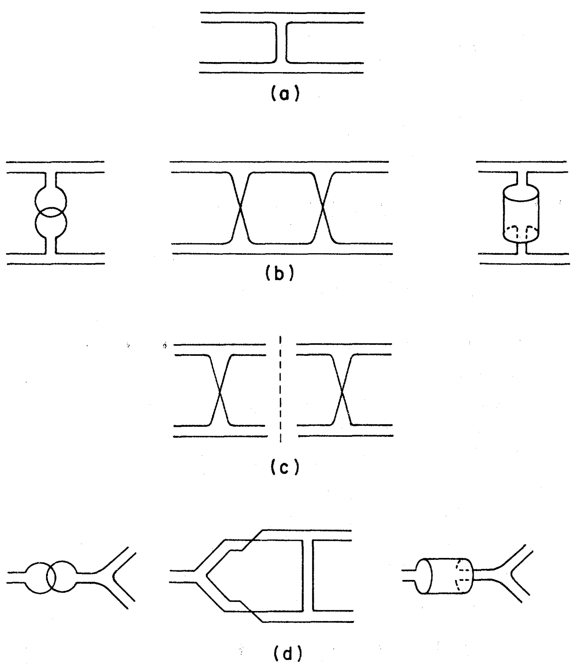


FIG. 1. (a) Dual diagram for Regge exchange. (b) Dual diagram for Pomeron exchange. (c) Pomeron as iteration of twisted diagrams. (d) Pomeronlike effects in $s$-channel decays.

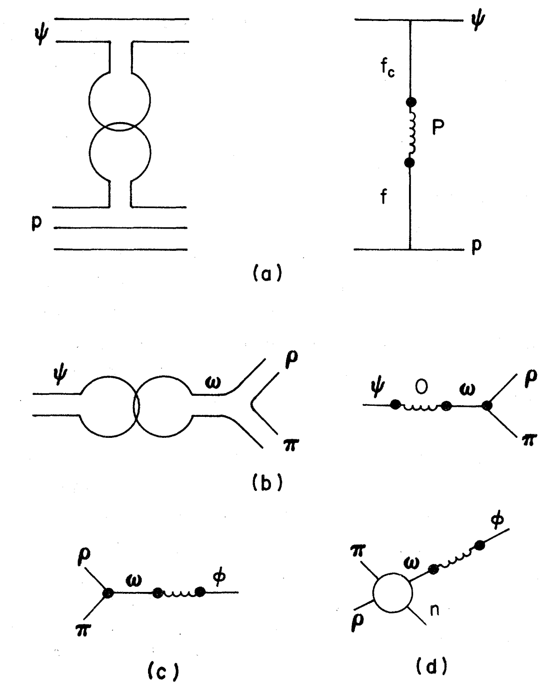


FIG. 2. (a) $f$-dominated Pomeron picture. (b) Corresponding decay diagrams. (c) Diagram for $\phi$ decay. (d) Diagram for $\phi$ production.

matrix formalism will be described which accomplishes this goal.

Historically, perhaps the first phenomenological treatment of the forbidden amplitudes occurred for spacelike transitions ($t$ channel) and is contained in the "$f$-dominated Pomeron" approach of Freund and Rivers[15] and Carlitz, Green, and Zee.[16] The Freund and Nambu model is in a sense simply the timelike implementation ($s$ channel) of that work.[17] Figures 2(a) and 2(b) make clear that connection, where normal trajectories are mediated by the Pomeron as normal states are mediated by Pomeron-related effects ($P$ daughters, $O$ mesons, bound gluon states, empty bags, cuts?) in the timelike forbidden transitions.

Important features of this implementation are the following:

(1) $s$- and $t$-channel "unitarity" effects are related. If the intermediary for timelike transitions is considered a Pomeron-daughter pole, its spectrum is controlled by the Pomeron trajectory; if it is a cut, its structure is related to Pomeron-Reggeon cuts. In any case a unified $s$- and $t$-channel description is feasible.

(2) Detailed exclusive processes can be estimated (a *single and asymptotic* energy is not necessary) and production or decay processes for OZI violations are multiplicably related to corresponding allowed processes [see Figs. 2(c) and 2(d), where an intermediate pole approximation has been used].

(3) Correspondingly, production *and* decay are related, independent of the detailed nature of the OZI-violating process; e.g., in Figs. 2(c) and 2(d), $f_{O\omega}^2 f_{O\phi}^2 (m_\phi^2 - m_O^2)^{-2} (m_\phi^2 - m_\omega^2)^{-2}$ may be eliminated to predict $\Gamma_{\phi \to 3\pi}/\Gamma_{\omega \to 3\pi}$ in terms of $\pi^- p \to \omega n/\pi^- p \to \phi n$. Both (2) and (3) are possible because of the factorizing nature of OZI violation in this picture.

(4) When OZI violation is small (e.g., $c\bar{c} \to$ normal quarks or $\bar{s}s \to$ normal in $1^-$ channel), a perturbative approach may be valid, similar to that employed in the $f$-dominated Pomeron picture. In fact, we shall argue below that the Pomeron in a sense itself is small relative to Regge exchanges and appears large only because it is all that remains after Regge exchanges are no longer important.

To summarize the model, OZI-violating processes, including Pomeron exchange, proceed via intermediate states which may be a pole or cut, whose spectrum is revealed both in timelike and spacelike forbidden transitions. The parameters of the intermediary are partially determined by that spectrum, with the determination completed by high-energy behavior or by the size of the OZI-forbidden production and decay processes. Once these parameters are determined, other processes can be calculated to check the consistency of the scheme.

This paper is organized as follows: In Sec. II we review and discuss $t$-channel–pole implementation, determine some basic parameters, and discuss the "size" of the Pomeron. We then review the timelike production and decay results, in the pole approximation. Questions of the perturbative or nonperturbative nature of the transition in various sectors of SU(4) are examined. Factorization and model independence are discussed. The problem of off-mass-shell behavior is reviewed.

In Sec. III the model is formulated in terms of a cut rather than a pole, motivated by the apparent experimental absence of $O$ mesons, by rising cross-section data, and by a need for stronger damping.

In Sec. IV we discuss, both in pole and cut pictures, the propagator-mass-matrix formulation in which a unified, highly constrained, and nonperturbative treatment is developed of masses, mixings, and OZI violation.

In Sec. V we comment on the topological expansion and point out some counting difficulties for our phenomenology.

## II. POLE MODELS

### A. $t$-channel–pole-model results

We first review the $f$-dominated Pomeron results, with a view toward estimating the size of the coupling between Regge trajectories and the Pomeron. This clarifies what we mean by the "smallness" of the Pomeron and the validity of a perturbative approach. Since these junctions, when continued to timelike $t$, are related to $O$-meson coupling parameters, this will enable us later to compare the size of the timelike and spacelike couplings.

We remark that it is not clear how, if at all, this $t$-channel picture of the Pomeron as an OZI-violating process can be incorporated in the quantum chromodynamics (QCD) picture.[18]

Consider first a rough fit to $pp$ and $p\bar{p}$ total cross sections. Neglecting the $A_2$ trajectory, which lies below $f$ and is diminished by a Clebsch-Gordan coefficient of $\frac{1}{9}$, the leading diagrams for the sum $\sigma_T(pp)+\sigma_T(p\bar{p})=\sigma_T^+$ are given by $f$ exchange and $f$-dominated Pomeron exchange, resulting in

$$\gamma_f^{\,2}\left\{\frac{F_{OT}^{\,2}}{[J-\alpha_f(t)]^2[J-\alpha_P(t)]}+\frac{1}{J-\alpha_f(t)}\right\} \tag{1}$$

whose Mellin transform, at $t=0$, is

$$\sigma_T^+=\gamma_f^{\,2}\left\{\frac{F_{OT}^{\,2}}{[\alpha_P(0)-\alpha_f(0)]^2}+s^{\alpha_f(0)-1}\right\}. \tag{2}$$

A hand fit to the data points ($\sigma_T^+\approx 102$ mb at $s=10$ GeV$^2$, 83 mb at $s=100$ GeV$^2$) yields

$$F_{OT}^{\,2}\cong\begin{cases}0.21 & \text{for } \alpha_f(0)=0.5,\\ 0.14 & \text{for } \alpha_f(0)=0.6\end{cases} \tag{3}$$

for the probability that the $f$ trajectory makes the transit to the Pomeron. Rougly speaking, this is a measure of the mixture of nonvalence quarks, at $t=0$, in the proton. Note $F_{OT}^{\,2}$ is a small dimensionless number. The major successes of this simple approach are given by[19]

$$\frac{\sigma_T(Kp)}{\sigma_T(\pi p)}=\frac{1}{2}\left[1+\frac{\alpha_P(0)-\alpha_f(0)}{\alpha_P(0)-\alpha_{f'}(0)}\right], \tag{4}$$

$$\frac{\sigma_T(\phi p)}{\sigma_T(\rho^0 p)}=\frac{\alpha_P(0)-\alpha_f(0)}{\alpha_P(0)-\alpha_{f'}(0)}, \tag{5}$$

$$\frac{\sigma_T(\psi p)}{\sigma_T(\phi p)}=\frac{\alpha_P(0)-\alpha_f(0)}{\alpha_P(0)-\alpha_{f_c}(0)}, \tag{6}$$

where the Pomeron has been assumed to be an SU(4) singlet ($F_{Pf'}=F_{Pf_c}=F_{Pf}/\sqrt{2}$).

With exchange-degenerate $\omega-f$, $\phi-f'$, and $\psi-f_c$ trajectories, $\alpha_f=1+\alpha'(t-m_\omega^{\,2})$, $\alpha_{f'}=1+\alpha'(t-m_\phi^{\,2})$, and $\alpha_{f_c}=1+\alpha'(t-m_\psi^{\,2})$, the results are the following:

| | Model | Exp. |
|---|---|---|
| $\frac{\sigma_T(Kp)}{\sigma_T(\pi p)}=$ | 0.8 | 0.84 |
| $\frac{\sigma_T(\phi p)}{\sigma_T(\rho^0 p)}=$ | 0.6 | 0.4 |
| $\frac{\sigma_T(\psi p)}{\sigma_T(\phi p)}=$ | 0.1 | 0.1 |

(7)

The difficulties with the small coupling of the Pomeron to normal states, within a QCD approach, seem quite serious. Since the squared mass carried by the Pomeron cluster is zero, asymptotic-free-field-theory results cannot be used to justify weak coupling. Even worse, the small coupling seems in direct contradiction to the large effective coupling needed to confine the quarks at large distances.

### B. $s$-channel pole model, $J^P=1^-$

Classifying the OZI-forbidden transition by the $J^P$ of the decaying state, the $\phi\to\rho\pi$ rate in the $J^P=1^-$ sector is the classic problem for which the OZI rule was formulated. If the forbidden transition is mediated by a pole—the $O$ meson—then in QCD this pole would be a bound state of gluons[20,21] and in the dual model this particle would be, following Freund and Nambu,[4] a particle on one of the Pomeron daughters. For the moment let us adopt the latter approach, which fixes the squared mass of the lowest contributing daughter at $(\alpha'_P)^{-1}$, implying $2 \leq m_O{}^2 \leq 3$ if the slope of the Pomeron is between $\frac{1}{2}$ and $\frac{1}{3}$. Interestingly enough, bag calculations have found bound gluon states in the same range.[20]

Referring to Fig. 2(c), the appropriate propagators can be written in ordinary or Regge form; the former is useful for comparing with the literature, the latter is useful for comparing with $t$-channel results or for introducing cuts or continuum states. [As before, we assume SU(4)-symmetric junctions.] The connection is

$$\frac{F_{\phi O} F_{\omega O}}{[1-\alpha_{P_1}(s)][1-\alpha_\omega(s)]} = \frac{F_{\phi O} F_{\omega O}}{[1-\alpha_{P_1}(0)-\alpha'_{P_1} s][1-\alpha_\omega(0)-\alpha'_\omega s]} = \frac{f_{\phi O} f_{\omega O}}{(m_O{}^2 - s)(m_\omega{}^2 - s)} \tag{8}$$

$$F_{\phi O} F_{\omega O} = \alpha'_P \alpha'_\omega f_{\phi O} f_{\omega O} \,. \tag{9}$$

By comparing[22] the OZI-violating rate $\phi \to \rho\pi$ with the allowed rate $\phi \to K_L K_S$, $f_{OV} = f_{O\phi} = f_{O\psi} = f_{O\omega}/\sqrt{2}$ has been determined as[23]

$$f_{OV} = \begin{cases} 0.155 & \text{for } m_O{}^2 = 2\,, \\ 0.222 & \text{for } m_O{}^2 = 3\,, \end{cases} \tag{10}$$

indicating that

$$F_{OV}{}^2 = \alpha'_P \alpha'_\omega f_{OV}{}^2 = \begin{cases} 0.010 & \text{for } m_O{}^2 = 2\,, \\ 0.013 & \text{for } m_O{}^2 = 3\,, \end{cases} \tag{11}$$

which is somewhat smaller than the Pomeron coupling $F_{OT}$ determined in Sec. II A. This value is close to the value determined from the breaking of exchange degeneracy of the $\rho$ and $\omega$ trajectories in Ref. 17, where an equivalent procedure appears together with a more refined fit.

Having established the $J^P = 1^-$ junction coupling from the rate $\phi \to \rho\pi / \phi \to K_L K_S$, one can make a number of predictions which test the model.

The rate ratio, for example $\sigma_{\pi^- p \to \phi n} / \sigma_{\pi^- p \to \omega n}$, has been calculated,

$$\frac{\sigma_{\pi^- p \to \phi n}\,(\text{on shell})}{\sigma_{\pi^- p \to \omega n}\,(g_\omega{}^2 = m_\phi{}^2)} = \frac{2 f_{OV}{}^4}{(m_\omega{}^2 - m_\phi{}^2)^2 (m_O{}^2 - m_\phi{}^2)^2} \cong 0.005 \pm 0.0005\,, \tag{12}$$

with excellent success (the experimental result of Ref. 24 is $0.0035 \pm 0.001$), if the small off-shell extrapolation is ignored.

It should be noted that this test survives any tampering with the $q^2$ propagator suppression of the model. The best way of seeing this is to express the ratio in terms of $\Gamma_{\phi \to \rho\pi} / \Gamma_{\phi \to K_L K_S}$, in which the explicit dependence on the form of the OZI suppression cancels out:

$$\frac{\sigma_{\pi^- p \to \phi n}}{\sigma_{\pi^- p \to \omega n}} = \frac{\Gamma_{\phi\rho\pi}}{\Gamma_{\phi \bar{K} K}} \frac{m_\rho{}^2}{6(m_\phi{}^2 - m_\omega{}^2)^2} \frac{(m_\phi{}^2 - 4 m_K{}^2)^{3/2}}{(m_\phi{}^2 - m_\rho{}^2)} \times m_\phi \left[1 - \frac{m_\pi{}^2}{(m_\phi - m_\rho)^2}\right]^{3/2}, \tag{13}$$

where we have used the SU(6) result $g_{\omega\rho\pi}{}^2 = 4 g_{\rho\pi\pi}{}^2 / m_\rho{}^2$. Experimentally, this relation gives (ignoring the off-shell effects in the cross sections)

$$0.0035 \pm 0.001 = 0.005 \pm 0.0005\,.$$

This gives a strong indication of the factorizable form of the OZI-rule suppression. We should also remark that this relationship is independent of the SU(4) structure.

To actually test the $q^2$ dependence of a specific model, one must go to a different regime, for example large $q^2$, as in $\psi$ decay.

The basic scale of OZI violation in the model, coming from the size of $f_{OV}$ and the propagator effect, *is* tested in the model's prediction for the signal-to-background ratio of $\psi$ production in the initial codiscovery experiment at Brookhaven National Laboratory. Referring to Fig. 3(a), where $e^+e^-$ from virtual-vector-dominated photon production has been used to estimate the background,[25] the prediction is

$$\frac{d^2\sigma(\psi)/d^3p_+ d^3p_-}{d^2\sigma(\text{bg})/d^3p_+ d^3p_-} = \frac{2 f_{OV}{}^4}{(m_\psi{}^2 - m_O{}^2)^2 m_\psi{}^2 \Gamma_\psi} \times \left[\left(\frac{\Gamma_{\omega \to e^+e^-} m_\psi}{\Gamma_{\psi \to e^+e^-} m_\omega}\right)^{1/2} + \left(\frac{\Gamma_{\phi \to e^+e^-} m_\psi}{\Gamma_{\psi \to e^+e^-} m_\phi}\right)^{1/2}\right]^{-2}. \tag{14}$$

When this is corrected for the 20-MeV resolution, the signal-to-background ratio is ~31 (180) for $m_O{}^2 = 2$ (3) GeV, which brackets the data.[26] This rate clearly tests the scale of the suppression and is independent of $q^2$ dependence concerning off-shell $\omega$, $\rho$, and $\phi$ production, which cancel in the ratio.

The best test of the scale of the suppression mechanism, which is independent of the size of $f_{OV}$ and depends only on the propagator damping effect, is given by the first prediction of Freund and Nambu,[4]

$$\frac{\Gamma_{\psi\to\rho\pi}}{\Gamma_{\phi\to\rho\pi}}=\frac{g_{\omega\rho\pi}{}^2(m_\psi{}^2)}{g_{\omega\rho\pi}{}^2(m_\phi{}^2)}\left[\frac{(m_\phi{}^2-m_O{}^2)(m_\phi{}^2-m_\omega{}^2)}{(m_\psi{}^2-m_O{}^2)(m_\psi{}^2-m_\omega{}^2)}\right]^2\left(\frac{p_{\psi\to\rho\pi}}{p_{\phi\to\rho\pi}}\right)^3$$
$$\approx[g_{\omega\rho\pi}{}^2(m_\psi{}^2)/g_{\omega\rho\pi}{}^2(m_\phi{}^2)]\times\begin{cases}0.02 & \text{for } m_O{}^2=2\\ 0.11 & \text{for } m_O{}^2=3\end{cases}$$
$$=0.14\times10^{-2}\text{ experimentally (Ref. 27).} \qquad (15)$$

which indicates either that the off-shell effect in $g_{\omega\rho\pi}$ is large,

$$\frac{g_{\omega\rho\pi}{}^2(m_\psi{}^2)}{g_{\omega\rho\pi}{}^2(m_\phi{}^2)}=\begin{cases}0.07 & \text{for } m_O{}^2=2,\\ 0.013 & \text{for } m_O{}^2=3,\end{cases} \qquad (16)$$

or that the suppression given by the $O$ poles is inadequate. We shall return to this question in Sec. III.

On the other hand, the ratio $\psi\to p\bar{p}/\psi\to\rho\pi$, which does not have the difficulty of the large-$q^2$ dependence, gives predictions in good agreement with the data,

$$\frac{\Gamma_{\psi p\bar{p}}}{\Gamma_{\psi\rho\pi}}=\frac{3}{16}\frac{m_\psi m_\rho{}^2}{p_\rho{}^3}\left(1-\frac{4m_\rho{}^2}{m_\psi{}^2}\right)^{1/2}\left(1+\frac{2m_\rho{}^2}{m_\psi{}^2}\right)$$
$$=0.11$$
$$=0.12\pm0.03\text{ experimentally (Ref. 27).} \qquad (17)$$

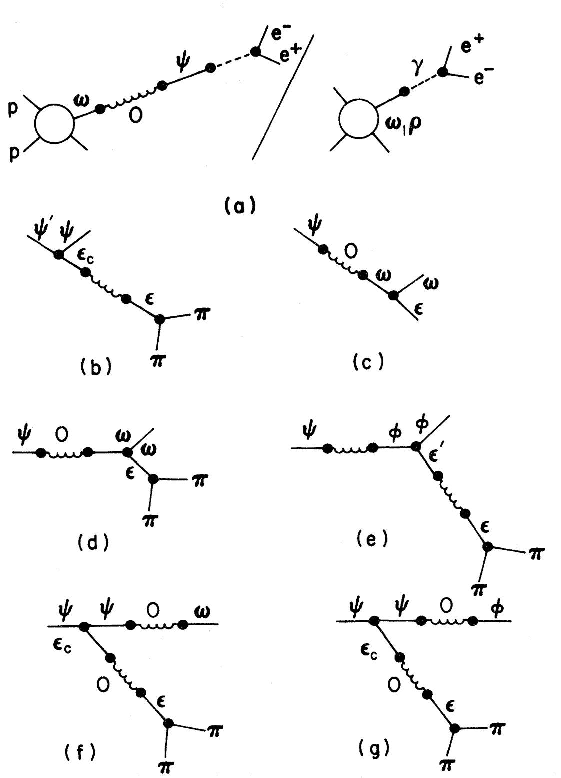

FIG. 3. (a) Signal-to-background diagrams for $\phi$ or $\psi$ production in hadronic collisions. (b)–(g) OZI-violating decay diagrams for various rates.

Another test of the model consists in its SU(4) symmetry and symmetry-breaking predictions for OZI-violating decays. The SU(4) general symmetric coupling is

$$\mathcal{L}=O_\mu(\sqrt{2}f_{O\omega}\omega^\mu+f_{O\phi}\phi^\mu+f_{O\psi}\psi^\mu), \qquad (18)$$

where $f_{O\omega}=f_{O\phi}=f_{O\psi}$ ($\equiv f_{OV}$) in the SU(4) limit. Mass differences in propagators induce some symmetry breaking, in addition to that which may be there at the two-particle junction. In the absence of the latter, $\psi$ then decays basically as an SU(3) singlet. For example, because $\psi\to K\bar{K}$ is forbidden by $G$ parity in the SU(3) limit, its rate is proportional to mass differences, and is small (~eV) when calculated using the summetric couplings.

Other predictions of this type are the branching ratios

$$\frac{\Gamma_{\psi\to K^{*-}K^+}}{\Gamma_{\psi\to\rho^-\pi^+}}=0.84\left[\frac{1}{2}+\frac{1}{2}\frac{f_{O\phi}}{f_{O\omega}}\left(\frac{m_\psi{}^2-m_\omega{}^2}{m_\psi{}^2-m_\phi{}^2}\right)\right]^2$$
$$=0.36\pm0.17\text{ experimentally (Ref. 27),} \qquad (19)$$

which is unity in the SU(3) limit. The phase-space–angular-momentum-barrier factor brings this down, as does the propagator effect, but not nearly enough to come into agreement with the data, which require considerable breaking of the SU(4) relations for the $f_{OV}$ couplings; even taking the upper limit for the data indicates $f_{O\phi}/f_{O\omega}\sim0.7$. If this is the case, $\psi$ is more $\omega$-like than $\phi$-like in its OZI-violating decays. (An argument which relates this to the $K$-yield dips at $\psi$ and $\psi'$ was advanced in Ref. 2.)

So far we have concentrated on the $J^P=1^-$ OZI violations in the $s$ channel. The best-known OZI-violations are here because the spectroscopy is clean and the multiplet is close to being ideally mixed, even in the SU(3) [U(3)] sector. The $O$-meson parameters are small here; a perturbative approach is valid.

In the case of the $J^P=0^+$ system, the picture is muddied by the uncertain status of the scalar spectroscopy. We believe that the scalars may be well mixed in the SU(3) sector, so that the perturbative mixing implied by simple $O$-meson diagrams is not valid; one must sum the diagrams and diagonalize the propagator matrix exactly.

This is clearly not the case in the transition from the $\epsilon_c(\bar{c}c)$ to the normal quark-model states because of the large energy difference in the denominators; the rate $\psi' \to \psi\pi^+\pi^-$ may then be used to estimate $f_{OS}$, assuming the $\pi^+\pi^-$ system is dominated by the $s$ wave [see Fig. 3(b)].

Assuming $g_{\psi'\psi\epsilon_c} \approx \sqrt{2}\, g_{\rho'\rho\epsilon}$ [SU(4)] and with $g_{\epsilon\pi\pi}$, determined from $\Gamma_{\epsilon\to\pi\pi}$, then $\Gamma_{\rho'\to\rho\pi\pi}$ determines $\Gamma_{\psi'\to\psi\pi\pi}$ up to the $f_{OS}^2$ scalar $O$-meson parameters. (For details, see Ref. 3.) Tensor coupling has been used at the $VVS$ vertex and chiral coupling at the $PPS$ vertex, the former to keep the diagram gauge invariant when vector dominance is used to allow the final $\psi$ to convert to a photon, the latter to cope with the experimentally rising $\pi^+\pi^-$ mass spectrum. The result is

$$f_{OS} = \begin{cases} 0.40 & \text{for } m_O^2 = 2\,, \\ 0.48 & \text{for } m_O^2 = 3\,. \end{cases} \tag{20}$$

Note that the OZI violation here is small, with the scale controlled by (in, for example, $\epsilon_c \to \pi\pi$)

$$\frac{f_{OS}}{(m_{\epsilon_c} - m_O^2)} \cong 0.04\,. \tag{21}$$

This again is nicely consistent with the QCD picture in that the $0^+$ violation of the OZI rule is expected in this picture to be the result of two-gluon exchange and therefore should be larger than the $1^-$ transitions which go by three gluons.

Assuming $g_{\psi'\psi\epsilon_c} \approx g_{\psi\psi\epsilon_c}$,[28] then SU(4) implies $g_{\omega\omega\epsilon} = g_{\rho'\rho\epsilon}$, and the rate $\psi \to \omega(\epsilon \to \pi\pi)$ may be estimated [see Fig. 3(c)] with the result

$$\Gamma_{\psi\to\omega\epsilon} / \Gamma_{\psi\to\rho\pi} \approx 0.60\,, \tag{22}$$

where to minimize the problem of the off-shell $\omega\omega\epsilon$ vertex we quote the result as a ratio, comparing it to $\psi \to \rho\pi$, in which the $\omega\rho\pi$ vertex is similarly off-shell, and should extrapolate in a qualitatively similar way. The result is in fair agreement with the data.[27]

Returning to the magnitude of $f_{OS}$ extracted from $\psi' \to \psi\pi^+\pi^-$, this implies, if the SU(4) coupling is assumed,

$$f_{OS}\, O(\sqrt{2}\,\epsilon + \epsilon' + \epsilon_c)\,, \tag{23}$$

that mixing effects in the SU(3) sector may be considerable, with the amplitude scaled controlled by

$$\frac{f_{OS}}{(m_\epsilon^2 - m_O^2)} \cong 0.27\,. \tag{24}$$

These effects are important when considering the rates $\psi \to \omega\pi^+\pi^-$ and $\psi \to \phi\pi^+\pi^-$, with lowest-order diagrams [Figs. 3(d), 3(e), 3(f), 3(g)]. Since the scalar $O$-meson coupling seems large, perturbation theory seems highly questionable when working in the SU(3) sector of OZI violation. This question has been asked and answered in Ref. 3, where the SU(3) sector of the $0^{++}$ nonet is handled nonperturbatively. The calculation suffers from the usual ambiguity of the scalar spectroscopy, but for a variety of plausible assignments it is fair to conclude from the detailed calculation of Ref. 3 that the ratio $\psi \to \phi\pi^+\pi^- / \phi \to \omega\pi^+\pi^-$ is expected to lie between 0.1 and 0.4, consistent with the data. This is despite the fact, as has often been noted, that $\psi \to \phi\pi^+\pi^-$ is doubly suppressed, with respect to the OZI rule, as is evident from the diagrams in Figs. 3(e) and 3(g), whereas $\psi \to \omega\pi^+\pi^-$ is singly suppressed, as seen in the diagram of Fig. 3(d).

It is interesting to note that these same types of considerations will seriously constrain models with more than four quarks that have a $\psi'$ made mainly of the higher-quark state.[7] In that case the decay of the $\psi' \to \psi\pi\pi$ will be a doubly–OZI-rule–suppressed process completely analogous to $\psi \to \phi\pi\pi$. The ratio of the rates, however, is of the order of $10^3$, and it is difficult to see how one can accomplish this in essentially identical processes, especially with phase space working in the wrong direction.

Finally, we mention some preliminary information[29] on $f_{OT}$, the "Pomeron" coupling, as measured in the $s$-channel decay $f' \to 2\pi$, indicating

$$f_{OT} = \begin{cases} 0.082 & \text{for } m_O^2 = 2\,, \\ 0.127 & \text{for } m_O^2 = 3\,, \end{cases} \tag{25}$$

with the notation

$$\mathcal{L}_{OT} = f_{OT}\, O_{\mu\nu}(\sqrt{2}\, f^{\mu\nu} + f'^{\mu\nu} + f_O^{\mu\nu}) \tag{26}$$

implying that the Regge form is

$$F_{OT}^2 = \begin{cases} 0.0033, & m_O^2 = 2 \\ 0.0054, & m_O^2 = 3 \end{cases} \tag{27}$$

as compared with $F_{OT}^2 \approx 0.21 - 0.14$ in the $t$ channel, as estimated in Sec. II A; the value of $F_{OT}^2$ determined in Eq. (27) is extremely sensitive to both the partial width of $f' \to \pi\pi$ and the mass and width of the $O$ meson. It is interesting to note, however, that the vector and tensor nonets seem to be similar in their departure away from ideal mixing.

## III. CUTS OR POLES?

Nothing which has been calculated so far prevents $f^2$ from being negative, that is, that the "$O$ meson" is a ghost or nonparticle state. (In fact, in certain mass matrix/wave function or trajectory mixing calculations, precisely this occurs.[17]) In such a case it appears that the $O$-meson pole is merely being used to approximate and parametrize a cut in the OZI-forbidden amplitudes. If, however, one is willing to forego the $O$ mesons as real particles, there are much more general cut trajectories which one may entertain in the OZI-violating and Pomeron transitions. One loses the simple picture of an intermediate particle state and the Pomeron as a simple pole, but is this picture plausible, in the absence of conspicuous $O$-meson states in the experimental spectrum and with respect to the asymptotic freedom arguments for multiple-gluon-exchange discontinuities? These two questions will be returned to in the next section; now we will be content with more mundane motivations for a cut rather than pole structure:

(a) A simple pole does not seem responsible for all the OZI damping. Off-shell effects in the OZI-allowed couplings at the end of the sequential pole decay chain may be the culprit. But is this not double counting?

(b) The Pomeron itself does not seem to be a simple pole, for many reasons. Thus the daughter trajectories below it should also be cuts, or alternatively, particle-Pomeron cuts.

(c) The total cross section is rising at high energies; can this rise be attributed to the same cut structure which succeeds in the stronger damping of the OZI mechanism?

We choose a simple form to test a cut hypothesis, namely, a branch point at the conventional pole position, with linear trajectories as before, with power $\nu$ to parametrize the cut. The branching ratio $\phi\to\rho\pi/\phi\to K_LK_S$ is then given by

$$\frac{\Gamma_{\phi\to\rho\pi}}{\Gamma_{\phi\to K_LK_S}}=3\left(\frac{g_{\omega\rho\pi}{}^2}{g_{\phi K\bar K}{}^2}\right)\frac{F_{OV}{}^4}{[\alpha_{P_1}(m_\phi{}^2)-1]^{2\nu}[\alpha_\omega(m_\phi{}^2)-1]^2}\times\frac{1}{m_\phi}[(m_\phi{}^2-m_\rho{}^2-m_\pi{}^2)^2-4m_\pi{}^2m_\rho{}^2]^{3/2}(m_\phi{}^2-4m^2)^{3/2} \tag{28}$$

and the ratio $\psi\to\rho\pi/\phi\to\rho\pi$ is given by

$$\frac{\Gamma_{\psi\to\rho\pi}}{\Gamma_{\phi\to\rho\pi}}=\left[\frac{\alpha_{P_1}(m_\phi{}^2)-1}{\alpha_{P_1}(m_\psi{}^2)-1}\right]^{2\nu}\left[\frac{\alpha_\omega(m_\phi{}^2)-1}{\alpha_\omega(m_\psi{}^2)-1}\right]\left(\frac{m_\phi}{m_\psi}\right)^3\left[\frac{(m_\psi{}^2-m_\rho{}^2-m_\pi{}^2)^2-4m_\rho{}^2m_\pi{}^2}{(m_\phi{}^2-m_\rho{}^2-m_\pi{}^2)^2-4m_\rho{}^2m_\pi{}^2}\right]^{3/2}, \tag{29}$$

where as before we make an SU(4) assumption for the trajectory couplings. $\alpha_{P_1}$ refers to the Pomeron first daughter. Equation (29) enables us to calculate the power $\nu$

$$\nu=-1.63,\quad m_O{}^2=2$$
$$\nu=-2.78,\quad m_O{}^2=3 \tag{30}$$

where $\nu=-1$ in a simple pole model. Then Eq. (28) determines $F_{OV}$:

$$F_{OV}=0.08\ . \tag{31}$$

Thus, the model is completely determined and one can see if the other $J^P=1^-$ predictions remain intact. The results are

$$\frac{\sigma_{\pi^-p\to\phi n}}{\sigma_{\pi^-p\to\omega n}}=4.8\times10^{-3}\ [\exp(5\pm0.5)\times10^{-3}], \tag{32}$$

$$\Gamma_{\psi\to\bar p p}=0.08\text{ keV}\ \ (\exp 0.106\pm0.032\text{ keV})\,. \tag{33}$$

Alternately, rather than parametrizing the cut via

$$\left[\frac{1}{J-\alpha(s)}\right]^\nu, \tag{34}$$

with $\alpha(s)$ determined by the daughter structure described above, one can regard $\alpha(s)$ as the Pomeron (as before) for $J^P=2^+$ transitions, which yields

$$\left[\frac{1}{2-\alpha_P(0)-\alpha'_Ps}\right]^\nu=\left[\frac{1/\alpha'_P}{1/\alpha'_P-1}\right]^\nu=\left[\frac{1/\alpha'_P}{m_O{}^2-s}\right]^\nu, \tag{35}$$

indicating $m_O^{(2^+)}=(1/\alpha'_P)^{1/2}=\sqrt{2}$ or $\sqrt{3}$, whereas for other $J^P$ transitions one can regard $\alpha(s)$ as an effective Pomeron-Reggeon cut, with trajectories described by the simple rules

$$\alpha_c=\alpha_P+\alpha_R-1,\quad\text{at }t=0 \tag{36}$$

$$\alpha_{c'}=\frac{\alpha_{P'}\alpha_{R'}}{\alpha_{P'}+\alpha_{R'}}, \tag{37}$$

where $\alpha_{c'}$ is the slope of the effective $P$-$R$ cut.

Then for $J^P=1^{--}$ ($P+\rho$ cut) we have (for simplicity we calculated only the case $\alpha'_P=\frac12$ corresponding to $m_O=\sqrt{2}$ above)

$$\alpha_c=\tfrac12,\quad \alpha_{c'}=\tfrac13,\quad m_O=(\tfrac32)^{1/2}\text{ GeV}, \tag{38}$$

whereas for $J^P=0^-$ ($P+\pi$ cut) we have

$$\alpha_c=0,\quad \alpha_{c'}=\tfrac13,\quad m_O=0\,. \tag{39}$$

We merely wish to emphasize here that when one divorces oneself from the strict Freund-Nambu model, both the location and the power of the branch point become negotiable, with indication from this simple Pomeron and Reggeon cut model that this location can vary from one $J^P$ multiplet to another. Moreover, if $\alpha_P(0)$ is taken to be greater than 1 [e.g., $\alpha_P(0)=1.1$], then

$$\alpha_{\pi P(0)} = 0.1$$

and (40)

$$m_O{}^2 = -0.3 \quad \text{at } \alpha_{P'} = \tfrac{1}{2}.$$

The moral here is that a negative $O$ mass is certainly respectable, even predicted in simple cut models, when the literal interpretation of the $O$ as a particle pole is given up. We make these remarks here to prepare the way for the next section, where we discuss the propagator-mass-matrix formalism. In that model the simple particle-pole picture must be given up, if the OZI violation above explains the deviation from ideal mixing in the mass formula.

## IV. MASS-PROPAGATOR-MATRIX FORMALISM AND RESULTS

A more ambitious approach, and one implied by certain pictures, is to relate directly the deviation from the ideal-mixing mass formula (or deviation from exact exchange degeneracy in a Regge approach) to the violation of OZI rule in transition rates. (The corresponding modifications implied for the exact Harari-Freund duality picture have been discussed in Ref. 17.)

For this discussion, applicable to any $J^{PC}$ nonet, let us generally denote the pure states by the quark content

$$\bar\epsilon = \frac{1}{\sqrt{2}}(u\bar u + d\bar d),\quad \bar\epsilon' = \bar s s,\quad \bar\epsilon_c = \bar c c, \tag{41}$$

with $\delta=\bar\delta$ and $\kappa=\bar\kappa$ the usual isovector and strange partners. The corresponding true states will be denoted by $\epsilon$, $\epsilon'$, $\epsilon_c$, $\delta$, and $\kappa$. Then the unmixed states satisfy the ideal-mixing mass formula

$$m_{\bar\epsilon}{}^2 = m_\delta{}^2,\quad 2m_\kappa{}^2 = m_{\bar\epsilon'}{}^2 + m_{\bar\epsilon}{}^2. \tag{42}$$

With the $O$ meson a real, independent particle state, corresponding to a quarkless or pure-gluon pole, as in the original scheme of Freund and Nambu, then implied by the SU(4)-invariant interaction

$$\mathcal{L} = fO(\sqrt{2}\,\epsilon + \epsilon' + \epsilon_c) \tag{43}$$

is a contribution to the propagator matrix

$$\begin{aligned}\pi &= P + PQP + PQPQP + \cdots\\ &= P(1-QP)^{-1}\\ &= (1+PQ)P(1-OP)^{-1}\\ &= (1+PQ)(P^{-1}-O)^{-1},\end{aligned} \tag{44}$$

where

$$P^{-1} = \begin{bmatrix} (s-m_{\bar\epsilon}{}^2)^2 & & & \\ & (s-m_{\bar\epsilon'}{}^2)^2 & & \\ & & (s-m_{\bar\epsilon_c}{}^2)^2 & \\ & & & (s-m_{\bar O}{}^2)^2 \end{bmatrix},\quad Q = \begin{bmatrix} 0 & 0 & 0 & \sqrt{2}f\\ 0 & 0 & 0 & f\\ 0 & 0 & 0 & f\\ \sqrt{2}f & f & f & 0\end{bmatrix}, \tag{45}$$

and

$$O = QPQ = \begin{bmatrix} \frac{2f^2}{(s-m_{\bar{O}}^2)} & \frac{\sqrt{2}f^2}{(s-m_{\bar{O}}^2)} & \frac{\sqrt{2}f^2}{(s-m_{\bar{O}}^2)} & 0 \\ \frac{\sqrt{2}f^2}{(s-m_{\bar{O}}^2)} & \frac{f^2}{(s-m_{\bar{O}}^2)} & \frac{f^2}{(s-m_{\bar{O}}^2)} & 0 \\ \frac{\sqrt{2}f^2}{(s-m_{\bar{O}}^2)} & \frac{f^2}{(s-m_{\bar{O}}^2)} & \frac{f^2}{(s-m_{\bar{O}}^2)} & 0 \\ 0 & 0 & 0 & \frac{2f^2}{(s-m_{\bar{\epsilon}}^2)} + \frac{f^2}{(s-m_{\bar{\epsilon}'}^2)} + \frac{f^2}{(s-m_{\bar{\epsilon}_c}^2)} \end{bmatrix}, \tag{46}$$

where rows and columns are labeled by $\bar{\epsilon}, \bar{\epsilon}', \bar{\epsilon}_c, \bar{O}$.

The shifted masses are roots of $|P^{-1} - O| = O$.

From the direct sum structure of $O = QPQ$ it is easy to show that the problem is equivalent to diagonalizing the 3×3 subsector. There are four independent mass relations among the nine variables $f, \epsilon, \epsilon', \epsilon_c, O$ and $\bar{\epsilon} = \delta$, $\bar{\epsilon} = 2\kappa - \delta$, $\bar{\epsilon}_c$, and $\bar{O}$; if the physical masses $(\epsilon, \epsilon', \epsilon_c, \delta, \kappa)$ are known, then the remaining four unknowns $(f, O, \bar{O}, \bar{\epsilon})$ can be determined, fixing all mixing parameters. Since

$$\pi_{ij} = \sum_\alpha \frac{V_i^\alpha V_j^\alpha}{\lambda_\alpha(s)}, \tag{47}$$

where $V_i^\alpha$ are the normed eigenvectors of $(P^{-1} - Q)^{-1} = (s - m_O^2 - Q)^{-1}$ and $\lambda_\alpha^{-1}(s)$, the corresponding eigenvalues, we have

$$(s - m_O^2 - Q)\, V^\alpha = \lambda_\alpha V^\alpha$$

or (48)

$$(m_O^2 + Q)\, V^\alpha = (s - \lambda_\alpha)\, V^\alpha$$

and diagonalizing the propagator matrix is equivalent to diagonalizing the mass matrix

$$m_O^2 + Q = \begin{bmatrix} m_{\bar{\epsilon}}^2 & 0 & 0 & \sqrt{2}f \\ 0 & m_{\bar{\epsilon}'}^2 & 0 & f \\ 0 & 0 & m_{\bar{\epsilon}_c}^2 & f \\ \sqrt{2}f & f & f & m_{\bar{O}}^2 \end{bmatrix}. \tag{49}$$

In this simple case $V^\alpha$ is $s$-independent and one has the simple form

$$\pi_{ij} = \sum_\alpha \frac{V_i^\alpha V_j^\alpha}{s - m_\alpha^2}, \tag{50}$$

with no "continuum" terms, a special property of the $O$-pole interaction. The rank-1 residue matrix yields, in this nonperturbative formulation, the decomposition of the $\alpha = (\epsilon, \epsilon', \epsilon_c, O)$ pole in terms of pure channels $(\bar{\epsilon}, \bar{\epsilon}', \bar{\epsilon}_c, \bar{O})$ whose lowest-order expansion in $f$ yield the $O$-meson factors used in earlier calculations. Thus orthogonality and completeness is realized in a 4×4 sense; this is equivalent to a classic function–mass-matrix–mixing model, like that studied in this context by Kazi and Kramer,[10] but with an extra channel. A particle is then conceived as decomposible into various quark species *plus* a quarkless piece. The prescription for a transition like $\psi \to \rho\pi$, which in the perturbative formalism is suppressed by the factor

$$\frac{2f_O v^4}{(m_\psi^2 - m_O^2)^2 (m_\psi^2 - m_\omega^2)^2}, \tag{51}$$

is to replace that by $|V_\omega^\psi|^2$, whose lowest-order approximation is

$$\frac{2f_O v^4}{(m_\psi^2 - m_{\bar{O}}^2)^2 (m_\phi^2 - m_{\bar{\omega}}^2)^2}. \tag{52}$$

Note that since $f^2$ and $m_O^2$ are parameters determined by the positions of the physical poles in the various particle multiplets, they may be either positive or negative numbers. In the latter case, the interpretation of the $O$-meson pole as a particle pole clearly breaks down and it may be that the $O$ pole is approximating a more complicated singularity, such as a multigluon state or some other quarkless intermediary which does not resonate.

Alternatively, one can formulate the problem in a general way by mixing the three bare channels via a second-order interaction

$$O = h(s) \begin{vmatrix} 2 & \sqrt{2} & 1 \\ \sqrt{2} & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = h(s) g g^\dagger. \tag{53}$$

For $h(s) = f^2/(s - m_O^2)$, with $f^2 > 0$, we recover the $O$-meson picture of Freund and Nambu. When we normalize our eigenvector in the 4×4 sense, the problem is identical to the previous 4×4 formalism. For a general function $h(s)$, the mass equation constraints become complicated; it will be shown elsewhere that one can normalize the eigenvectors in the 3×3 sense, with a consistent inter-

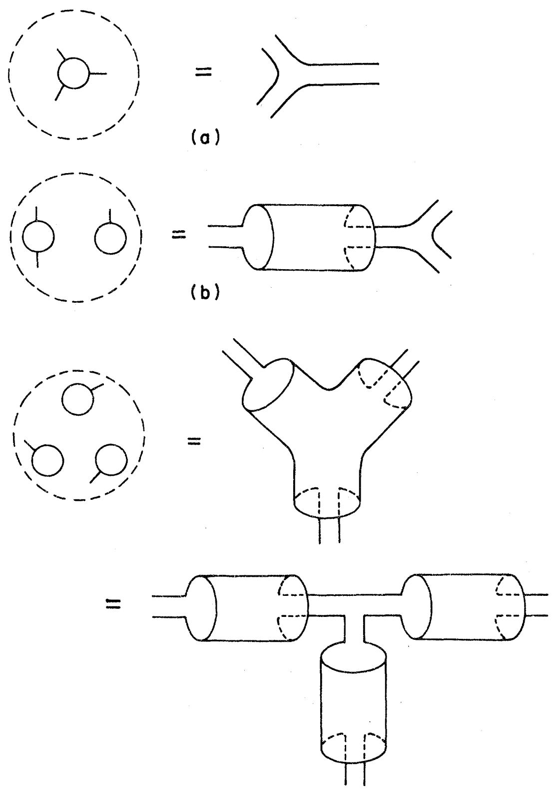


FIG. 4. (a) $O(N^0)$, (b) $O(N^{-1})$, and (c) $O(N^{-2})$ vertex dual diagrams in terms of quark lines (right) and in terms of a cut sphere (left) with particles attached to (solid lines) holes in the sphere (dotted lines).

pretation for $f^2<0$ and $m_O{}^2<0$. Moreover, it can be shown that there exists an $h(s)$ which generates precisely those masses which appear in the 4×4 problem, but with completeness now implemented in a 3×3 sense, and no $O$ pole. The procedure is lengthy and technical and will be described elsewhere.[30] The moral is that one can live comfortably with negative "$O$"-meson mass and negative $f^2$. It is only the particle interpretation of the $O$ meson which fails.

## V. $O(1/N^2)$ OZI VIOLATION IN VERTICES

In the previous development, an OZI-violating decay or amplitude has been calculated by inserting an intermediate cut or pole which joins single-particle or trajectory states of different quark species. In terms of the topological expansion[5,31] for an $n$-point function we have

$$A_n(g,n)=n^{1-n/2}\sum_{b=1}^{n} N^{1-b}(1/N^2)^h A_{n,b,h}(\lambda^2)\,, \qquad (54)$$

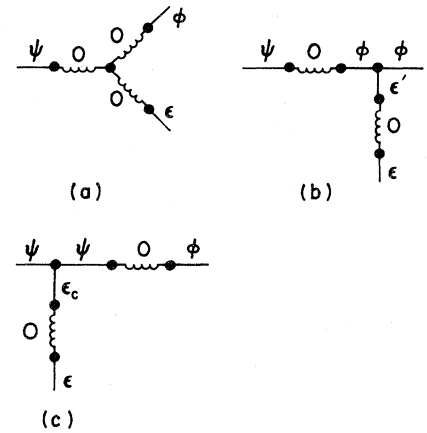


FIG. 5. (a), (b), and (c). $O$-meson diagrams corresponding to Fig. 4(c).

with $\lambda^2=g^2N$, fixed, where $N$ is the number of quarks in U($N$), $b$ the number of boundaries to which external lines are attached, and $h$ the number of handles of the two-dimensional surface ($h=0$, sphere; $h=1$, torus; etc.) in which the graph can be imbedded. These planar vertices have $h=0$, $b=1$, and are of order $N^0$ (ignoring the fixed $N^{1-n/2}$ factor). Single-cylinder corrections have $h=0$, $b=2$, and are of order $N^{-1}$. The diagrams of Figs. 4(a) and 4(b) represent two topologies, with the dashed line the surface $S^{(h)}$, and the solid closed circles, with external lines attached, the boundaries of $S^h$. To the right of each we draw the corresponding quark diagram. In Fig. 4(c) we have the double-cylinder diagram $O(1/N)$ down from the single-cylinder and $O(1/N^2)$ from the planar diagram. It is this diagram which is important for "double" OZI violation as in $\psi\to\phi\pi^+\pi^-$.

At this point a problem of principle occurs when the topologically equivalent forms are saturated by single-particle or single-trajectory states. For example, when applied to $\psi\to\phi\epsilon(\to\pi\pi)$, one has the three alternatives: Figs. 5(a), 5(b), and 5(c). Are there separate independent diagrams, each of which contributes to the topological graph, or rather is each a single-term estimate to three *different alternative* expansions of the *single* topological graph? If the former is the case, one should *add* the diagrams in the amplitudes; if the latter is the case, one should choose one of the diagrams, hopefully the one which best estimates the single topological graph. In Ref. 3, in the spirit of the topological expansion, we took the latter point of view. We still believe this to be the correct one. It is possible to argue that the topology above does not imply the equivalence we demanded, and this question seems not to be resolved at this time.[32]

## VI. SUMMARY

We have reviewed, expanded, modified, and commented on our previous approaches to OZI violation. A good deal of data from both the timelike and spacelike transits have been correlated. Difficulties with the simple pole approach, off-shell behavior, and the possible disease of double counting have been revealed. More work remains to be done. In particular, we are currently studying the $J^P = 0^-$ and $2^+$ OZI violation, the latter intimately connected with the Pomeron singularity, allowing the most direct test of the conjecture that OZI violation and the Pomeron are one and the same phenomenon.

In addition we are considering more general 3×3 propagator-mass matrix mixings, in which the $h(s)$ of Eq. (53) plays the role of an OZI-violating potential which can be fit to reproduce the particle masses. General formulas emerge for the particle mixings which depend only on observed or observable masses, and reveal the model independence of many of the $O$-meson results while dispensing with the literal particle-pole interpretation. Results of these efforts will be reported shortly.

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*Research supported in part by the U. S. Energy Research and Development Administration under Contract No. AT(11-1)-1545.

[1] J. F. Bolzan, K. A. Geer, W. F. Palmer, and S. S. Pinsky, Phys. Rev. Lett. 35, 419 (1975).

[2] J. F. Bolzan, K. A. Geer, W. F. Palmer, and S. S. Pinsky, Phys. Lett. 59B, 351 (1975).

[3] W. F. Palmer and S. S. Pinsky, Phys. Rev. D 14, 1916 (1976).

[4] P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975).

[5] G. Veneziano, Nucl. Phys. B74, 365 (1974); Phys. Lett. 52B, 220 (1974).

[6] G. Zweig, CERN Report No. 8419/TH412, 1964 (unpublished); S. Okubo, Phys. Lett. 5, 165 (1963); I. Iizuka, K. Okuba, and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).

[7] H. Fritzsch and P. Minkowski, Caltech Report No. CALT-68-492 (unpublished).

[8] C. Rosenzweig and G. F. Chew, Phys. Lett. 58B, 93 (1975); G. F. Chew and C. Rosenzweig, Phys. Rev. D 12, 3907 (1975); C. Rosenzweig, *ibid.* 13, 3080 (1976).

[9] Chan Hong-Mo, Ken-ichi Konishi, J. Kwiecinski, and R. G. Roberts, Phys. Lett. 60B, 469 (1976); P. Aurenche *et al.*, Rutherford Laboratory Report No. RL-75-084/T.121 (unpublished).

[10] A. Kazi, G. Kramer, and O. H. Schiller, DESY Report Nos. DESY 75/11 and DESY 75/10.

[11] L. Clavelli, Phys. Lett. 57B, 257 (1975).

[12] R. C. Brower and J. R. Primack, Report No. UCSC 74/103-104 (unpublished); Phys. Rev. D 12, 237 (1975).

[13] H. Harari, Phys. Rev. Lett. 20, 1385 (1968); P. G. O. Freund, *ibid.* 20, 235 (1968).

[14] G. Veneziano, Nucl. Phys. B108, 285 (1976). This paper casts some doubt on the compatibility of the work of Harari and Freund and the topological expansion.

[15] P. O. Freund and R. J. Rivers, Phys. Lett. 29B, 510 (1969).

[16] R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. D 4, 3439 (1971).

[17] S. S. Pinsky and D. R. Snider, Phys. Rev. D 13, 1470 (1976).

[18] H. Fritzsch, Caltech Report No. CALT-68-524, to be published in Fortschr. Phys.

[19] V. Barger, in Proceedings of the McGill Institute of Physics International Summer School, 1975 (unpublished).

[20] R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976). For details of the MIT bag, see K. Johnson, CTP Report No. CTP 494, Lectures at the XVth Cracow School of Theoretical Physics, Zakopane, Poland, 1975 (unpublished).

[21] J. F. Willemsen, Phys. Rev. D 13, 1327 (1976).

[22] See Ref. 4 for details.

[23] This assumes $g_{\phi K\bar{K}}/g_{\rho\pi\pi}$ is given by the SU(3) Clebsch-Gordan coefficient, and the SU(6) result $4g_{\rho\pi\pi}^2 = m_\rho^2 g_{\omega\rho\pi}^2$, or alternately one can determine $g_{\omega\rho\pi}$ from $\omega \to 3\pi$. The numerical results do not differ significantly.

[24] D. S. Ayres, R. Diebold, A. F. Greene, S. L. Kramer, J. S. Lefine, A. J. Pawlicki, and A. B. Wicklund, Phys. Rev. Lett. 32, 1463 (1974).

[25] A detailed study of the use of the vector dominance model in this process has been made by S. Chavin and J. D. Sullivan, Phys. Rev. D 13, 2990 (1976).

[26] S. C. Ting *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[27] G. Abrams, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Stanford, 1975,* edited by W. T. Kirk (SLAC, Stanford, 1976).

[28] For a detailed discussion of the assumption, see L. Clavelli and S. Nussinov, Phys. Rev. D 13, 125 (1976).

[29] A. J. Pawlicki *et al.*, Phys. Rev. Lett. 37, 971 (1976).

[30] A detailed account of this work is given elsewhere [J. Bolzan, W. F. Palmer, and S. Pinsky, Phys. Rev. D 14, 1920 (1976)].

[31] For a comprehensive review of the topological expansion and application, see Proceedings of the Sixth International Colloquium on Multiparticle Reactions, Oxford, 1975, Rutherford Laboratory Report No. RL-75-143 (unpublished).

[32] There are a number of examples in the literature where topologically equivalent diagrams are added: Ref. 8, Ref. 21, and M. Chaichian and M. Hayashi, Phys. Lett. 61B, 178 (1976).