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Hadronic production of large-$p_\perp$ leptons and of large-Q^2 lepton pairs

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We study, within the framework of the constituent-interchange model, the contribution of the subprocess meson + quark → massive photon + quark to the production of massive lepton pairs and of large-$p_\perp$ leptons. Special attention is paid to the problem of gauge invariance. Our numerical results indicate that this subprocess may give, for the massive-pair production, a greater contribution than that coming from the Drell-Yan mechanism in the range $1 \lesssim Q^2 \lesssim 9$ GeV^2. Its contribution to the large-$p_\perp$ lepton production may become observable for $p_\perp \gtrsim 3$–4 GeV/c. We also argue that the best way to study the origin of lepton pairs of momentum Q consists in looking at $d\sigma/dQ^2 d\vec{Q}$; a calculation of $d\sigma/dQ_\perp$ within our model agrees well with the available data.

I. INTRODUCTION

A great number of experimental results are by now available in the field of lepton production in high-energy hadronic collisions. Most of them come from the observation of a single lepton produced with a large $p_\perp$ ($0.54 \lesssim p_\perp \lesssim 5.4$ GeV/c), in proton-proton collisions[1] or in proton-nucleus collisions.[2] Some experiments have also studied the production of lepton pairs with a large invariant mass. The earliest one[3] gives the cross section $d\sigma/dM_{l^+l^-}$ in the range $1 \lesssim M_{l^+l^-} \lesssim 6$ GeV/c^2, whereas the others[4,5] have mainly studied the $J(\Psi)$ region ($M_{l^+l^-} \simeq 3.1$ GeV/c^2).

The comparison of the experimental results with theoretical predictions[6-8] shows that the most simple models cannot account for all the data. The most popular is that of Drell and Yan,[6] who proposed a scheme in which a massive photon is produced through a $q\bar{q}$ annihilation, with the quark and the antiquark coming from the incident hadrons. This picture leads to a cross section which "scales," $d\sigma/dQ^2 = (1/Q^4) f(Q^2/s)$, where $\sqrt{s}$ is the c.m. energy of the incident hadrons and $Q^2 = M_{l^+l^-}{}^2$. Many calculations[9] led always to numerical results lower than the data. A "renormalization" of the data, recently proposed by Farrar[10] to take nuclear effects into account, brings theories closer to experiments. But a factor-of-10 disparity still exists between theoretical predictions and data for $M_{l^+l^-} \gtrsim 4.0$ GeV/c^2.

When only a single lepton is detected, the Drell-Yan model leads to a prediction which is one or two orders of magnitude below the data.[9] Since the inclusive single-lepton cross section is proportional to that of the pion over a large range of $p_\perp$, it is also tempting to consider the lepton as a decay product of a vector meson.[8] But the ρ, ω, and φ production at high $p_\perp$ seems to be too weak to fit the data,[11,12] and explanations founded on the $J(\Psi)$ contribution[12,13] appear to be inconsistent with a recent experiment.[1]

Thus the production of leptons is still an open question, and it is necessary to study other mechanisms which may contribute to it; it is also important to propose experimental ways which could allow us to distinguish them from the Drell-Yan and vector-meson-dominance (VMD) pictures.

Many reactions involving large-$p_\perp$ or large-invariant-mass production have received an explanation within models where hadron constituents (partons) undergo a "hard scattering" in which large momenta are exchanged.[9,14,15] These models give, for hadron production, results in good agreement with experiment. For instance, the subprocess $\pi + q \to \pi + q$ leads to a good fit of the CERN ISR data for the production of π with large $p_\perp$ (see below).

In this paper we shall study, within the framework of "hard-scattering" models, the subprocess (M stands for meson)

$$M + q \to \gamma + q \,, \tag{1}$$

in which a massive photon is produced which subsequently decays into a pair of leptons. We expect, according to counting-rules arguments,[16] that this subprocess has a non-negligible contibution to the lepton production cross section.

The bound state M will be described by a covariant wave function, and particular attention will be paid to the problem of gauge invariance. Arbitrary constants arising from the meson wave function will be determined by fitting the ISR data for the inclusive π production with the subprocess $\pi + q \to \pi + q$. Our results are the following:

(i) The massive-pair-production cross section "scales":

$$\frac{d\sigma}{dQ^2} = \frac{1}{Q^6} g\left(\frac{Q^2}{s}\right). \tag{2}$$

The numerical results indicate that the contribution of subprocess (1) may be greater, for $Q^2 \lesssim 9$ GeV^2, than that calculated in the Drell-Yan model.

(ii) For large-$p_\perp$ processes, we find ($\vec{K}$ is the lepton momentum)

$$E\frac{d\sigma}{d\vec{K}} = \frac{1}{|\vec{K}_\perp|^6} h(x_\perp)$$

with

$$x_\perp = \frac{2|\vec{K}_\perp|}{\sqrt{s}}. \tag{3}$$

The numerical results indicate that the scaling law (3) may become observable for $K_\perp \gtrsim 4$ GeV/c and small $x_\perp$, superseding the $K_\perp^{-8}$ behavior observed for $K_\perp \lesssim 4$ GeV/c.

We dwell on the fact that subprocess (1) may be experimentally distinguished from that of Drell and Yan by measuring the transverse momentum of the massive lepton pair. In the Drell-Yan picture, the photon is emitted with a transverse momentum $|\vec{Q}_\perp| \lesssim 0.7$ GeV/c, whereas process (1) leads to a power-law behavior of the cross section

$$\frac{Q^2 d\sigma}{dQ^2 d\vec{Q}_\perp} = \frac{1}{|\vec{Q}_\perp|^6} k\left(x_\perp, \frac{Q^2}{s}\right) \text{ for } Q^2 < \vec{Q}_\perp{}^2, \tag{4}$$

which allows the production of large-$\vec{Q}_\perp$ lepton pairs. It may also be distinguished from the vector-dominance model in that no bumps appear in $d\sigma/dQ^2$ for $Q^2 \simeq m_V{}^2$ and in $E d\sigma/\vec{K}$ at $\vec{K} \simeq m_V/2$ (for large m_V).

(iii) Another interesting feature of the cross section $Q^2 d\sigma/dQ^2 d\vec{Q}_\perp$, calculated from subprocess (1), is its flat behavior in $|\vec{Q}_\perp|$, at fixed Q^2, as long as $|\vec{Q}_\perp|^2 \lesssim \frac{1}{2}Q^2$. Such a flat behavior seems to be observed in the experiment of Christenson *et al.*[3]

Other subprocesses may also contribute to the lepton production, but we expect, according to counting-rules arguments, contributions smaller than that coming from subprocess (1). The only subprocess which might be competitive is the fusion one, $q + \bar{q} \to M + \gamma$. It has been studied by Esobar,[17] who found that its contribution to the lepton production at large $K_\perp$ is negligible. However, Escobar has only calculated the lepton production coming from the decays of photons with small mass $(Q^2)^{1/2}$ and large transverse momentum $Q_\perp$. It seems also that he did not allow the mass $(Q^2)^{1/2}$ to reach the kinematical limit $2m_l$ (m_l is the lepton mass), which leads to a term proportional to $\log(m_l{}^2)$ in the cross section. On the other hand, in this paper we integrate over all the allowed range for $(Q^2)^{1/2}$.

Sachrajda and Blankenbecler have classified the different subprocesses according to the behavior of their cross sections[16]:

$$Q^4 \frac{d\sigma}{dQ^2} \propto (Q^2)^{-n}\left(1 - \frac{Q^2}{s}\right)^k. \tag{5}$$

For the subprocess (1), they get $n = 1$ and $k = 3$, in disagreement with our result $k = 2$. This discrepancy, as a matter of fact, illustrates that "spin complications" may change the counting rules. Indeed, Sachrajda and Blankenbecler used a $\lambda\phi^4$ model with scalar quarks, whereas we use a quark-gluon model with spinor quarks.

The plan of this paper is the following. In Sec. II, we calculate the Born terms of subprocess (1) with a quark-gluon field theory, with particular attention paid to the problem of gauge invariance. In Sec. III, we study, within the same framework, the contribution of subprocess $\pi + q \to \pi^0 + q$ to the inclusive production of pions. Sections IV and V are devoted to our results on massive-lepton-pair production and on large-$p_\perp$ single-lepton production. In Sec. VI, we give and discuss the cross section $d\sigma/dQ^2\vec{Q}_\perp dQ_\parallel$ for the production of a lepton pair with large $\vec{Q}_\perp$ in proton-proton and meson-proton collisions. Section VII is a conclusion.

II. THE SUBPROCESS $M + q \to \gamma + q$

In this section we calculate the amplitude $M + q \to \gamma + q$, where M is a $q\bar{q}$ pseudoscalar bound state (Fig. 1). We have the usual definition $\hat{s} = (K+p)^2$, $\hat{t} = (K - K')^2$, $\hat{u} = (K' - p)^2$ with $\hat{s} + \hat{t} + \hat{u} = K'^2$, where the quark and meson masses are neglected.

We use a quark-gluon field theory, known to lead to the counting rules,[18] and we assume that the Born terms are dominant in the asymptotic region where $\hat{s}$, $\hat{t}$, and $\hat{u}$ are large. Thus we have to consider the four Feynman graphs of Fig. 2. Graph 2(c) represents the Drell-Yan mechanism and graph 2(d) is a final-state correction to it. The only difference from the original Drell-Yan picture is that the massive photon may be produced with a large $\vec{K}'_\perp$. These corrections to the Drell-Yan picture have already been considered,[19] so, in this paper, we shall study a completely different production mechanism described by graphs 2(a) and 2(b). Owing to the bound state M, graphs 2(a) and 2(b) are generally not gauge-invariant. They become gauge-invariant only in the limit where the $q\bar{q}$ bound state is described by the product of free quark and antiquark wave functions, that is, in the limit where we do not take into account the internal motion and the off-mass-shell effects due to the binding. Therefore, we guess that in the case where the binding effects are negligible we shall have a gauge-invariant result. An example of such a situation happens when $\hat{s}$, $\hat{t}$, and $\hat{u}$ are large;

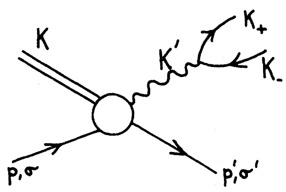

FIG. 1. The subprocess $M+q\to l^+l^-+q$.

this is precisely the kinematical region explored in the large-$p_\perp$ or large-Q^2 phenomena. Let us examine this point in detail.

If $\chi(K,k)$ is the covariant wave function of M with total momentum K and relative momentum k, we get from graphs 2(a) and 2(b)

$$\mathfrak{M}_a^\mu=\bar{u}(p',\sigma')\gamma^\mu\frac{1}{\not p'+\not K'}\Gamma\int d^4k\frac{\chi(K,k)}{(p+\frac{1}{2}K-k)^2}\times\Gamma u(p,\sigma)\,, \tag{6a}$$

$$\mathfrak{M}_b^\mu=\bar{u}(p',\sigma')\Gamma\int d^4k\frac{1}{\frac{1}{2}\not K+\not k-\not K'}\gamma^\mu\frac{\chi(K,k)}{(\frac{1}{2}K+p-k)^2}\times\Gamma u(p,\sigma)\,, \tag{6b}$$

where Γ is a Dirac matrix coming from the quark-gluon coupling. If the binding effects are negligible in the asymptotic region, we can make the substitution

$$\int d^4k\frac{\chi(K,k)}{(p+\frac{1}{2}K-k)^2}\Rightarrow\frac{1}{(p+\frac{1}{2}K)^2}\int d^4k\,\chi(K,k)\,, \tag{7a}$$

$$\int d^4k\frac{1}{\frac{1}{2}\not K+\not k-\not K'}\gamma^\mu\frac{\chi(K,k)}{(\frac{1}{2}K+p-k)^2}\Rightarrow\frac{1}{\frac{1}{2}\not K-\not K'}\gamma^\mu\frac{1}{(p+\frac{1}{2}K)^2}\int d^4k\,\chi(K,k)\,, \tag{7b}$$

and thus recover the free-quark case. But the crucial point is the form of the covariant wave function. If we describe, for instance, the meson M by

$$\chi(K,k)=(\tfrac{1}{2}\not K+\not k)^{-1}\gamma^5(\tfrac{1}{2}\not K-\not k)^{-1}G(k,K)\,, \tag{8}$$

with $G(\not k,K)\propto(k^2-\alpha^2+i\epsilon)^n$ and $n\geq 2$, substitution (7) is not allowed, as shown in Appendix A, and $m^\mu=m_a^\mu+m_b^\mu$ is not gauge-invariant. The physical reason for this phenomenon is the Lorentz dilatation of k introduced by the propagators of (8) when $\vec{K}$ becomes large. This dilatation does not permit us to neglect the relative momentum. But if the wave function of M is

$$\chi(K,k)=\not K\gamma^5G(k,K)\,, \tag{9}$$

with $G(k,K)\propto(k^2-\alpha^2+i\epsilon)^n$ and $n\geq 3$, substitution (7) is allowed (see Appendix A), and we get for the dominant contribution of graphs 2(a) and 2(b) (here and in the following we take $\Gamma=1$; analogous results are obtained with vector gluons)

$$\mathfrak{M}^\mu=\bar{u}(p',\sigma')\left[\frac{\gamma^\mu\gamma^5}{\hat{s}}+\frac{2\gamma^\mu\gamma^5\hat{s}-2\not K'\gamma^5(K'^\mu+p'^\mu+p^\mu)}{\hat{s}(K'^2+\hat{t})}\right]\times u(p,\sigma)2\tilde{G}(0)\,, \tag{10}$$

with $\tilde{G}(0)=\int d^4k\,G(k,K)$. We have included the quark-gluon coupling constants in $\tilde{G}(0)$. Expression (10) is exactly gauge-invariant (we have set the meson, quark, and gluon masses equal to zero).

Let us note that we did not assume that the $q\bar{q}$ wave function was a solution of the Bethe-Salpeter equation deduced within the framework of a quark-gluon field theory. We used such a theory only to describe reactions with large momenta exchanged. Quark confinement, internal motion, etc., may proceed from another scheme which has no consequences for our calculations. Indeed, the effect of the wave function amounts to that of a multiplicative parameter.

During this work, we shall use the amplitude (10) to describe the process $M+q\to\gamma+q$ in the asymptotic region of large $\hat{s}$, $\hat{t}$, and $\hat{u}$. However, we see that the second term in the square bracket has a pole at $\hat{t}=-K'^2$, which corresponds to a quark on its mass shell. For this value of $\hat{t}$, our result is no longer correct. Indeed, when $\hat{t}\simeq-K'^2$, the integration over k in (6) replaces the pole by an enhancement of the amplitude, but the result depends strongly on the form of the wave function and substitution (7) is no longer allowed. Moreover, the model we use may be justified when the particles are far from their mass shell, but we do not know what occurs when we are in the vicinity of a pole. All that we can say is that an enhancement of the amplitude must occur in the region $\hat{t}\simeq-K'^2$. In order to describe this enhancement in a gauge-invariant way, we assume that it has a width $2d$ and, for $|K'^2+\hat{t}|\leq d$, we replace (10) by

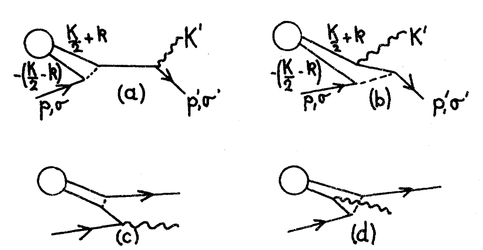

FIG. 2. The Born terms of subprocess $M+q\to\gamma+q$.

$$\mathfrak{M}^\mu = \bar{u}(p',\sigma')\left[\frac{\gamma^\mu\gamma^5(K'^2+\hat{t})}{\hat{s}d}+\frac{2\gamma^\mu\gamma^5\hat{s}-2K\!\!\!/'\gamma^5(K'^\mu+p'^\mu+p^\mu)}{\hat{s}d}\right]u(p,\sigma)\epsilon(K'^2+\hat{t})2\tilde{G}(0)\,, \qquad (11)$$

which extrapolates (10) in a smooth way. The parameter d depends on the radius of the wave function and on the behavior of the quark propagator close to the pole. It expresses the fact that we cannot neglect some masses in this kinematical region and it must have the value of a characteristic $(\text{mass})^2$ of the $q\bar{q}$ bound state.

The other arbitrary parameter is $\tilde{G}(0)$. We shall fix its value by calculating the subprocess $M+q \to \pi^0+q$ within the same model and by fitting the ISR results.

III. PRODUCTION OF π^0 AT LARGE $p_\perp$ IN $M+q\to\pi^0+q$

In this section, we study the contribution of the subprocess

$$M+q\to\pi^0+q \qquad (12)$$

to the inclusive meson production in proton-proton collision, and we shall assume that it is dominant at low $x_\perp$. The counting rules[18] lead indeed to a $(p_\perp)^{-8}$ behavior of the cross section in agreement with the ISR results.[22] The fusion process[20] also leads to a $(p_\perp)^{-8}$ behavior and its contribution may be as important as that considered here. However, we only need a rough estimate of $|\tilde{G}(0)|^2$, and by neglecting the fusion process the results cannot be changed by a factor of more than 2.

The amplitude $M+q\to\pi^0+q$ is calculated from the Feynman graphs of Fig. 3, which are analogous to graphs 2(a) and 2(b). We get

$$E\frac{d\sigma^{(M+q\to\pi^0+q)}}{d\vec{p}}=\frac{|2\tilde{G}(0)|^4}{(4\pi)^2}\,\frac{(\hat{s}+\hat{t})}{\hat{s}^2}\left(\frac{1}{\hat{s}}+\frac{2}{\hat{t}}\right)^2 \times\delta(\hat{s}+\hat{t}+\hat{u})\,. \qquad (13)$$

We have now to do the convolution[15] of $d\sigma^{(M+q\to\pi^0+q)}$ with the structure functions $G_{M/p}(x)$ and $G_{q/p}(x)$, which describe the probabilities of finding a meson (a quark) of momentum x in the incident proton. Let P_1 and P_2 be the momenta of the colliding protons and p be that of the π^0; with $s=(P_1+P_2)^2$, $t=(P_1-p)^2$, $\hat{s}=x_1x_2s$, $\hat{t}=x_1t$, and $x_\perp=2|\vec{p}_\perp|/\sqrt{s}$, we get for a π^0 produced at 90°

$$E\frac{d\sigma}{d\vec{p}}=\frac{3|2\tilde{G}(0)|^4}{(4\pi)^2s^4}\int dx_1dx_2\,\delta(x_1x_2-\tfrac{1}{2}x_\perp(x_1+x_2))\,G_1(x_1)\left[G_{\mathcal{P}/p}(x_2)+G_{\mathfrak{N}/p}(x_2)+G_{\lambda/p}(x_2)\right] \times\left[\frac{1}{x_1{}^3x_2{}^2}(x_2-\tfrac{1}{2}x_\perp)\left(\frac{1}{x_2}-\frac{4}{x_\perp}\right)^2\right], \qquad (14)$$

where $G_1(x)$ is the momentum distribution of an $\mathfrak{N}\bar{\mathcal{P}}$ pseudoscalar state in a proton (see Appendix B). The structure functions $G_{q/p}(x)$ may be extracted from the e-proton and e-neutron deep-inelastic scattering data. We take here those deduced by Gunion.[21] In order to determine $G_1(x)$, we assume that the quarks of the sea are produced through "hadronic bremsstrahlung,"[15]

$$G^{\text{sea}}_{q/p}(x)=\sum_M\int_x^1\frac{dz}{z}G_{M/p}(z)G_{q/M}(x/z)\,, \qquad (15)$$

and that the sum over M may be limited to the pseudoscalar mesons. The fact that we omit other contributions in (15) is roughly counterbalanced by also neglecting these contributions in subprocesses (1) and (12).

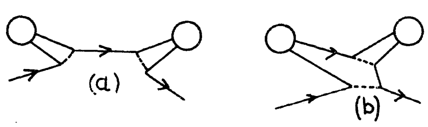

FIG. 3. The Born terms of subprocess $M+q\to M+q$.

The "Regge" and diffractive components of $G_{\mathcal{P}/p}(x)$ given by Gunion,

$$G^{\text{sea}}_{\mathcal{P}/p}(x)=G^R_{\mathcal{P}/p}(x)+G^D_{\mathcal{P}/p}(x) =1.88\frac{(1-x)^7}{\sqrt{x}}+0.2\frac{(1-x)^7}{x}\,, \qquad (16)$$

are related to $G_1(x)$ through (15). We describe the pseudoscalar structure function by $G_{q/M}(x)=6(1-x)x$ with $\int_0^1 dx\,G_{q/M}(x)=1$, thereby taking only the valence quarks into account. We shall not try to find for $G_1(x)$ an expression which verifies (15) for each x, but we write

$$G_1(x)=a_D\frac{(1-x)^5}{x}+a_R\frac{(1-x)^5}{\sqrt{x}} =G_1^D(x)+G_1^R(x)\,, \qquad (17)$$

where the behavior as x tends to zero (one) is justified[21] by "Regge arguments" (counting rules), and we fix a_D and a_R through the condition

$$\int_0^1 dx\,x\,G^R_{\mathcal{P}/p}(x)=6\int_0^1 dz\,z\,G_1^R(z)\int_0^1 dy\,y\,G_{\mathcal{P}/M}(y)\,, \qquad (18)$$

$$\int_0^1 dx\, x\, G^D_{\mathcal{P}/p}(x) = 3\int_0^1 dz\, z\, G^D_1(z) \int_0^1 dy\, y\, G_{\mathcal{P}/M}(y)\,. \tag{19}$$

In this way we get

$$a_R = 0.41\,,\quad a_D = 0.1\,. \tag{20}$$

From these numbers, we find that the part of the proton momentum carried away by a π^- is $\int_0^1 dx\, G_1(x) x = 0.04$.

We now adjust the parameter $|2\tilde{G}(0)|$ in order to fit the results of Büsser *et al.*[22] For $|2\tilde{G}(0)|^2 = 1.5\times 10^2$ GeV^2 we have an excellent agreement with the experimental data which is shown in Fig. 4.

In a recent experiment at Fermilab,[23] the ratio $R = \pi^- p \to \pi^0 X / pp \to \pi^0 X$ was found to be close to unity. This fact may be called an indication that the subprocess $M+q \to M+q$ is not dominant compared to others such as $q+\bar{q} \to M+\text{jet}$[24] or $q+qq \to q+qq$.[25] We want to point out here that constituent-model predictions for pion-beam experiments are highly model-dependent. To illustrate this point, we have calculated the contribution of the subprocess $M+q \to M+q$ to the inclusive cross section $pp \to \pi^0 X$ and to the ratio R within three models: the spinor quark model used in this paper, a Φ^4 model, and a model in which the scaled squared amplitude $|\hat{s}A|^2 = f(\hat{t}/\hat{s})$, is assumed to be a constant. For $\sqrt{s} = 23.5$ GeV and $x_\perp = 0.4$, we get inclusive cross sections in the ratio 7.6/1/1 and for the R values we get 0.6, 15.5, and 1. We see that the cross sections are not too model-dependent, but that the ratio R is very sensitive to the model used for the calculations. This is due to the fact that pion beam kinematics fixes the value $x_1 = 1$ in (14) [we have $G(x_1) = \delta(1-x_1)$], where strong differences between models may exist. In the reaction $pp \to \pi^0 X$, these differences are reduced by the integration over x_1. The model used in this paper, leading to $R = 0.6$ at $x_\perp = 0.4$, is not in disagreement with experiment. We have not taken into account the fragmentation ($q+M \to q+M^* \to q+M+X$), but a recent analysis of correlation data[26] indicates that the direct production of mesons is comparable (or higher) to the production of mesons through a fragmentation mechanism. Thus we believe that our conclusions should be unchanged when introducing the fragmentation mechanism.

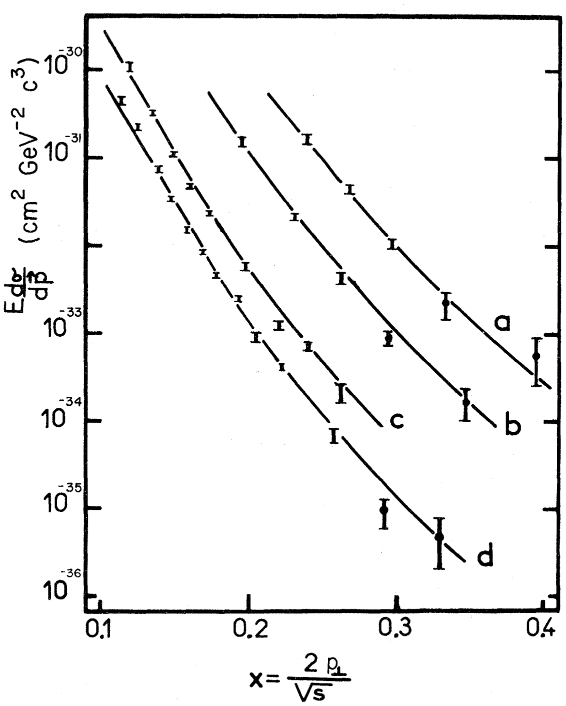

FIG. 4. Fit to the large-$p_\perp$ pion production data at curve (a) $\sqrt{s} = 23.5$ GeV; curve (b) $\sqrt{s} = 30.6$ GeV; curve (c) $\sqrt{s} = 44.8$ GeV; curve (d) $\sqrt{s} = 52.7$ GeV. The experimental points are those of Büsser *et al.*, Ref. 22.

IV. THE PRODUCTION OF MASSIVE LEPTON PAIRS

We now have everything we need to calculate $d\sigma/dQ^2$. Let us define

$$T^{\mu\nu} = \tfrac{1}{2}\sum_{\sigma,\sigma'} m^\mu m^\nu$$

and

$$t^{\mu\nu} = 4(K_+^\mu K_-^\nu + K_+^\nu K_-^\mu - g^{\mu\nu}\tfrac{1}{2}K'^2)$$

(see Fig. 1). The cross section for the subprocess of Fig. 1 is

$$d\sigma^{(s)} = \frac{(4\pi\alpha)^2}{2\hat{s}(2\pi)^5}\int \frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}\,\delta^4(p+K-K_+-K_--p')\times\frac{d\vec{K}_+}{2E_+}\frac{d\vec{K}_-}{2E_-}\frac{d\vec{p}'}{2E'}\,, \tag{21}$$

which can be rewritten

$$d\sigma^{(s)} = \frac{2\alpha^2}{\hat{s}(2\pi)^3}\int dQ^2 d^4K' d^4p'\,\theta(p'^0)\,\delta(Q^2-K'^2)\,\delta(p'^2)\times\delta^4(p+K-p'-K')\frac{T^{\mu\nu}\tau_{\mu\nu}}{Q^4}\,, \tag{22}$$

where

$$\tau_{\mu\nu} = \int\frac{d\vec{K}_+}{2E_+}\frac{d\vec{K}_-}{2E_-}\,\delta^4(K'-K_+-K_-)t_{\mu\nu} = \tfrac{2}{3}\pi(K'^\mu K'^\nu - g^{\mu\nu}K'^2)\,. \tag{23}$$

Using gauge invariance, we get

$$\frac{d\sigma^{(s)}}{dQ^2}=\frac{2}{3}\frac{\alpha^2}{(2\pi)^2}\frac{1}{\hat{s}}\int d^4K'd^4p'\theta(p'^0)\delta(p'^2)\delta(Q^2-K'^2)\times\delta^4(p+K-K'-p')\left(-\frac{T^\mu{}_\mu}{Q^2}\right). \tag{24}$$

The integrals over the δ functions in (24) may be easily calculated with the result

$$\int d^4K'\,d^4p'\,\theta(p'^0)\delta(Q^2-K'^2)\delta(p'^2)\delta^4(p+K-p'-K')=\frac{\pi}{2}\int_{\hat{\tau}-1}^{0}d\sigma\,, \tag{25}$$

where we have defined $\sigma=\hat{u}/\hat{s}$ and $\hat{\tau}=Q^2/\hat{s}$. At this stage of the calculation we can introduce, in a straightforward way, a longitudinal or transverse cutoff over the massive-pair momentum, which changes the integration limits in (25). Finally, we get (without cutoff)

$$\frac{d\sigma^{(s)}}{dQ^2}=\frac{\alpha^2}{12\pi}\frac{\hat{\tau}^2}{Q^6}\int_{\hat{\tau}-1}^{0}d\sigma(-\tilde{T}^\mu{}_\mu)\,, \tag{26}$$

with

$$\tilde{T}^\mu{}_\mu=\frac{2\,|2\tilde{G}(0)|^2}{(2\tilde{\tau}-1-\sigma)^2}[4\tilde{\tau}^3-8\tilde{\tau}^2\sigma+\tilde{\tau}(5\sigma^2+2\sigma-3)-(1+\sigma+\sigma^2+\sigma^3)]\equiv-2\,|2\tilde{G}(0)|^2 I\left(\tilde{\tau},\sigma,\frac{d}{\hat{s}}\right). \tag{27}$$

We recall that in (27), $(2\tilde{\tau}-1-\sigma)$ must be replaced by $d/\hat{s}$ if $|2\tilde{\tau}-1-\sigma|\leq d/\tilde{s}$.

In the limits where $\tilde{\tau}\to 1$, it is easy to verify that

$$Q^4\frac{d\sigma^{(s)}}{dQ^2}\propto\frac{(1-\tilde{\tau})^2}{Q^2}\,. \tag{28}$$

This result is different from that of Sachrajda and Blankenbecler. Here we have an illustration of how "spin complications" may change the counting rules of Ref. 16.

The convolution of $d\sigma/dQ^2$ with $G_{M/p}(x_1)$ and $G_{q/p}(x_2)$ gives (see Appendix B)

$$\frac{d\sigma}{dQ^2}=\frac{\alpha^2}{3\pi}\frac{|2\tilde{G}(0)|^2}{Q^6}\tau^2\int\frac{dx_1dx_2}{(x_1x_2)^2}\theta(x_1x_2-\tau)G_1(x_1)\times[G_{\mathcal{O}/p}(x_2)+G_{\mathfrak{N}/p}(x_2)+G_{\lambda/p}(x_2)]\times\int_{\hat{\tau}-1}^{0}d\sigma I\left(\hat{\tau},\sigma,\frac{d}{\hat{s}}\right), \tag{29}$$

where we have assumed that the incident hadrons were protons. We have defined $\tau=Q^2/s$. Let us note that we have taken colorless quarks with fractional charges.

We present numerical results for two values of s: $s=57$ GeV^2, for which exist experimental results,[3] and $s=2500$ GeV^2, which is a typical ISR value. To compare with the data, we have to introduce a longitudinal cutoff present in the experiment ($K'_{1\text{ab}\parallel}\geq 12$ GeV/c). Our results, for $d=0.5$, 1, and 2 GeV^2, are shown in Fig. 5. We observe that the variation of $d\sigma/d(Q^2)^{1/2}$ as a function of d is roughly linear. The parameter d, following the assumption of Sec. II, must have the value of a characteristic (mass)^2 of the $q\bar{q}$ bound state. It may be close, for instance, to the central (mass)^2 of the 35-plet or to the inverse of the Regge slope (which can be connected to the radius of the $q\bar{q}$ wave function[27]); the value $d=1-2$ GeV^2 appears to be reasonable.

Our calculation shows that the contribution of the subprocess $M+q\to\gamma+q$ may be important for $Q^2\lesssim 9$. For larger Q^2, we note a steep decrease of $d\sigma/d(Q^2)^{1/2}$ connected to the very weak probability of finding a pseudoscalar with x_1 close to 1 in the proton. We also present, as a comparison, the contribution of the Drell-Yan mechanism, calculated with the structure functions of Gunion, for color-colorless quarks.

The results for $s=2500$ GeV^2 are given in Fig. 6. We observe that for small τ, the mechanism $M+q\to\gamma+q$ may be more important than that of Drell and Yan. We also show the sensitiveness of $d\sigma/d(Q^2)^{1/2}$ to a transverse cutoff of 1 GeV/c on the pair momentum, a cutoff which leaves the Drell-Yan contribution unchanged. We have thus an experimental way to distinguish the contribution of the Drell-Yan scheme from that of the $M+q\to\gamma+q$ scheme.

We conclude this section with a result on the production of real photons. The calculation proceeds following the usual steps and we find that a good fit to our numerical result is given, at $\sqrt{s}=23.5$ GeV, by

$$E'd\sigma/d\vec{K}'=1.3\times10^{-30}(1-x_\perp)^8/|\vec{K}'_\perp|^6\ (\text{cm}^2\ \text{GeV}^{-2}c^3)\,.$$

This result is close (smaller by a factor of 2.3) to that obtained by Escobar[17] for the production of real photons in the fusion process.

V. THE PRODUCTION OF LARGE-$p_\perp$ LEPTONS

Since the calculation of the cross section $E_+d\sigma/d\vec{K}_+$ is tedious, we describe here only the main points and invite the reader to refer to Appendix C for more details.

We have to pay special attention to the region where $K'^2\simeq 4m_l^2$ (m_l is the lepton mass). Indeed, for $K'^2=4m_l^2$, the leptons produced by the decay of the photon are collinear and we have $K'^\mu=2K_+^\mu=2K_-^\mu$. Therefore, the lepton tensor becomes $t^{\mu\nu}=K'^\mu K'^\nu-g^{\mu\nu}K'^2$ and by gauge invariance we

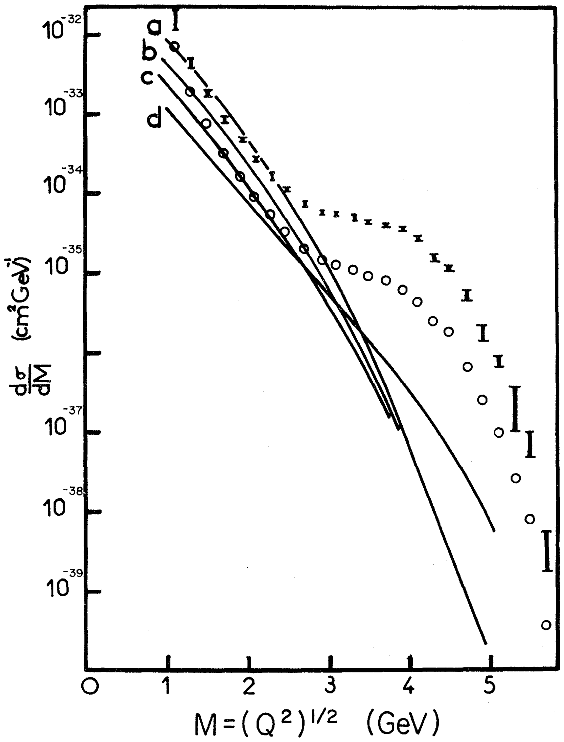

FIG. 5. The cross section $d\sigma/dM$ at $s=57$ GeV^2 for curve (a) $d=0.5$ GeV^2; curve (b) $d=1$ GeV^2; curve (c) $d=2$ GeV^2. Curve (d) is the Drell-Yan contribution. The experimental points are those of Christenson *et al.*, Ref. 3. The open circles are the "renormalized" data (see Ref. 10).

get

$$\frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}\propto\frac{1}{K'^2}\,, \tag{30}$$

leading to a logarithmic dependence on $m_l{}^2$ of the cross section.

The cross section is given by

$$E_+\frac{d\sigma^{(s)}}{d\vec{K}_+}=\frac{\alpha^2}{\hat{s}(2\pi)^3}\int d^4K'\,\delta((K'-K_+)^2-m_l{}^2)\times\delta(\hat{s}+\hat{t}+\hat{u}-K'^2)\frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}\,. \tag{31}$$

We calculate (31) in the center of mass of the subprocess

$$E_+\frac{d\sigma^{(s)}}{d\vec{K}_+}=\frac{\alpha^2}{8(2\pi)^3\hat{s}^{3/2}K_+}\int dQ^2d\phi'\,\theta(\hat{s}-Q^2)\theta(Q^2-m_l{}^2)\times\theta\left(1-\left|\frac{\sqrt{\hat{s}}\,Q^2-(\hat{s}+Q^2)K_+^0}{(\hat{s}-Q^2)K_+}\right|\right)\times\frac{T^{\mu\nu}t_{\mu\nu}}{Q^4}\,, \tag{32}$$

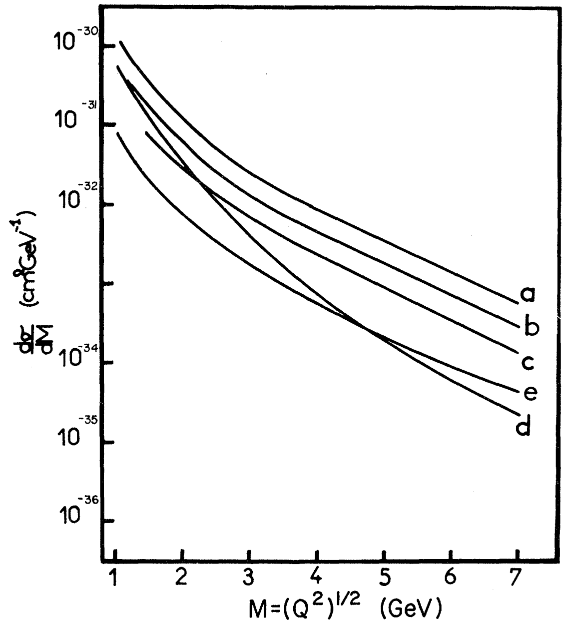

FIG. 6. The cross section $d\sigma/dM$ at $s=2500$ GeV^2 for curve (a) $d=0.5$ GeV^2; curbe (b) $d=1$ GeV^2; curve (c) $d=2$ GeV^2; curve (d) $d=1$ GeV^2 and a transverse cutoff of 1 GeV/c on the pair momentum. Curve (e) is the Drell-Yan contribution.

where K_+^0, $K_+=|\vec{K}_+|$ are defined in the cms; $Q^2=K'^2$ and ϕ' is the angle defined in Fig. 7.

The functions θ put limits on the integration domain of Q^2 in (32). For instance, we have

$$Q_{\min}{}^2=\frac{\hat{s}(K_+^0-K_+)}{\sqrt{\hat{s}}-K_+^0-K_+}\simeq\frac{m_l{}^2}{\hat{x}(1-\hat{x})}$$

for $\hat{x}=2K_+/\sqrt{\hat{s}}$ not close to 1.

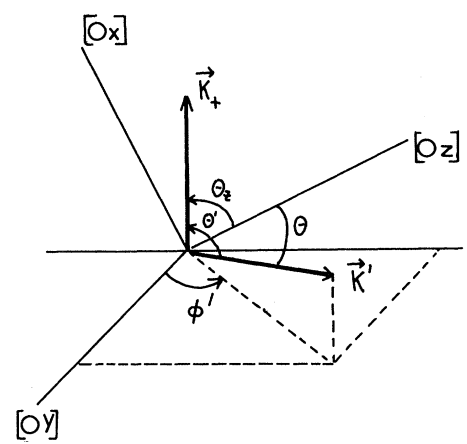

FIG. 7. Kinematics of the large-$|\vec{K}_\perp|$ lepton pair production.

The integration over Q^2 in the region $Q_{\min}^2$ cannot be done with a computer, owing to the Q^2/Q^4 behavior of the integrand. Therefore, we expand $T^{\mu\nu}t_{\mu\nu}$ in a power series of Q^2 and m_l^2, keeping only the lowest orders. The terms proportional to $(Q^2)^0$ or $(m_l^2)^0$ cancel out and we get after integration $E_+ d\sigma^{(s)}/d\vec{K}_+ \propto \ln(m_l^2)$.

The convolution of $E_+ d\sigma^{(s)}/d\vec{K}_+$ with the structure functions gives

$$E_+ \frac{d\sigma}{d\vec{K}_+} = 2 \int dx_1 dx_2 G_1(x_1)[G_{\mathcal{P}/p}(x_2) + G_{\mathfrak{N}/p}(x_2) + G_{\lambda/p}(x_2)] \times E_+ \frac{d\sigma^{(s)}}{d\vec{K}_+}, \tag{33}$$

where we have assumed that the incident hadrons were protons. For a lepton at 90°, it is easy to check from formula (33) and formula (C8) of Appendix C that we get the "scaling law"

$$E_+ \frac{d\tau}{d\vec{K}_+} = \frac{1}{|\vec{K}_+|^6} h(x_\perp). \tag{34}$$

We now present our numerical results. We expect a difference between the cross sections for electron or muon production. This difference, as explained above, comes from the low-Q^2 integration region and is proportional to

$$\int_{Q_{\min}^2(e)}^{m^2} dQ^2 - \int_{Q_{\min}^2(\mu)}^{m^2} dQ^2, \tag{35}$$

where we have chosen $m^2 = 20 m_\mu^2$. For $Q^2 \gtrsim m^2$, the difference (35) is negligible [the lepton (mass)^2 becomes negligible compared to Q^2 and the other variables] and for $Q^2 \lesssim m^2$ our expansion of $T^{\mu\nu}t_{\mu\nu}$ is valid, thus allowing an analytic integration.

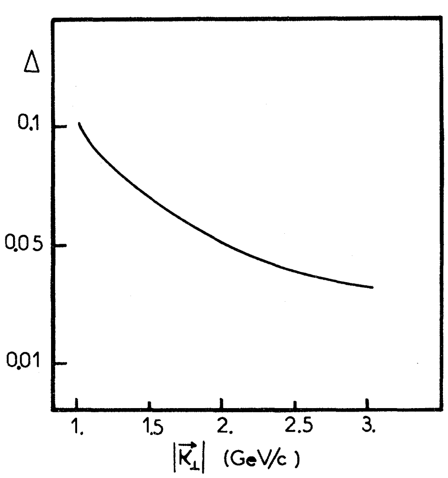

FIG. 8. The ratio Δ at $s = 564$ GeV^2 for $d = 2$ GeV^2.

We define

$$\Delta = \frac{E_+ d\sigma(e)/d\vec{K}_+ - E_+ d\sigma(\mu)/d\vec{K}_+}{E_+ d\sigma(\mu)/d\vec{K}_+}. \tag{36}$$

Figure 8 shows the result of our calculations for $s = 564$ GeV^2 and $d = 2$ GeV^2. The ratio Δ is negligible for $K_{+\perp} \gtrsim 1$ GeV/c, but it steeply increases as $K_{+\perp}$ decreases and may become observable for $K_{+\perp} < 1$ GeV/c. At $K_{+\perp} = 1$ GeV/c, we have

$$\Delta \left(E_+ \frac{d\sigma(\mu)}{d\vec{K}_+} \right) = 2.3 \times 10^{-34}\ (\text{cm}^2\ \text{GeV}^{-2}).$$

However, we cannot give Δ for $K_{+\perp} < 1$ GeV/c, owing to the limitation of our asymptotic model valid only for large $K_{+\perp}$ or Q^2.

In Figs. 9 and 10, we compare our results with ISR and Fermilab data. We remind the reader that the normalization of $E_+ d\sigma/d\vec{K}_+$ has been obtained by fitting large-$p_\perp$ pion production data and by using the structure functions extracted by Gunion from the e-N deep-inelastic reactions. Therefore, the normalization only depends on the parameter d, which may be reasonably chosen in the range 1–2 GeV^2; moreover, we observe in Figs. 9 and 10 that this dependence is weak. With these assumptions, we get a numerical result indicating

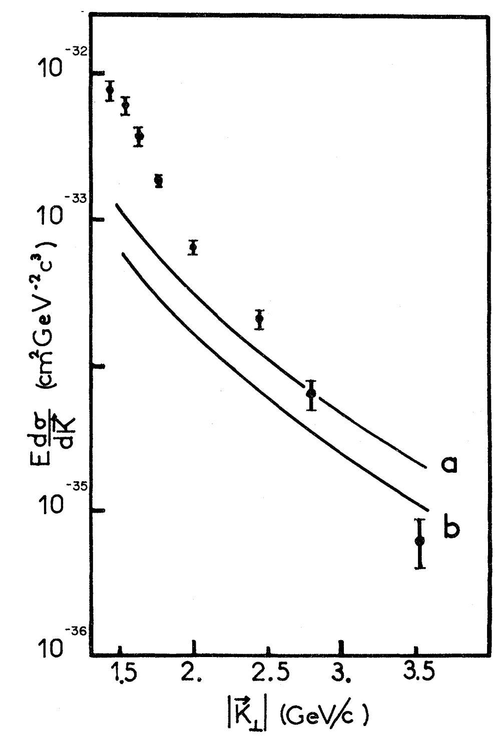

FIG. 9. Cross section for the large-$|\vec{K}_\perp|$ lepton production at $\sqrt{s} = 52.7$ GeV for curve (a) $d = 1$ GeV^2; curve (b) $d = 2$ GeV^2. The experimental points are those of Büsser *et al.*, Ref. 1.

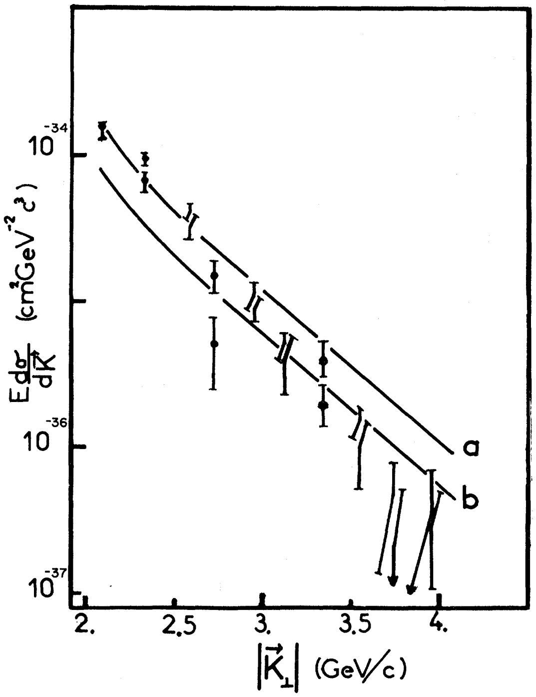

FIG. 10. Cross section for the large-$|\vec{K}_\perp|$ lepton production at $\sqrt{s}=23.7$ GeV for curve (a) $d=1$ GeV^2; curve (b) $d=2$ GeV^2. The experimental points are those of Appel *et al.*, Ref. 2.

that the contribution of subprocess (1) may become important for $K_{+\perp}>3-4$ GeV/c. The way to see if such a contribution appears for $K_{+\perp}>4$ GeV/c would be to fit the data with the expression $E_+ d\sigma/d\vec{K}_+ \propto h(x_\perp)/|\vec{K}_{+\perp}|^n$ and to observe if the exponent n changes from the value $n \gtrsim 8$ for $K_{+\perp}<4$ GeV/c (the present experimental status) to the value $n\simeq 6$ for $K_{+\perp}>4$ GeV/c, predicted by the scaling law (34).

VI. PRODUCTION OF MASSIVE PAIRS WITH LARGE TRANSVERSE MOMENTUM

A very important feature of the experiments where a massive lepton pair is detected with a large total transverse momentum is the prossibility of probing a kinematical region which cannot be reached in large-$p_\perp$ hadron production. Indeed, the massive photon may be far from its mass shell, thus allowing a more complete exploration of hard-scattering dynamics.

Let us examine this point by studying the cross section for the production of massive pairs $d\sigma/dQ^2d\vec{Q}_\perp dx$, where Q^2 is the (mass)^2 of the pair, $\vec{Q}_\perp$ is its transverse momentum, and $x=Q_3/P$ (P is the momentum of an incident hadron in the total center of mass). The experimental knowledge of $d\sigma/dQ^2d\vec{Q}_\perp dx$ would be very valuable to determine the various subprocesses which contribute to the pair production. For instance, in the Drell-Yan picture, a strong cutoff in $\vec{Q}_\perp$ is expected, whatever Q^2 may be; in the VMD model, we must obseve bumps in Q^2 at the meson masses.

We now study the contribution of subprocess (1),

$$\frac{d\sigma^{(s)}}{dQ^2d\vec{Q}_\perp dx}=\frac{\alpha^2}{3(2\pi)^3\hat{s}}\frac{1}{(4\tau+x_\perp{}^2+x^2)^{1/2}} \times \delta(\hat{s}+\hat{t}+\hat{u}-Q^2)\left(-\frac{T^\mu{}_\mu}{Q^2}\right), \qquad (37)$$

where $x_\perp=2|\vec{Q}_\perp|/\sqrt{s}$ and $x=2Q_3/\sqrt{s}$; s is the square of the total energy in the c.m. of the hadron-hadron system. Let $P(P,0,0,P)$ and $\tilde{P}(P,0,0,-P)$ be the momentum of the incident hadrons, and let K and p be given by $K=x_1P$, $p=x_2\tilde{P}$. The convolution of (37) with the proton structure functions leads to

$$\frac{Q^2d\sigma}{dQ^2d\vec{Q}_\perp dx}=\frac{\alpha^2}{3(2\pi)^2s^3}\frac{1}{(4\tau+x_\perp{}^2+x^2)^{1/2}}\int\frac{dx_1dx_2}{(x_1x_2)^2}G_1(x_1)[G_{\mathcal{P}/p}(x_2)+G_{\mathfrak{N}/p}(x_2)+G_{\lambda/p}(x_2)] \times\delta(\tau+x_1x_2-(x_1+x_2)(\tau+\tfrac{1}{4}x_\perp{}^2+\tfrac{1}{4}x^2)^{1/2}+(x_1-x_2)\tfrac{1}{2}x)(-\tilde{T}^\mu{}_\mu), \qquad (38)$$

with $\tilde{T}^\mu{}_\mu$ given by (27). To take the case where $K=x_1\tilde{P}$ and $p=x_2P$ into account, we do the substitution $x \longleftrightarrow -x$ in (38). Figure 11 shows some numerical results for $Q^2d\sigma/dQ^2d\vec{Q}_\perp dx$ obtained with $Q^2=1, 10, 50$, and 100 GeV^2 at $x=0.0$ and $\sqrt{s}=52.7$ GeV (the cross section does not vary for more than 20% for $0.0\le x\le 0.08$). It is interesting to note that for large Q^2, the cross section does not decrease steeply with $|\vec{Q}_\perp|$; the expected $|\vec{Q}_\perp|^{-6}$ $f(x_\perp,\tau)$ behavior only settles for $|\vec{Q}_\perp|^2\simeq\frac{1}{2}Q^2$. This variation with Q^2 of the slope in $|\vec{Q}_\perp|$ is interesting feature of subprocess (1) which can be experimentally tested.

Up to now, no experimental results exist for $d\sigma/dQ^2d\vec{Q}_\perp dx$. However, Christenson *et al.*[3] give the cross section $d\sigma/dQ_\perp$ for $Q^2\gtrsim 1$ GeV^2 and $0.25 \le Q_\perp \le 1.75$ GeV/c. The interesting feature of this result is the flat behavior of the cross section up to $Q_\perp\simeq 1$ GeV/c and then its steep decrease. The contribution of subprocess (1) to $d\sigma/dQ_\perp$ is obtained

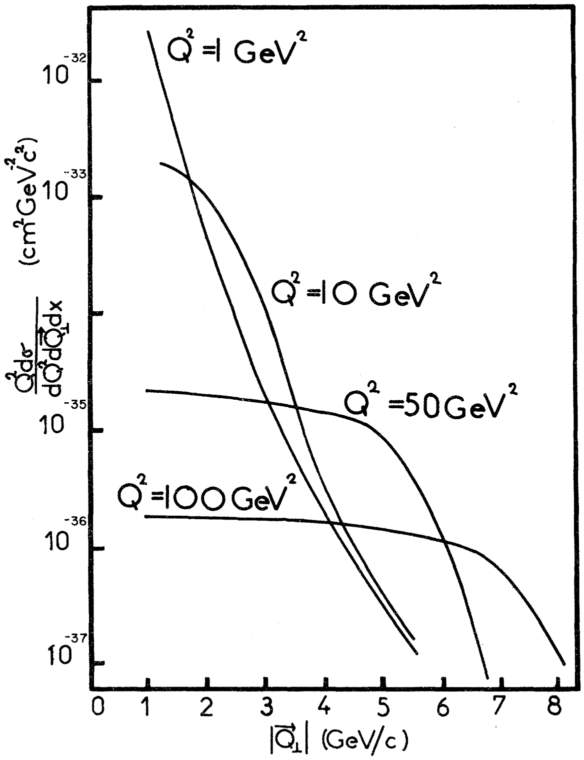

FIG. 11. The cross section $Q^2d\sigma/dQ^2d\vec{Q}_\perp dx$ at $x=0.0$ and $\sqrt{s}=52.7$ GeV for $Q^2=1$, 10, 50, and 100 GeV^2.

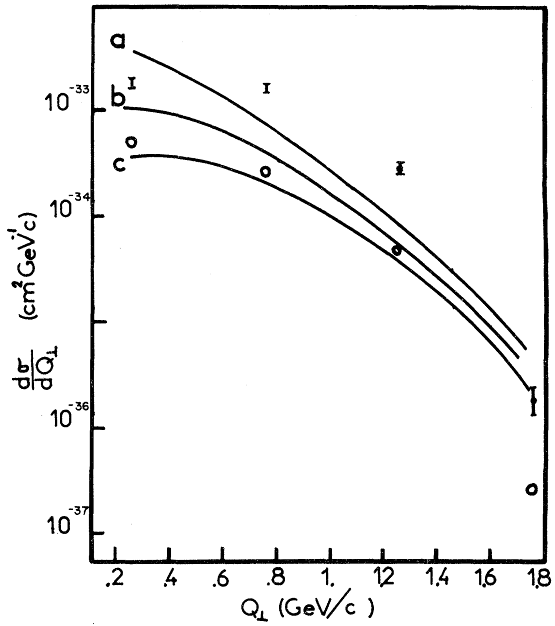

FIG. 12. The cross section $d\sigma/dQ_\perp$ at $s=57$ GeV^2 for curve (a) $Q_{min}{}^2=0.25$ GeV^2; curve (b) $Q_{min}{}^2=1$ GeV^2; curve (c) $Q_{min}{}^2=2$ GeV^2. The experimental points are those of Christenson *et al.*, Ref. 3. The open circles are the "renormalized" data.

by integrating (38) in Q^2 and x with the appropriate cutoff. In the experiment of Ref. 3, the detected dimuons have a laboratory longitudinal momentum greater than 12 GeV/c and a mass greater than 1 GeV. Our results are shown in Fig. 12. The plateau up to $Q_\perp \simeq 1$ GeV/c is correctly reproduced and then the cross section steeply decreases. The agreement with the experiment is still more striking if we take the "renormalization" of the data proposed by Farrar[10] into account.

However, we remind the reader that the experimental cross section is deduced from the data by a Monte Carlo calculation, and that the result depends on the model chosen to describe the dimuons production. Thus important stystematic errors may be present in the result of Christenson *et al.* With these reservations in mind, the agreement between theories and experiments leads us to conclude that, in this kinematical region, the massive-pair production is dominated by the contribution of subprocess (1). The important point in our calculation is the cutoff at $Q_{min}{}^2 \simeq 1$ GeV^2, leading to a flat cross section for $\vec{Q}_\perp{}^2 \lesssim \frac{1}{2}Q^2$. This behavior of the cross section has already been noted in Fig. 11.

In the case of a nucleon-nucleon collision, dynamics of the subprocess is screened by the convolution (38). Therefore, the best way to disentangle the subprocess cross section from the nucleon structure function consists in performing experiments with pion beam. In such a case, $G_1(x_1)$ must be replaced by $\delta(1-x_1)$, leading to the simple expression (for a π^+-proton collision)

$$\frac{Q^2 d\sigma}{dQ^2 d\vec{Q}_\perp dx} = \frac{4}{9}\frac{\alpha^2}{3(2\pi)^2 s^3}\frac{1}{(4\tau + x_\perp{}^2 + x^2)^{1/2}} \times \frac{G_{\mathfrak{N}/p}(x_2)}{x_2{}^2[1-\frac{1}{2}x-(\tau+\frac{1}{4}x_\perp{}^2+\frac{1}{4}x^2)^{1/2}]}(-\tilde{T}^\mu{}_\mu), \tag{39}$$

with

$$x_2 = \frac{-\tau + (\tau + \frac{1}{4}x_\perp{}^2 + \frac{1}{4}x^2)^{1/2} - \frac{1}{2}x}{1-\frac{1}{2}x-(\tau+\frac{1}{4}x_\perp{}^2+\frac{1}{4}x^2)^{1/2}}.$$

We can now directly observe, for instance, the enhancement of $T^\mu{}_\mu$, which occurs when the following relation is verified:

$$Q_\perp{}^2 = Q^2(4\tau + 2x - 1). \tag{40}$$

It provides an observable effect, as shown in Fig. 13, and illustrates the fact that the best way to study the hard-scattering subprocesses consists in looking at $d\sigma/dQ^2 d\vec{Q}_\perp dx$ in pion-beam experiments.

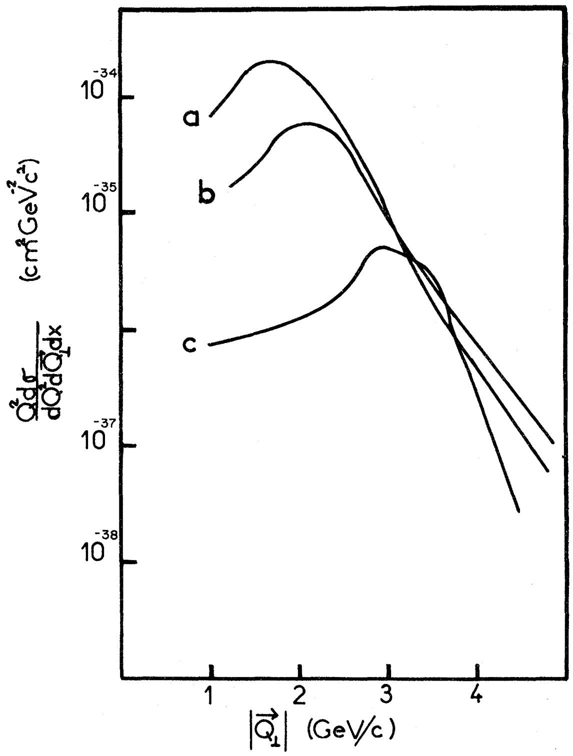

FIG. 13. The cross section $Q^2 d\sigma/dQ^2 d\vec{Q}_\perp dx$ in a π^+-proton collision at $E_{\pi^+} = 150$ GeV and for $d = 2$ GeV^2; curve (a) $x = 0.4$ and $Q^2 = 25$ GeV^2; curve (b) $x = 0.2$ and $Q^2 = 50$ GeV^2; curve (c) $x = 0.0$ and $Q^2 = 80$ GeV^2.

VII. CONCLUSION

In this work, we have studied the contribution of the subprocess $\pi + q \to \gamma + q$ to lepton pair production in hardonic collisions.

Our numerical results indicate that this subprocess gives an important contribution to $d\sigma/dQ^2$, the cross section for massive-pair production; this contribution may be greater than that coming from the Drell-Yan mechanism in the range $1 \leq Q^2 \lesssim 9$ GeV^2. When only a single lepton with large $K_\perp$ is detected, we found that subprocess $\pi + q \to \gamma + q$ gives for $K_\perp \simeq 4$ GeV$/c$ an observable contribution which may become dominant for $K_\perp > 4$ GeV$/c$.

The subprocess $\pi + q \to \gamma + q$ leads to scaling laws for the cross sections which may be experimentally tested. We expect, for instance, that, at the ISR energy ($\sqrt{s} \approx 50$ GeV) (or small $x_\perp$), $E\, d\sigma/d\vec{K} = (1/|\vec{K}_\perp|^n) k(x_\perp)$ (with $n = 6$) for $|\vec{K}_\perp| \gtrsim 4$ GeV$/c$. However, at Fermilab energy ($\sqrt{s} \approx 20$ GeV) (or large $x_\perp$), other subprocesses[16] with higher values of the exponent n, but with a less steeply decreasing function $k(x_\perp)$ as $x_\perp \to 1$, may become important and hide the $|\vec{K}_\perp|^{-6}$ behavior. But the best way to study the origin of leptons in hadronic collisions consists in looking at $d\sigma/dQ^2 d\vec{Q}_\perp dx$. Indirect information on this cross section is given by the experiment of Christenson *et al.*,[3] which measures the cross section $d\sigma/dQ_\perp$. We have calculated in our model

$$\frac{d\sigma}{dQ_\perp} = 2\pi Q_\perp \int dx \int dQ^2 \frac{d\sigma}{dQ^2 d\vec{Q}_\perp dx}$$

by introducing the various cutoff present in the experiment. The agreement obtained between theories and experiments led us to the conclusion that the subprocess $\pi + q \to \gamma + q$ may explain the experimental cross section $d\sigma/dQ^2$ for small Q^2. A more complete study of subprocess $\pi + q \to \gamma + q$ should include graphs (c) and (d) of Fig. 2, that is, the Drell-Yan mechanism. But let us notice that here, contrary to the original Drell-Yan picture, the massive photon may be produced with a large transverse momentum. Let us emphasize the fact that the scheme we propose here enables us to get gauge-invariant amplitudes. The procedures which enlarge the Drell-Yan scheme by introducing a transverse momentum distribution in the nucleon structure function are incorrect; they, lead to nongauge-invariant results by neglecting terms $O(\vec{k}_\perp^2/Q^2)$ which become important for values of the transverse quark momentum $|\vec{k}_\perp|$ equal to or higher than $\sqrt{Q^2}$.

ACKNOWLEDGMENTS

The author would like to thank Professor A. P. Contogouris and Dr. D. Schiff for helpful conversations.

APPENDIX A

In this appendix, we discuss gauge invariance and wave functions in the limit where $\hat{s}, \hat{t}, \hat{u} \to \infty$. Firstly, let us study the wave function given by expression (8),

$$\chi(K,k) = \frac{(\frac{1}{2}\not{K} + \not{k} + m)\gamma^5(\frac{1}{2}\not{K} - \not{k} - m)}{[(\frac{1}{2}K + k)^2 - m^2][(\frac{1}{2}K - k)^2 - m^2]} G(k,K)\,, \tag{A1}$$

where m is the quark mass and $K^2 = M^2$. Its substitution in (6b) leads us to study terms such as

$$I_b = \int \frac{d^4k}{[(\frac{1}{2}K + k - K')^2 - m^2][\frac{1}{2}K + p - k)^2][(\frac{1}{2}K + k)^2 - m^2][(\frac{1}{2}K - k)^2 - m^2][k^2 - \alpha^2]^2}\,. \tag{A2}$$

Its dominant contribution is

$$I_b \simeq \frac{i\pi^2}{p\cdot K} \times\int_0^1 \frac{dx_1dx_2(1-x_1-x_2)\theta(1-x_1-x_2)}{(1-x_1+x_2)[\frac{1}{4}M^2(x_1-x_2)^2+(m^2-\frac{1}{4}M^2)(x_1+x_2)+\alpha^2(1-x_1-x_2)]^2[(1-x_1+x_2)p\cdot K-2p\cdot p'-p'\cdot K(1-x_1+x_2)]}\,. \tag{A3}$$

For the amplitude (6a), we get the corresponding expression

$$I_a \simeq \frac{i\pi^2}{p\cdot K}\int_0^1 \frac{dx_1dx_2(1-x_1-x_2)\theta(1-x_1-x_2)}{(1-x_1+x_2)[\frac{1}{4}M^2(x_1-x_2)^2+(m^2-\frac{1}{4}M^2)(x_1+x_2)+\alpha^2(1-x_1-x_2)]^2}\,. \tag{A4}$$

To compare with the quasifree case, we define

$$\begin{aligned}\tilde{\chi}(0) &= \int \frac{d^4k}{[(\frac{1}{2}K-k)^2-m^2][(\frac{1}{2}K+k)^2-m^2](k^2-\alpha^2)^2}\\ &= i\pi^2\int_0^1 \frac{dx_1dx_2(1-x_1-x_2)\theta(1-x_1-x_2)}{[\frac{1}{4}M^2(x_1-x_2)^2+(m^2-\frac{1}{4}M^2)(x_1+x_2)+\alpha^2(1-x_1-x_2)]^2}\,.\end{aligned} \tag{A5}$$

The result expected in the quasifree case is

$$I_a \simeq \frac{1}{p\cdot K}\tilde{\chi}(0)\,, \tag{A6}$$

$$I_b \simeq \frac{1}{p\cdot K(p\cdot K-2p\cdot p'-p'\cdot K)}\tilde{\chi}(0)\,. \tag{A7}$$

We observe that expressions (A2), (A3), and (A5) look like those of the quasifree case. They would be even closer to (A6) and (A7) if we were to impose $\alpha^2 \ll m^2, M^2$, a condition which favors the small k^2 or the $x_1\simeq x_2 \simeq 0$ contributions in the integrals.

Before doing the calculations with the expression (9) for χ, let us comment on its spin wave function. It may be obtained either from the spin wave function of expression (A1),

$$(\tfrac{1}{2}\not{K}+\not{k}+m)\gamma^5(\tfrac{1}{2}\not{K}-\not{k}-m)\simeq -\not{K}\gamma^5 m\,, \tag{A8}$$

or by noting that in the free-quark case

$$\sum_{\sigma,\sigma'} c(\sigma,\sigma')u(\tfrac{1}{2}K,\sigma)\bar{\sigma}(\tfrac{1}{2}K,\sigma')\approx \gamma^5\tfrac{1}{2}\not{K}\,, \tag{A9}$$

where $c(\sigma,\sigma')$ is the appropriate Clebsch-Gordan coefficient to get a pseudoscalar state. Let us now substitute the wave function (9) in (6a) and (6b). It is easy to verify that the dominant contributions are

$$I'_a = -\frac{i\pi^2}{2\alpha^2}\frac{1}{p\cdot K}\,, \tag{A10}$$

$$I'_b = -\frac{i\pi^2}{2\alpha^2}\frac{1}{(K\cdot p-2p\cdot p'-p'\cdot K)}\frac{1}{p\cdot K}\,, \tag{A11}$$

and that

$$\int \frac{d^4k}{(k^2-\alpha^2+i\epsilon)^3} = -\frac{i\pi^2}{2\alpha^2}\,. \tag{A12}$$

Therefore, we recover the quasifree case and gauge invariance. Let us now study the kinematical region where $K\cdot p - K\cdot p' - 2pp' = Q^2+\hat{t}\simeq 0$. The dominant contribution I'_b has a pole. This is due to the simple form (9) we took for the calculations; a more sophisticated form,

$$G(k,K)\propto \int_{\nu^2}^{\infty}\frac{d\alpha^2\sigma(\alpha^2)}{(k^2-\alpha^2+i\epsilon)^n}\,, \tag{A13}$$

would lead to an enhancement of I'_b instead of a pole; the width of this enhancement is of the order of magnitude of the (mass)^2 which appears in this problem i.e., m^2, M^2, $\int_{\nu_2}^{\infty} d\alpha^2\sigma(\alpha^2)\alpha^2$.

APPENDIX B

In this appendix, we give some details on how the various structure functions appear in convolution (14) and (29). We consider only the case of incident protons and define by $G_1(x)$ the probability of finding an ($\mathfrak{N}\bar{\mathcal{P}}$) system in a proton. We assume that (R is for Regge and D for diffractive)

$$\begin{aligned}G^R_{\mathfrak{N}\bar{\mathcal{P}}} &= G^R_{\mathfrak{N}\bar{\mathfrak{N}}} = G^R_{\mathfrak{N}\bar{\lambda}} = G^R_1\,,\\ G^R_{\mathcal{P}\bar{\mathcal{P}}} &= G^R_{\mathcal{P}\bar{\mathfrak{N}}} = G^R_{\mathcal{P}\bar{\lambda}} = 2G^R_1\,,\end{aligned} \tag{B1}$$

and $G^D_{q\bar{q}} = G^D_1$, whatever q and $\bar{q}$ may be. The probabilities of creating a π^0 through graphs of Figs. 3(a) and 3(b) are then

$$[(G^R_{\mathcal{P}\bar{\mathcal{P}}} + G^R_{\mathcal{N}\bar{\mathcal{P}}})G_{\mathcal{P}/p} + (G^R_{\mathcal{P}\bar{\mathcal{N}}} + G^R_{\mathcal{N}\bar{\mathcal{N}}})G_{\mathcal{N}/p} + (G^R_{\mathcal{P}\bar{\lambda}} + G^R_{\mathcal{N}\lambda})G_{\lambda/p}]\left(\frac{1}{\sqrt{2}}\right)^2 \tag{B2}$$

and

$$[(G^D_{\mathcal{P}\bar{\mathcal{P}}} + G^D_{\mathcal{N}\bar{\mathcal{P}}})G_{\mathcal{P}/p} + (G^D_{\mathcal{P}\bar{\mathcal{N}}} + G^D_{\mathcal{N}\bar{\mathcal{N}}})G_{\mathcal{N}/p} + (G^D_{\mathcal{P}\bar{\lambda}} + G^D_{\mathcal{N}\bar{\lambda}})G_{\lambda/p}]\left(\frac{1}{\sqrt{2}}\right)^2 . \tag{B3}$$

The $\sqrt{2}$ comes from the π^0 wave function; the sum of (B2) and (B3) leads to

$$\tfrac{3}{2}(G^R_1 + \tfrac{2}{3}G^D_1)(G_{\mathcal{P}/p} + G_{\mathcal{N}/p} + G_{\lambda/p}) . \tag{B4}$$

For the production of a massive photon, we have the probabilities

$$\tfrac{4}{9}(G^R_{\mathcal{P}\bar{\mathcal{P}}}G_{\mathcal{P}/p} + G^R_{\mathcal{P}\bar{\mathcal{N}}}G_{\mathcal{N}/p} + G^R_{\mathcal{P}\bar{\lambda}}G_{\lambda/p}) + \tfrac{1}{9}(G^R_{\mathcal{N}\bar{\mathcal{P}}}G_{\mathcal{P}/p} + G^R_{\mathcal{N}\bar{\mathcal{N}}}G_{\mathcal{N}/p} + G^R_{\mathcal{N}\bar{\lambda}}G_{\lambda/p}) = G^R_1(G_{\mathcal{P}/p} + G_{\mathcal{N}/p} + G_{\lambda/p}) . \tag{B5}$$

An analogous calculation for G^D_1 leads to the final result

$$(G^R_1 + G^D_1)(G_{\mathcal{P}/p} + G_{\mathcal{N}/p} + G_{\lambda/p}) . \tag{B6}$$

We have to multiply by 2 to take into account the configurations where the $q\bar{q}$ system is emitted by the projectile and by the target.

APPENDIX C

The starting point for the calculation of the subprocess cross section is the expression

$$E_+\frac{d\sigma^s}{d\vec{K}_+} = \frac{\alpha^2}{\hat{s}(2\pi)^3}\int \frac{d\vec{K}_-}{2E_-}\frac{d\vec{p}\,'}{2E'}\delta^4(p+K-K_+-K_--p')\frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}, \tag{C1}$$

which may be rewritten

$$E_+\frac{d\sigma^s}{d\vec{K}_+} = \frac{\alpha^2}{\hat{s}(2\pi)^3}\int d^4K'\frac{d\vec{K}_-}{2E_-}\delta^4(K'-K_+-K_-)\delta(\hat{s}+\hat{t}+\hat{u}-K'^2)\frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}$$

$$= \frac{\alpha^2}{\hat{s}(2\pi)^3}\int d^4K'\delta((K'-K_+)^2 - m_l{}^2)\delta(\hat{s}+\hat{t}+\hat{u}-K'^2)\frac{T^{\mu\nu}t_{\mu\nu}}{K'^4}. \tag{C2}$$

We have neglected all hadronic and quark masses; m_l is the leptonic mass. We calculate (C2) in the center of mass of the subsystem, and the integrals over the δ function become

$$\int dQ^2\delta(K'^2-Q^2)\int d^4K'\delta(Q^2-2K'\cdot K_+)\delta(\hat{s}+Q^2-2\sqrt{\hat{s}}K'^0)$$

$$= \frac{1}{8\hat{s}}\int dQ^2(\hat{s}-Q^2)\int d\Omega_{K'}\delta\left(Q^2 - \left(\frac{\hat{s}+Q^2}{\sqrt{\hat{s}}}\right)K^0_+ + \left(\frac{\hat{s}-Q^2}{\sqrt{\hat{s}}}\right)|\vec{K}_+|\cos\theta'\right)$$

$$= \frac{1\sqrt{\hat{s}}}{8\hat{s}|\vec{K}_+|}\int dQ^2\int_0^{2\pi} d\phi'\,\theta((\hat{s}-Q^2)|\vec{K}_+| - |\sqrt{\hat{s}}Q^2 - (\hat{s}+Q^2)K^0_+|) . \tag{C3}$$

The angles θ' and ϕ' are defined in Fig. 7. By symmetry, we get $\int_0^{2\pi}d\phi' = 2\int_{-\pi/2}^{+\pi/2}d\phi'$. The tensor product $T^{\mu\nu}t_{\mu\nu}$ depends on the variables (see Fig. 1) $S=\hat{s}$, $T=(K-K_+)^2$, $U=(p-K_+)^2$, and $\hat{u}=(p-K')^2$; the variable $\hat{u}$ may be rewritten with the angular variables defined in Fig. 7:

$$\hat{u} = \frac{Q^2-\hat{s}}{2}(1-\cos\theta'\cos\theta_z) + \frac{\hat{s}-Q^2}{2}\sin\theta'\sin\theta_z\sin\phi' . \tag{C4}$$

The expression of the tensor product is

$$T^{\mu\nu}t_{\mu\nu}=\frac{8\,|2\tilde{G}(0)|^2}{\hat{s}^2(2Q^2-\hat{s}-\hat{u})^2}F(S,T,U,\hat{u})\,,$$

$$\begin{aligned}F(S,T,U,\hat{u})=&\hat{u}^3(\tfrac{1}{2}Q^2+m_l^2)+\hat{u}^2[-\tfrac{3}{2}Q^4+Q^2(T+\tfrac{1}{2}S-5m_l^2)+(T+U)^2-Sm_l^2]\\&+\hat{u}[Q^6+Q^4(2U-3T-S+8m_l^2)+Q^2(-3T^2+US-4TU-TS+\tfrac{1}{2}S^2+2Sm_l^2)+2US(T+U)-S^2m_l^2]\\&+Q^6(-4U+2T+S-4m_l^2)+Q^4(-4U^2+2T^2+3TS+\tfrac{3}{2}S^2)\\&+Q^2[S^2U+ST(T+S)+\tfrac{1}{2}S^3+3S^2m_l^2]+S^2U^2+m_l^2S^3\,.\end{aligned}\qquad\text{(C5)}$$

To study the limit $Q^2\to 4m_l^2$, it is necessary to replace in (C5) T and U by $(T-m_l^2)$ and $(U-m_l^2)$ [these m_l^2 have been neglected in (C5)] and to notice that, in this limit, $\hat{u}\to 2U+2m^2$ and $S\to -2(T+U)$. Then we can verify that

$$T^{\mu\nu}t_{\mu\nu}\xrightarrow[Q^2\to 4m_l^2]{}-2Q^2T^{\mu}_{\ \mu}\,,\qquad\text{(C6)}$$

as expected from the colinearity of the leptons [$T^{\mu}_{\mu}=\tilde{T}^{\mu}_{\mu}/\hat{s}$ is given in (27)]. For Q^2 close to $4m_l^2$, we expand for fixed S, T, and U, $T^{\mu\nu}t_{\mu\nu}$ in a power series of m_l^2 and Q^2. A cancellation occurs among the terms proportional to $(m_l^2)^0$ and among those proportional to $(Q^2)^0$, leading to the behavior

$$\begin{aligned}T^{\mu\nu}t_{\mu\nu}\propto\ & m_l^2 a(S,T,U,\sin\phi')\\&+Q^2 b(S,T,U,\sin\phi')\,.\end{aligned}\qquad\text{(C7)}$$

The cross section may finally be written

$$E_+\frac{d\sigma^{(s)}}{d\vec{K}_+}=\frac{\alpha^2}{(2\pi)^3}\frac{1}{4\hat{s}}\frac{\sqrt{\hat{s}}}{|\vec{K}_+|}\times\int_{\hat{\tau}_{\min}}^{\hat{\tau}_{\max}}\frac{d\hat{\tau}}{\tau^2}\int_{-\pi/2}^{\pi/2}d\phi'\,T^{\mu\nu}t_{\mu\nu}\,,\qquad\text{(C8)}$$

where $T^{\mu\nu}t_{\mu\nu}$ is given by (C5) and $\hat{\tau}=Q^2/\hat{s}$. Let us notice that the tensor product, except for the factor $|2G(\tilde{0})|^2$, is dimensionless. If we have, in convolution (33), $K=x_1P$ and $p=x_2\tilde{P}$ with $P(P,0,0,P)$ and $\tilde{P}(P,0,0,-P)$, S, T, and U become

$$S=x_1x_2s\,,\quad T=-\tfrac{1}{2}x_\perp x_1\,,\quad U=-\tfrac{1}{2}x_\perp x_2\qquad\text{(C9)}$$

for a lepton detected at 90° with $x_\perp=2|\vec{K}_+|/\sqrt{s}$ and $s=4P^2$.

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