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Hadronic production of the ψ/J meson

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It is proposed that the production of ψ/J mesons in hadronic collisions occurs via processes involving gluons, and that reactions with ^3P charmonium intermediate states play a significant role. The clarity of the observed ψ/J signal and the differences between production rates of ψ/J in pion- and nucleon-induced reactions are explained naturally. We calculate the total and differential cross section for producing ψ/J + anything, and compare to data. Also, we have calculated the decay rates of the ^3P states into two gluons; these results may be interesting in their own right.

I. INTRODUCTION

In this paper we try to understand the main features of the hadronic production of ψ/J particles. This investigation was stimulated by the realization that "conventional schemes"[1] do not explain these processes.[2-6] To orient the reader, we should point out the several unusual features of ψ/J hadronic production.

(a) The observed signal is surprisingly clean. The experiments that currently have the best mass resolution (BNL-MIT,[2] ~20 MeV) show a singnal-to-background ratio of about 200/1 for hadronic ψ/J production followed by its decay into $\mu^+\mu^-$. Were we to use the e^+e^- colliding-beam results[7] as a guide, we would expect this ratio to be roughly $\frac{1}{2}$ at the given mass resolution.

(b) Pions seem to be 5–7 times more effective than nucleons[3] for producing ψ/J particles with large momentum fraction x. The total cross sections, however, are comparable because the bulk of the cross section is at small x.

(c) The transverse-momentum distribution of the ψ/J is much flatter[4] than what is typical of other hadronic reactions. One observes $\langle P_\perp\rangle_\psi \approx 0.7$ to 1.0 GeV, compared to the usual $\langle P_\perp\rangle \approx 0.35$ GeV.

There are two key ideas in our model. The first is that the ψ/J particles are produced from the gluon component of the hadron's wave function. This follows Einhorn and Ellis.[8] Considerations that will be given below then lead us to propose the second idea: that the ψ/J is not produced directly, but rather *the ψ/J is produced via ^3P intermediate states*, which decay into a ψ/J and a photon.

Also, we have had to calculate the couplings of two gluons to each of the $^3P_J(J=0,1,2)$ states. This is conveniently expressed as the width of the $^3P \to 2g$ decay, and these results, which are interesting in their own right, are given in Eq. (15) below.

In Sec. II we explain our model, in Sec. III we discuss the decays of the ^3P states into ψ/J and γ and into two gluons, and in Sec. IV we compare our results to the data. Some concluding remarks are made in Sec. V.

II. THE MODEL

In all our considerations we assume that the ψ/J particles are bound states of quarks of a new flavor. This is convincingly supported by the observed psion spectroscopy.[9-11] We will also assume, in the standard fashion,[9] that each quark comes in three colors, but physical hadrons are color singlets. The color group is $SU(3)_c$.

The interaction between quarks is mediated by 8 massless vector gluons. The Lagrangian of strong interactions is a non-Abelian Yang-Mills Lagrangian, which exhibits asymptotic freedom.[12] Because of this last property, when the momentum scale or mass scale becomes large, the effective coupling constant becomes small enough so that rates calculated using free-field behavior and lowest-order perturbation theory are meaningful.

The deep-inelastic electron-nucleon scattering data requires that about $\frac{1}{2}$ of the momentum of the incoming hadron be carried by neutral constituents.[13] We identify these neutral constituents with the gluons of the field theory.

We pursue the idea that gluon-gluon interaction[8] is a natural candidate for producing ψ/J-like particles. Quantum-number ($J^{PC}=1^{--}$) considerations require at least 3 gluons in order to produce a ψ/J directly. However, the arguments based on asymptotic freedom suggest that the effective coupling constant is small and therefore direct production of ψ/J is unlikely. This argument is in agreement with the narrow experimental hadronic width of the ψ/J ($\sim 59 \pm 14$ keV).[14]

A much more likely process is the production of a heavier intermediate state[15] that can be produced by two gluons and which decays with a reasonable width into ψ/J's. Possible intermediate states for this process include the ^3P $c\bar{c}$ bound states. The recently found[16] $C=+$ states around $M \sim 3.5$ GeV could be identified with the ^3P states.

In Fig. 1 we display the relevant process for P-wave production and subsequent decay into $\psi/J + \gamma$. (Observe that if the P states have a mass of 3.5 GeV they cannot decay hadronically into ψ/J. One-π decay is forbidden by isospin, two-π decay by G parity, and everything else by phase space.) Reactions like the one shown in Fig. 1 were considered by Einhorn and Ellis,[8] who considered its application to the production of η_c ($J^{PC}=0^{-+}$).

The process diagramed in Fig. 1 is an extension of the Drell-Yan mechanism. In order to do the calculation we need detailed knowledge of the gluon probability distributions. We will assume that

$$f_g(x) = C_n \frac{1}{x}(1-x)^n , \tag{1}$$

where $f_g(x)$ is the momentum probability distribution and is the same for each type of gluon. Since the gluons carry half the momentum and there are eight types of gluons, we have

$$\int_0^1 x f_g(x) dy = \tfrac{1}{16} \tag{2}$$

and $C_n = \frac{1}{16}(n+1)$. The counting-rule arguments[17] suggest that n is twice the minimum number of particles left behind if one gluon is removed, minus 1, so that $n=5$ for nucleons and $n=3$ for pions is preferred.

The differential cross section in longitudinal momentum is

$$\frac{d\sigma}{dx_{\text{c.m.}}} = \frac{8\pi^2}{M_P{}^3} \frac{\Gamma(^3P_J \to 2g)\Gamma(^3P_j \to \psi/J+\gamma)}{\Gamma_{\text{total}}(^3P_J)} \tau \times \frac{M_P{}^2}{M_P{}^2 - M_\psi{}^2} \int_{x_L}^{x_U} \frac{dx}{x^2+\tau} f_g(x) f_g\left(\frac{\tau}{x}\right) . \tag{3}$$

The integration variable x is the momentum fraction of the gluon from the projectile; the first f_g above comes from the projectile, the second from the target. If $p_\parallel$ is the longitudinal momentum of the ψ/J in the overall c.m., then $x_{\text{c.m.}} = 2p_\parallel/\sqrt{s}$. A sum over $J=0$, 1, 2 for the P states is understood with a $(2J+1)$ weighting factor, $\tau = M_P{}^2/s$, and

$$x_L = \frac{M_\psi{}^2}{M_P{}^2} x_U = \tfrac{1}{2}\left[(x^2 + 4M_\psi{}^2/s)^{1/2} + x\right] . \tag{4}$$

III. CALCULATIONS OF THE WIDTHS

In this section we determine the various widths that we need.

The calculation of the annihilation rate of $c\bar{c}$ into two gluons is more delicate for the ^3P_J states than for the $^1S_0$ states because the P-state wave functions vanish at the origin. To obtain the leading nonvanishing result, one must expand the amplitude for $c\bar{c} \to 2g$ to first order in p/m, where m is the mass of the charmed quark. We shall also make an approximation by neglecting effects due to the fact that the quarks are off the mass shell.

The well-known Feynman diagrams are drawn in Fig. 2, and we write the amplitudes as[18]

$$A_{rs} = -i\bar{v}_r(p_+)\mathcal{Q}u_s(p_-) \tag{5}$$

and

$$\mathcal{Q} = \not{\epsilon}_2 \frac{\not{p}_- - \not{k}_1 + m}{-2p_-\cdot k_1} \not{\epsilon}_1 + \not{\epsilon}_1 \frac{\not{p}_- - \not{k}_2 + m}{-2p_-\cdot k_2} \not{\epsilon}_2 . \tag{6}$$

To first order in $p = |\vec{p}\,|$ we can write the momenta as

$$p_- = (m, \vec{p}) , \quad k_1 = (m, \vec{k}) , \qquad p_+ = (m, -\vec{p}) , \quad k_2 = (m, -\vec{k}) . \tag{7}$$

Then we expand $\mathcal{Q}$ to first order obtaining,

$$\mathcal{Q} = \frac{1}{m^2}\left[m\vec{\epsilon}_1\cdot\vec{\epsilon}_2 + i\gamma_0\gamma_5\vec{k}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_2)\right] - \frac{\vec{p}\cdot\vec{k}}{m^4}\left[im\vec{\sigma}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_1) + \vec{\gamma}\cdot\vec{k}\vec{\epsilon}_1\cdot\vec{\epsilon}_2\right] - \frac{1}{m^2}\left[\vec{\gamma}\cdot\vec{\epsilon}_2\vec{p}\cdot\vec{\epsilon}_1 + \vec{\gamma}\cdot\epsilon_1\vec{p}\cdot\vec{\epsilon}_2 - \vec{\epsilon}_1\cdot\vec{\epsilon}_2\vec{\gamma}\cdot\vec{p}\right] . \tag{8}$$

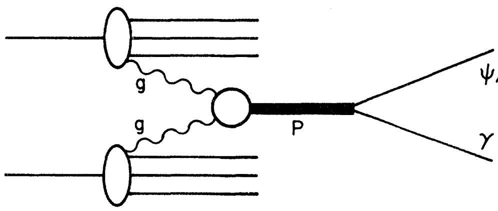

FIG. 1. Process of producing P states from two gluons. The P state subsequently decays into ψ/J plus γ.

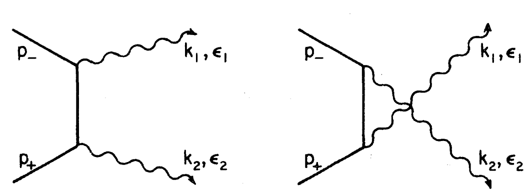

FIG. 2. Feynman diagrams for fermion-antifermion annihilation into two gluons (or two photons).

The first of the above three terms has no p dependence, and it alone is needed to obtain an accurate result for $^1S_0$ annihilation.[19] It also contributes to the ^3P_J case, as the spinors will give terms of $O(p/m)$. If we reduce the result in terms of two-spinors $\chi_\pm$, keeping only terms of $O(p/m)$, we have

$$A_{rs} = -\frac{2i}{m}\chi_r^T(i\sigma_2) \times\left(\frac{\vec{p}\cdot\vec{k}}{m^2}\vec{\epsilon}_1\cdot\vec{\epsilon}_2\vec{\sigma}\cdot\vec{k}+\vec{p}\cdot\vec{\epsilon}_1\vec{\sigma}\cdot\vec{\epsilon}_2+\vec{p}\cdot\vec{\epsilon}_2\vec{\sigma}\cdot\vec{\epsilon}_1\right)\chi_s . \tag{9}$$

The matrix elements for annihilation from a ^3P_J bound state may be written out in terms of the above A_{rs}. Since the width for a given angular momentum and projection (J,M) is independent of M, we shall take $M=0$. The three required matrix elements may then be given in terms of $\mathfrak{M}_{\pm,0}$, where

$$\mathfrak{M}_+ = \mathfrak{M}_- = \int\frac{d^3p}{(2\pi)^3}\phi(p)Y_{1,-1}(\hat{p})A_{++}$$

and (10)

$$\mathfrak{M}_0 = \int\frac{d^3p}{(2\pi)^3}\phi(p)Y_{10}(\hat{p})\frac{1}{\sqrt{2}}(A_{-+}+A_{+-}) .$$

The momentum-space P-state wave function above is related to the radial wave function $\phi(r)$ [normalized as $\int|\phi(r)|^2r^2dr=1$] by

$$\phi(p) = 4\pi\int_0^\infty r^2dr\, j_1(pr)\phi(r) , \tag{11}$$

and, of particular utility here,

$$\int_0^\infty p^3dp\,\phi(p) = 6\pi^2\frac{d\phi}{dr}(0) . \tag{12}$$

The matrix elements for the ^3P_J states are then

$$\begin{aligned}
\mathfrak{M}(^3P_0) &= \frac{1}{\sqrt{3}}(\mathfrak{M}_+ - \mathfrak{M}_0 + \mathfrak{M}_-)\\
&= -\frac{3i}{m}\left(\frac{2}{\pi}\right)^{1/2}\vec{\epsilon}_1\cdot\vec{\epsilon}_2\frac{d\phi}{dr}(0) ,\\
\mathfrak{M}(^3P_1) &= \frac{1}{\sqrt{2}}(\mathfrak{M}_+ - \mathfrak{M}_-)\\
&= 0 ,\\
\mathfrak{M}(^3P_2) &= \frac{1}{\sqrt{6}}(\mathfrak{M}_+ + 2\mathfrak{M}_0 + \mathfrak{M}_-)\\
&= -\frac{3i}{m\sqrt{\pi}}[\vec{\epsilon}_1\cdot\vec{\epsilon}_2(1-\hat{k}_z^2) - 2\epsilon_{1z}\epsilon_{2z}]\frac{d\phi}{dr}(0) .
\end{aligned} \tag{13}$$

Substituting into

$$\Gamma(^3P_J) = \left(\frac{2}{3}\right)\frac{\alpha_g^2}{16m^2}\int d\Omega_\gamma\sum_{\text{pol}}|\mathfrak{M}(^3P_J)|^2 , \tag{14}$$

we at last obtain

$$\begin{aligned}
\Gamma(^3P_0) &= \left(\frac{2}{3}\right)9\frac{\alpha_g^2}{m^4}\left|\frac{d\phi}{dr}(0)\right|^2 ,\\
\Gamma(^3P_1) &= 0 ,\\
\Gamma(^3P_2) &= \left(\frac{2}{3}\right)\frac{12}{5}\frac{\alpha_g^2}{m^4}\left|\frac{d\phi}{dr}(0)\right|^2 .
\end{aligned} \tag{15}$$

The factor of $\frac{2}{3}$ in each of the above equations is due to color, which had been neglected until now. The same formulas are applicable to the annihilation of positronium from a ^3P_J state (provided, of course, that the $\frac{2}{3}$ is dropped, $\alpha_g \to \alpha = \frac{1}{137}$, and $m \to$ electron mass).

The wave functions can by now be gotten from a number of sources. We have chosen to work with analytic wave functions obtained by assuming gaussian form (times r^L, where L is the orbital angular momentum) for the $2\,^3P$ and $1\,^3S$ states, and determined the parameters by a variational calculation using a linear potential.[10,20] The linear potential is $V(r) = r/a^2 - V_0$, where $a = 1.94$ GeV^{-1} and $V_0 = 1.37$ GeV, and the mass of the new quark is $m = 1.84$ GeV.[21] These constants give the observed masses of the ψ/J (3095) and the ψ'(3684), as well as $M_P = 3.44$ GeV.

Explicitly, the $2P$ and $1S$ (which we shall need below) radial wave functions are

$$\phi_{1S}(r) = 2\left(\frac{\beta_S^3}{\pi}\right)^{1/4}e^{-\beta_S r^2/2}$$

and (16)

$$\phi_{2P}(r) = 2\left(\frac{4}{9}\frac{\beta_P^5}{\pi}\right)^{1/4}re^{-\beta_P r^2/2} ,$$

with $\beta_S = 0.322$ GeV^2 and $\beta_P = 0.277$ GeV^2. This gives $2\,^3P_J$ widths as

$$\begin{aligned}
\Gamma(^3P_0 \to 2g) &= 1.29 \text{ MeV} ,\\
\Gamma(^3P_2 \to 2g) &= 0.34 \text{ MeV} .
\end{aligned} \tag{17}$$

Incidentally, a similar calculation for the η_c $(1\,^1S_0) \to 2g$ gives $\Gamma(\eta_c \to 2g) = 3.26$ MeV, so that the decays of the P states are not so much suppressed relative to the S states as one might guess.

We also require the rate for $P \to \psi/J + \gamma$.[10] This is a standard $E1$ radiative transition calculation, and if we use our wave functions we get

$$\Gamma(^3P_J \to \psi/J + \gamma) = 270 \text{ keV}$$

independent of $J = 0, 1, 2$, if the charge of the new quark is $\frac{2}{3}e$. Given these results, the width factor $\sum_J(2J+1)\Gamma(^3P_J \to 2g)\Gamma(^3P_J \to \psi/J+\gamma)/\Gamma_{\text{total}}(^3P_J)$ becomes 0.98 MeV.

After completing our work, we learned that the two-gluon results had also been obtained by Barbieri, Gatto, and Kögerler.[22] Their formulas agree with our Eqs. (15)[23]; our numerical values disagree somewhat because of the differences in the wave functions we have worked with.

IV. COMPARISON TO DATA

We may now calculate $d\sigma/dx_{\text{c.m.}}$ and σ_{total}. The total cross section as a function of p_{lab} or s is plotted in Fig. 3. We have chosen to concentrate on $n = 3$ and 5 for the proton and $n = 1$ and 3 for the pion. The curves seem to be too low, typically by a factor of 4, although direct comparison of σ_{total} to experiment is model dependent since no experiment measures $d\sigma/dx$ for all x.

A factor of ~4 is not at present a major difficulty, since the errors in the experiments may be as large as a factor of 2, and further differences could well be due to uncertainties in the $P \to \psi/J + \gamma$ rates or in the value of the coupling constant. Also, it may mean that our process is significant but not dominant. We will comment on the normalization after we have examined $d\sigma/dx$ and chosen best values for n.

Plots of $d\sigma/dx_{\text{c.m.}}$ for $s = 280$ GeV^2 ($p_{\text{lab}} \approx 150$ GeV) are given in Fig. 4, superimposed upon the data of the Chicago-Princeton group.[5] We are interested in checking the shape of the spectrum and have adjusted our results to match the data at small x. A similar plot is given in Fig. 5 for the Northeastern data[3] ($s = 380$ GeV^2 for π and 430 GeV^2 for p). The curves seem to fall too quickly for $n_p = 5$ and $n_\pi = 3$. The agreement is better if we choose $n_p = 3$ and $n_\pi = 1$, evidence that, surprisingly enough, the gluon probability functions have the same dependence on momentum fraction as the valence quarks.[24]

Our normalizations, as mentioned, are low. We shall compare to several experiments, assuming that the $\psi/J \to \mu\bar{\mu}$ branching ratio is 0.069.[7] The experiments cover different dissimilar ranges of x, so we organize our comparisons as the following: (i) The CERN-ISR experiment of Büsser *et al.*[6]

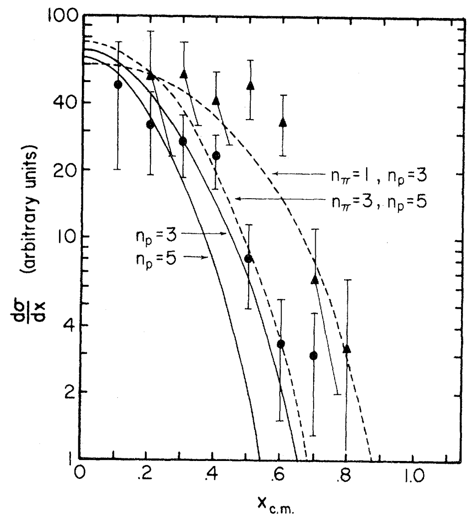

FIG. 4. $d\sigma/dx_{\text{c.m.}}$ for pions and protons at $s = 280$ GeV^2. The data are from Ref. 5; the circles are for $p + N \to \psi/J$ +anything and the triangles for $\pi + N \to \psi/J$ +anything. The shape and relative normalization of the four curves are taken from our calculation, but the overall normalization is adjusted to the data.

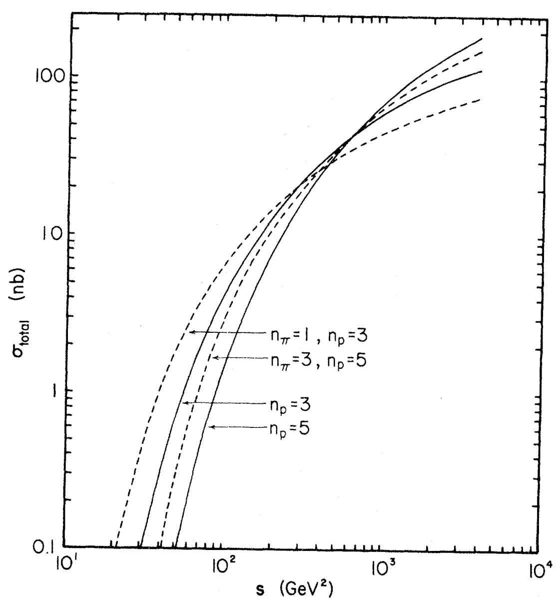

FIG. 3. Total cross sections for $p + p \to \psi/J$ +anything (solid curves) and $\pi + p \to \psi/J$ +anything (dashed curves).

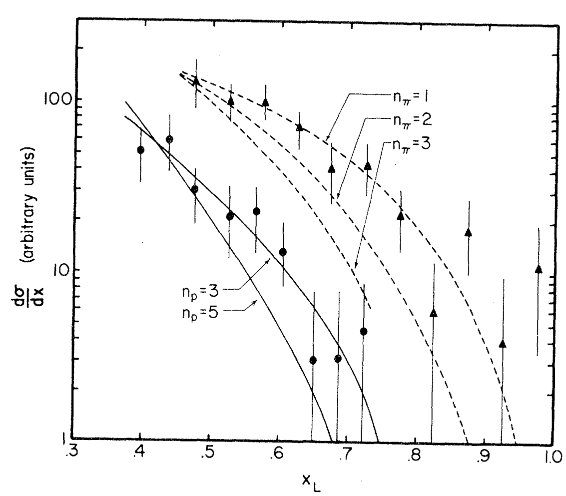

FIG. 5. $d\sigma/dx_L$ for pions at $s = 280$ GeV^2 and protons at $s = 430$ GeV^2. The data are from Ref. 3. The pion and proton curves are separately normalized at the intersection points. For large x, the shape of the pion curves is insensitive to n_p.

measures $d\sigma/dy$ (y = rapidity = $\ln\{[p_{\parallel}+(p_{\parallel}^2+M_\psi^2)^{1/2}]/M_\psi\}$) for $|y|\leq 0.32$. Their result can be given as $\sigma(|y|\leq 0.32)=69\pm 23$ nb. We obtain for $\sqrt{s}=50$ GeV, $\sigma(|y|\leq 0.32)=17$ nb for $n=3$ and 28 nb for $n=5$. [Our result is only slightly dependent on s for the s range of their experiment, because $d\sigma/dx$ ($x=0$) increases with s but the range of x decreases.] (ii) The Fermilab experiment of Knapp *et al.*[4] measured $\sigma(|x_L|\geq 0.24)=52$ nb at $s=470$ GeV^2. [They have doubled their number for $\sigma(x_L \geq 0.24)$.] We get $\sigma(|x_L|\geq 0.24)=14$ nb for $n=3$ and 10 nb for $n=5$. (iii) The Fermilab experiment of Anderson *et al.*[5] measures $\sigma(x_{\text{c.m.}}\geq 0.05)=45\pm 23$ nb for the proton and $\sigma(x_{\text{c.m.}}\geq 0.05)=74\pm 32$ nb for the pion. For the same $x_{\text{c.m.}}$ range, we would have $\sigma_p=7.5$ nb and $\sigma_\pi=11.1$ nb for $n_p=3$, $n_\pi=1$, and $\sigma_p=5.4$ nb and $\sigma_\pi=8.7$ nb for $n_p=5$, $n_\pi=3$. In the various experiments, our calculation, including just the 2^3P intermediate states, is low by a factor of $\frac{5}{2}$ to $\frac{17}{2}$.

It is to be emphasized that the difference between pion- and proton-induced ψ/J production at high $x_{\text{c.m.}}$ are due to differences, which are expected, between the pion and proton wave functions. This would be true even if the ψ/J production came from the quarks rather than the gluons. In particular, the pion proton data *are not* evidence that a quark-antiquark interaction is stronger than a quark-quark interaction.

In the remainder of this section we will discuss the transverse-momentum distribution of ψ/J particles. As pointed out in the Introduction, the mean transverse momentum of the produced ψ/J particles is large. It is encouragining to observe that the production mechanism here described shows a similar trend. To give a rough estimate of the effect, let us consider, for simplicity, the case where the gluon component of the wave function has a Gaussian distribution in transverse momentum that we take to be $\exp(-6\ \text{GeV}^{-2}k_\perp^2)$ [such a distribution gives the same $\langle k_\perp^2\rangle$ as the conventional $\exp(-6\ \text{GeV}^{-1}k_\perp)$].

The differential cross section for the subprocess $gg\to\psi/J+\gamma$ is the gg c.m. frame is given by

$$\frac{d\sigma}{dp_\perp^2}=\frac{\sigma(gg\to\psi/J+\gamma)}{2p_0^2}\left(1-\frac{p_\perp^2}{p_0^2}\right)^{-1/2}, \tag{18}$$

where

$$p_0=\frac{M_P^2-M_\psi^2}{2M_P}. \tag{19}$$

After folding this elementary cross section with the gluon transverse momentum distributions, one obtains

$$\frac{d\sigma}{dP_\perp^2}\propto e^{-3P_\perp^2}\int_0^{p_0}dp_\perp^2 e^{-3p_\perp^2}I_0(6P_\perp p_\perp)\left(1-\frac{p_\perp^2}{p_0^2}\right)^{-1/2}, \tag{20}$$

where I_0 is the modified Bessel function, units are GeV, and $P_\perp$ is the observed transverse momentum of the ψ/J. The resulting $\langle P_\perp^2\rangle^{1/2}$ is 0.64 GeV. It should be clear that the Gaussian wave function, if anything, tends to underestimate the contribution from large $k_\perp$. If, for example, one follows the wisdom of the constituent-interchange model (cf. Chu and Gunion, Ref. 24), a natural selection for the gluon probability distribution becomes

$$\frac{df(x,k_\perp^2)}{dk_\perp^2}\propto\frac{(1-x)^{2p-1}}{(k_\perp^2+\sigma^2)^{2p}},$$

with the best values $2p-1=3$ for the proton case and $=1$ for the pion case, and σ^2 is an arbitrary parameter expected to be about 1 GeV^2. With this distribution the transverse momentum can be better accomodated.

Given, in addition, the fact that the particular channel considered does not saturate the ψ/J cross section, we expect sizable contributions from the next excited P state: the 3^3P near 3.9 GeV. For purely kinematic reasons [see Eq. (20)] the transverse momentum of the ψ/J will be larger in the decay $3\,^3P\to\psi/J+\gamma$ than for the $2\,^3P$. The competitive channel $3\,^3P\to\psi/J+\omega$ can be calculated using our model and will have a $\langle P_\perp^2\rangle$ almost identical to the $2\,^3P\to\psi/J+\gamma$. In conclusion, the $P_\perp$ dependence of the hadronic ψ/J production can be understood in the model where the ψ/J are produced via P-wave intermediate states which subsequently undergo a two-body decay.

V. CONCLUDING REMARKS

We thus have a viable model in which the ψ/J is produced by decays of the ^3P states, which were produced by two gluons from the hadron wave functions. It is unlikely that ψ/J could be produced directly because of the small coupling constant and the need to involve three gluons.

Clearly, if our process is correct, the photons must also be present in the final state. The momentum the photon receives from the decay of the P state is the same as the received by the ψ/J and the transverse-momentum distributions must be the same. Also the momenta must satisfy $(P_\psi+P_\gamma)^2=M_P^2$. It is worth pointing out that we have not made a startling numer of numerical assumptions. In particular, the strong coupling constant α_g does not enter our calculation directly, and the neutral partons (gluons) must certainly be in the wave function. The decay rate $\Gamma(P\to\psi/J+\gamma)$ still needs to be measured. The experimental decay rate[14] for the ψ' into $P\gamma$, it is true, is smaller than calculated,[10] but some explanations have been offered,[25] and this need not have any bearing on the decay P into $\psi/J+\gamma$. We do believe that our calculated cross sections can be taken seriously, so that even if we do not have a complete explanation of ψ/J production, nonetheless, there should be photons present an appreciable fraction of the time. Also, we considered only the $2\,^3P$ states; there are also $3\,^3P$ (~3900 MeV), states that lead to ψ's in the final state and also have the possibility of decaying into ψ + hadrons. A particularly attractive channel to consider is the hadronic decay $3\,^3P \to \psi/J+\omega$.

Some comments on other possible processes are in order. Two ψ/J's or a ψ/J and charmed particles could be produced using only two gluons, but this reaction will probably have a small rate because of its involving two ψ/J wave functions and because of threshold effects. Rates for processes involving directly the quarks in the hadron wave function are, we think, smaller than those involving gluons. The usual Drell-Yan process[26] (with a photon intermediate state) is definitely too small. There is a variation of the Drell-Yan process wherein c and $\bar{c}$ quarks from the core of the hadrons combine[27] to form a ψ/J. However, its success would depend on using a value of $g_{\psi c\bar{c}}$ that is rather large, when one considers the idea of asymptotic freedom, and also the c and $\bar{c}$ parts of the hadron wave function may be substantially smaller than that suggested by SU(4) symmetry. Two other problems with this variation of the Drell-Yan model are also noteworthy: the $P_\perp$ distribution is too narrow and the expected x distribution for the ψ/J should be like $(1-x)^7$ for the proton and $(1-x)^5$ for the pion, if we use the Brodsky-Farrar rules.[17]

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