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$\bar{K}^{*0}$(890) decay density matrix elements and partial conservation of strangeness-changing vector current

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We show that the zeros in the density matrix elements $\rho^t_{1,-1}$ and $\mathrm{Re}\rho^s_{10}$ for $K^-p \to \bar{K}^{*0}n$ ($K^+n \to K^{*0}p$) are correlated and imply fundamental relationships among the Ball amplitudes following from partial conservation of strangeness-changing vector currents and the smoothness hypothesis.

In a recent paper[1] we considered ρ^0-meson decay density matrix elements in the process $\pi^-p \to \rho^0 n$ and found that two of these, $\rho^t_{1,-1}$ (Gottfried-Jackson frame) and Re ρ^s_{10} (s-channel helicity frame), had remarkable energy-independent features. $\rho^t_{1,-1}$ is consistent with being zero from $|t|=0$ to at least up to $8m_\pi{}^2$ for incident pion lab momenta of 2.7 GeV/c to 17 GeV/c. $\mathrm{Re}\rho^s_{10}$ goes through zero at $t \approx -m_\pi{}^2$ over the same energy range. Although the zero of $\mathrm{Re}\rho^s_{10}$ can be obtained in a gauge-invariant (electric) Born model[2,3] or absorption models (especially the Williams model[4]), we found that these zeros imply specific, to some extent, model-independent relations among the invariant Ball amplitudes for the process. $\rho^t_{1,-1}=0$ together with the smoothness relations of Cho and Sakurai[2,3] and Achasov and Shestakov[5] led to the zero of $\mathrm{Re}\rho^s_{10}$ at $t\approx -m_\pi{}^2$ in a natural way without using any specific exchange model.

Recently high-statistics data for the corresponding strange-particle reactions $K^-p \to \bar{K}^{*0}(890)n$ and $K^+n \to K^{*0}(890)p$ became available.[6-8] We do not exhibit the data here since they have been already published or will be published in the near future. Also $\rho^t_{1,-1}$ and $\mathrm{Re}\rho^s_{10}$, with which we will be mainly concerned in this paper, have almost identical features to the ρ^0 data presented in I. One again has $\rho^t_{1,-1}\approx 0$ for a large range of energy and extended range of momentum transfer, and $\mathrm{Re}\rho^s_{10}$ passes through zero at $t\approx -m_\pi{}^2$. If we pursue an analysis similar to I, interestingly enough, the zero is found near $t\approx -m_K{}^2$ (K-meson mass)^2 and not $t\approx -m_\pi{}^2$ since in this analysis only the external particle masses enter. The shift in the zero from $-m_K{}^2$ to $-m_\pi{}^2$ is explained here in a natural way by invoking partial conservation of strangeness-changing vector currents (PCVC).[9,10] PCVC has been used extensively in K_{l3}-decay analysis.[11,12] It is very interesting to find its confirmation in purely strong processes such as the present one.

Since essentially all the equations of I (except the ones explicitly mentioned here) hold for the present work with minor changes ($m_\rho{}^2 \to m_{K^*}{}^2$, $\mu^2 \to m_K{}^2$) we will not repeat them.

The matrix element for the process $K^-(q)+p(p')\to \bar{K}^{*0}(k)+n(p)$ is given by

$$\epsilon_\mu\langle n|J^{K^*}_\mu|p,K^-\rangle=\bar{u}(p',\lambda_p)\gamma_5[B_1(\gamma\cdot\epsilon)(\gamma\cdot k)+2B_2(\epsilon\cdot P)+2B_3(\epsilon\cdot q)+2B_4(\epsilon\cdot k)-B_5(\gamma\cdot\epsilon)+B_6(\epsilon\cdot P)(\gamma\cdot k)+B_7(\epsilon\cdot k)(\gamma\cdot k)+B_8(\epsilon\cdot q)(\gamma\cdot k)]u(p,\lambda_n). \tag{1}$$

$J^{K^*}_\mu$ is the source current and ϵ is the polarization vector for $\bar{K}^{*0}$ and $P=\frac{1}{2}(p+p')$. Note that according to the notation of Ref. 3, which we are using, p' and p are the momenta of the *initial* and *final* nucleons, respectively. Then $s=(p+k)^2=(p'+q)^2$, $t=(p'-p)^2=(q-k)^2$.

If we assume that $\bar{K}^{*0}$ is coupled to a conserved current ($\partial_\mu J^{K^*}_\mu=0$) then as in Ref. 3 we are led to the following (incorrect) relation (in the high-s limit):

$$k^2B_1+sB_2-(t-m_K{}^2-k^2)B_3+2k^2B_4=0. \tag{2}$$

There is a similar relation between B_5, B_6, B_7, and B_8 but these amplitudes are not needed in this paper.

Now, if the Ball amplitudes are independent of k^2, one obtains the following (incorrect) relations:

$$sB_2-(t-m_K{}^2)B_3=0, \tag{3}$$

$$B_1+B_3+2B_4=0. \tag{4}$$

These are the so-called smoothness relations.

Then, following along the same lines as in I, we find that $\rho^t_{1,-1}=0$ would require $B_1=-B_2$. (Actually the equation is somewhat different, since although $m_\pi{}^2$ is negligible compared to $m_\rho{}^2$, $m_K{}^2$ is not so when compared to $m_{K^*}{}^2$. A more careful treatment is given in Appendix A.) This results in the amplitude D_- (defined in I) as

$$D_- = -\frac{1}{\sqrt{2m}}(-sB_1 - 2tB_3 + 2mB_5)$$

$$\approx -\frac{1}{\sqrt{2m}}(sB_2 - 2tB_3). \qquad (5)$$

In deriving Eq. (5) terms of the order of $1/s$ have been neglected.[1-3] Also as in I, the contribution of the B_5 term has been justifiably neglected. First of all from exchange diagrams it is seen to be small. Also the coefficient in front ($2m$ against s for B_1) is small. The coefficient of $B_3(t)$ is also small but B_3 receives a dominant one-π-exchange contribution with a very small denominator. Using Eq. (3) we see that D_- passes through zero at $t \approx -m_K^2$. Now E q. (14) of I gives

$$6\,\mathrm{Re}\rho_{10}^s = \mathrm{Re}(D_+T_+^* + D_-T_-^*). \qquad (6)$$

Again the $D_+T_+^*$ (nucleon-nonflip) term can be expected to be very small as compared to the $D_-T_-^*$ (nucleon-flip) term. Hence $\mathrm{Re}\rho_{10}^s$ will pass through zero at $t \approx -m_K^2$ in complete disagreement with experiments.[6,7]

This difficulty can be immediately traced to the current-conservation condition which led to Eq. (3). As pointed out in Refs. 9 and 10 and subsequent works by several authors, because of the breaking of SU_3 symmetry we cannot expect conservation of the strangeness-changing vector current. However, on the positive side, as with PCAC, it would be partially conserved (or "conserved as exactly as possible"). Then we have

$$\partial_\mu S_\mu = g_\kappa M_\kappa^2 \kappa, \qquad (7)$$

where S_μ is the strangeness-changing vector current to which the $\bar{K}^{*0}$ meson is coupled. κ is the κ-meson field operator. g_κ is defined by

$$\langle 0|S_\mu|\bar{\kappa}^0\rangle = ig_\kappa k_\mu. \qquad (8)$$

Similarly g_{K^*} can be defined by

$$\langle 0|S_\mu|\bar{K}^{*0}\rangle = g_{K^*}\epsilon_\mu. \qquad (9)$$

Then instead of (2) we have

$$\bar{u}(p')\gamma_5 \frac{g_{K^*}[k^2B_1 + sB_2 - (t - m_K^2 - k^2)B_3 + 2k^2B_4 + \cdots]}{m_{K^*}^2 - k^2} u(p) = \frac{g_\kappa m_\kappa^2}{m_\kappa^2 - k^2} T_{K^-p\to\bar{\kappa}_0 n}, \qquad (10)$$

where $T_{K^-p\to\bar{\kappa}_0 n}$ is the matrix element for the process $K^-p \to \bar{\kappa}^0 n$, which can be defined by

$$T_{K^-p\to\bar{\kappa}^0 n}(s,t) = \bar{u}(p')\gamma_5[A(s,t) + C(s,t)\gamma\cdot k]u(p). \qquad (11)$$

Omitted terms in Eq. (10) involve $\gamma_5\gamma_\mu$ with B_5, B_6, B_7, and B_8. Now we assume that B's are independent of k^2, set $k^2 = 0$, and obtain

$$\frac{g_{K^*}}{m_{K^*}^2}[sB_2 - (t - m_K^2)B_3] = g_\kappa A(k^2 = 0). \qquad (12)$$

Writing the B_3 term as

$$-(t - m_K^2)B_3 = -(t - m_\pi^2)B_3 + (m_K^2 - m_\pi^2)B_3, \quad (13)$$

we see that the (correct) smoothness condition can be written as

$$sB_2 - (t - m_\pi^2)B_3 = 0 \qquad (14)$$

provided that $(m_K^2 - m_\pi^2)(g_{K^*}/m_{K^*}^2)B_3$ agrees with the right-hand side of (12).

Now consider the couplings of these currents to the K-π system. This would then correspond to taking the one-pion-exchange contribution to B_3. Baryon poles do not contribute to B_3 appreciably. From Appendix B

$$B_3^\pi = \frac{g_{K^*K\pi}}{t - m_\pi^2}, \quad A^\pi = \frac{g_{\kappa K\pi}}{t - m_\pi^2}. \qquad (15)$$

Hence the cancellation of π-pole parts requires the condition

$$(m_K^2 - m_\pi^2)\frac{g_{K^*}g_{K^*K\pi}}{m_{K^*}^2} = g_\kappa g_{\kappa K\pi}. \qquad (16)$$

But this is precisely the condition obtained (Appendix C) from application of PCVC to the matrix element of S_μ between K and π states.

Another equivalent way of looking at the result follows from the original paper on PCVC.[9] The implication is that the K^*-production matrix element develops a pole in k^2 at $k^2 = m_\kappa^2$, and for zero mass κ this will prevent B_4 from being smooth in k^2. This gives

$$2B_4k_\mu = \frac{fk_\mu A}{m_\kappa^2 - k^2}, \qquad (17)$$

where the coupling f is defined by the Lagrangian density $fK_\mu^*\partial_\mu\kappa$. The current-conservation condition now becomes

$$k^2B_1 + sB_2 - (t - m_K^2 - k^2)B_3 + \frac{fk^2A}{m_\kappa^2 - k^2} + 2k^2\tilde{B}_4 = 0, \qquad (18)$$

where $\tilde{B}_4$ does not have a κ pole. Reference 9 gives a Goldberger-Treiman-type relation (in our notation)

$$f = \frac{g_{K^*K\pi}(m_K^2 - m_\pi^2)}{g_{\kappa K\pi}}. \qquad (19)$$

In the limit $m_\kappa^2 \to 0$, the f term again changes the coefficient of B_3^π from $-(t - m_K^2 - k^2)$ to $-(t - m_\pi^2 - k^2)$ and thus the condition (14) is obtained by assuming independence of k^2.

In both methods, we have shown this cancellation for the (dominant) pion-pole parts of B_3 and A. In Appendix B we show that the smoothness condition (14) and $B_1=-B_2$ are satisfied by the baryon- (Λ^0, Σ^0) pole contribution to B_2 and the pion-pole contribution to B_3 if Sakurai's principle of universality of ρ couplings, ρ-meson current conservation, and SU_3 symmetry with F-type couplings for vector-meson–baryon–baryon vertex are used.[13] Thus in such models the correct condition (14) can be readily understood. Equation (14) together with $B_1=-B_2$ produces a zero in D_- at $t\approx -m_\pi^2$ which in turn gives rise to a zero in $\mathrm{Re}\rho_{10}^s$ near this point.

In absorption or Regge-cut models one would obtain this zero by modifying π-pole diagrams. This again qualitatively we can understand this effect in terms of PCVC.

As for the existence of κ as a bona fide particle, the experimental situation is still not clear.[14] Either it is a resonance with very large width or just a complicated K-π s-wave phenomenon in which the phase shift may or may not be rising through 90° in the region of 1200 to 1400 MeV. Since our discussion does not depend on the actual value of m_κ, g_κ, or $g_{\kappa K\pi}$ we can just regard the right-hand side of the PCVC relation as some effective way of representing SU_3-symmetry breaking by this s-wave phenomenon.

Finally we note that more accurate treatment given in Appendix A gives the zero in $\mathrm{Re}\rho_{10}^s$ at $t=-0.53\,m_\pi^2$. It is very interesting that the recent experiment at SLAC[6] gives the zero at $t\approx -0.011$ GeV^2, which is remarkably close to our value -0.010 GeV^2. In ρ production the corresponding zero occurs at $t\approx -m_\pi^2=-0.019$ GeV^2 and thus there is a shift toward a smaller $|t|$ value in the K^* data in agreement with our analysis.

Thus we can justifiably conclude that the zeros in $\rho_{1,-1}^t$ and $\mathrm{Re}\rho_{10}^s$ indicate confirmation of the smoothness condition and partial conservation of strangeness-changing vector current. To a certain extent there are hints of model-independent properties of the fundamental amplitudes of such processes involving vector mesons.

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APPENDIX A: MORE PRECISE LOCATION OF THE ZERO OF $\mathrm{Re}\rho_{10}^s$

In the text, it was mentioned that, unlike the case of ρ production where m_π^2 is negligible compared to m_ρ^2, m_K^2 is not so when compared to $m_{K^*}^2$ in the present case. Hence the equations have to be somewhat modified. $\rho_{1,-1}^t=0$ still leads to the equation [Eq. (13) of I]

$$(1-\cos x)(-sB_1)+\sin x\frac{\sqrt{-t}}{m_{K^*}}(-sB_2)+\left[2t\cos x+\frac{\sin x\sqrt{-t}}{m_{K^*}}(t+m_{K^*}^2-m_K^2)\right]B_3=0. \tag{A1}$$

But the crossing angles are given in a better approximation by

$$\cos x=\frac{m_{K^*}^2-m_K^2+t}{m_{K^*}^2-m_K^2-\dfrac{t(m_{K^*}^2+m_K^2)}{(m_{K^*}^2-m_K^2)}}$$

and (A2)

$$\sin x=\frac{2\sqrt{-t}\,m_{K^*}}{m_{K^*}^2-m_K^2-\dfrac{t(m_{K^*}^2+m_K^2)}{(m_{K^*}^2-m_K^2)}}.$$

The coefficient of B_3 still vanishes, but exact cancellation of the B_1, B_2 terms demands that

$$B_1=-\frac{m_{K^*}^2-m_K^2}{m_{K^*}^2}B_2=-0.7B_2 \tag{A3}$$

instead of $B_1=-B_2$ as in the previous case. Then Eq. (5) gives

$$D_-=-\frac{1}{\sqrt{2}\,m}\left(s\frac{m_{K^*}^2-m_K^2}{m_{K^*}^2}B_2-2tB_3\right). \tag{A4}$$

Using smoothness Eq. (14), we obtain

$$D_-=-\frac{1}{\sqrt{2}\,m}\left[(t-m_\pi^2)\frac{(m_{K^*}^2-m_{K^*}^2)}{m_{K^*}^2}-2t\right]B_3, \tag{A5}$$

which gives zero at

$$t=-m_\pi^2\frac{(m_{K^*}^2-m_K^2)}{(m_{K^*}^2+m_K^2)}$$
$$=-0.53m_\pi^2 \tag{A6}$$

From Eq. (6) it follows that the zero in $\mathrm{Re}\rho_{10}^s$ will be obtained close to this t value.

APPENDIX B: BARYON- AND PION-POLE TERMS

The s-channel ($K^-p\to\bar{K}^{*0}n$) Λ pole and the t-channel ($K^-K^{0*}\to n\bar{p}$) π pole are given by

$$g_{KN\Lambda}g_{K^*N\Lambda}\bar{u}(p')\frac{\gamma_5}{s-m_\Lambda^2}\Big[(m_\Lambda-m)(\gamma\cdot\epsilon)-(\gamma\cdot\epsilon)(\gamma\cdot k)+\epsilon\cdot k+2\epsilon\cdot P+\epsilon\cdot q\Big]u(p), \tag{B1}$$

$$g_{K^*K\pi}g_{\pi NN}\bar{u}(p')\gamma_5\frac{(2\epsilon\cdot q-\epsilon\cdot k)}{t-m_\pi^2}u(p). \tag{B2}$$

The s-channel Σ^0 pole gives a similar contribution. Charge labels have been suppressed. For large s,

$$B_2^\Lambda + B_2^\Sigma \approx \frac{g_{KN\Lambda}g_{K^*N\Lambda} + g_{KN\Sigma}g_{K^*N\Sigma}}{s}, \tag{B3}$$

$$B_3^\pi = \frac{g_{K^*K\pi}g_{\pi NN}}{t - m_\pi^2}, \tag{B4}$$

and

$$B_1 = -B_2.$$

Now the $K^*N\Lambda, K^*N\Sigma$ couplings can be readily related to the $K^*K\pi$ couplings by using SU_3 symmetry with only F-type coupling for vector-meson–baryon–baryon vertex and universal couplings of the ρ meson. Also $KN\Lambda, KN\Sigma$ couplings are similarly related to πNN coupling for any value of the F/D ratio.[13] It is found that

$$g_{KN\Lambda}g_{K^*N\Lambda} + g_{KN\Sigma}g_{K^*N\Sigma} = g_{K^*K\pi}g_{\pi NN} \tag{B5}$$

for the appropriate charged states and any F/D ratio for pseudoscalar-meson–baryon–baryon vertex.

From (B3) and (B4) it is easily seen that the three terms satisfy the smoothness Eq. (14) with $(t - m_\pi^2)$ as the coefficient of B_3.

In a similar way, the s-channel (Λ, Σ^0) and t-channel (π) pole terms for $K^-p \to \bar{K}^0n$ are given by

$$g_{\kappa N\Lambda}g_{KN\Lambda}\bar{u}(p')\frac{\gamma_5(m + m_\Lambda + \gamma\cdot k)u(p)}{s - m_\Lambda^2} \tag{B6}$$

and

$$g_{\kappa K\pi}g_{\pi NN}\bar{u}(p')\frac{\gamma_5}{t - m_\pi^2}u(p) \tag{B7}$$

with a similar term for Σ^0.

Note that the baryon-pole contribution to the "A" term of $T_{K^-p\to\bar{K}^0n}$ goes down by a factor of $1/s$ as compared to the π-pole term. Hence it can be consistently neglected.

APPENDIX C: PCVC BETWEEN K AND π STATES

A number of authors have used PCVC in connection with K_{l3} decay.[11,12] The matrix element of S_μ between K and π states is given by

$$\langle\pi(q_2)|S_\mu|K(q_1)\rangle = i[f_+(t)(q_1+q_2)_\mu + f_-(t)(q_1-q_2)_\mu]. \tag{C1}$$

Hence

$$\begin{aligned}\langle\pi(q_2)|\partial_\mu S_\mu|K(q_1)\rangle &= f_+(t)(m_K^2 - m_\pi^2) + f_-(t)t \\ &= f(t)\end{aligned} \tag{C2}$$

with $t = (q_1 - q_2)^2$. If we use PCVC Eq. (7) and assume that the form factors obey unsubtracted dispersion relations, saturation of $f_+(t)$ with K^* and $f(t)$ with κ, leads to

$$\begin{aligned}f(0) &= \frac{(m_K^2 - m_\pi^2)g_{K^*}g_{K^*K\pi}}{m_{K^*}^2} \\ &= g_\kappa g_{\kappa K\pi}.\end{aligned} \tag{C3}$$

The relation is just Eq. (16) in the text. In connection with the K_{l3} problem a number of authors have verified that even if subtractions are introduced in f's, when use of other inputs such as $SU_3 \times SU_3$ spectral-function sum rules, Callan-Treiman relations, etc. is made, results close to (C3) are obtained.

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