Neutrino identity and the second-class current*

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If the initial and final neutrino in muonless weak reactions are different, the neutral leptonic weak current is not Hermitian. On the other hand, the isospin third component of the $\Delta I = 1$, $\Delta S = 0$ charged weak current is not Hermitian if it contains a second-class component. We investigate conditions in which these two non-Hermiticities are related.

This note is a comment on the paper by Wolfenstein[1] and the paper by Kingsley *et al.*[2]

Despite the success of the conventional weak-interaction theory, the question of whether there are only two neutrinos, ν_e and ν_μ, has not yet been settled experimentally.[3] So questions such as whether the neutrinos from π decay are the same as those of K decay, and how these neutrinos are related to those participating in μ decay, etc. still await answers, although the conventional theory gives unambiguous answers.

The neutrino-identity problem is completely open in the case of muonless neutrino reactions[4] because we know little about them both experimentally and theoretically. These reactions have been instinctively interpreted as reactions of the type

$$\nu + \text{target} \to \nu + \text{other particles}. \tag{1}$$

However, they might as well be interpreted as

$$\nu + \text{target} \to \nu' + \text{other particles}, \tag{2}$$

where the initial neutrino ν and final "neutrino" ν' are different or partially different;

$$|\langle\nu'|\nu\rangle| \leq 1. \tag{3}$$

Let us assume that reactions (1) and (2) are neutral-current events and originate in the effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\frac{1}{2}\frac{G'}{\sqrt{2}}(J_\mu^{(0)} J_\mu^{(0)\dagger} + \text{H.c.}). \tag{4}$$

G' is of the same dimension as the Fermi constant G_F, but it is not necessarily the same numerically. $J_\mu^{(0)}$ is the neutral weak current consisting of a leptonic part and a hadronic part. Then there is an interesting relation between the neutrino-identity problem and that of the second-class current[5] (charged). This problem was first discussed by Wolfenstein.[1] His discussion is mainly concerned with non-Hermitian currents which carry some kind of charges such as strangeness and charm. In this note we will discuss the case of pure neutral non-Hermitian currents (the non-Hermitian neutral currents which do not carry any charge) using gauge-model ideas. To illustrate this, we first consider the neutral leptonic current l_μ. The fact that the reaction $\bar\nu_\mu + e^- \to \bar\nu'_\mu + e^-$ has been observed implies that $J_\mu^{(0)}$ contains a term of the form $l_\mu = \bar\nu'\Gamma_\mu\nu$, where Γ_μ is an appropriate Dirac matrix. We assume $\Gamma_\mu = \gamma_\mu(1+\gamma_5)$. As we do not know whether ν' is distinct from ν, we express ν' in the following form:

$$\nu' = a\nu + b\nu_1$$

with

$$|a|^2 + |b|^2 = 1, \ \langle\nu_1|\nu\rangle = 0 \tag{5}$$

With this ansatz for ν' we obtain for $\langle\nu'|\nu\rangle$

$$\langle\nu'|\nu\rangle = a. \tag{6}$$

The leptonic current l_μ can be expressed as a sum of a Hermitian current l_μ^h and an anti-Hermitian one l_μ^a:

$$l_\mu = l_\mu^h + l_\mu^a,$$

with

$$\begin{aligned} l_\mu^h &\equiv \tfrac{1}{2}(a+a^*)\bar\nu\Gamma_\mu\nu + \tfrac{1}{2}b\bar\nu\Gamma_\mu\nu_1 + \tfrac{1}{2}b^*\bar\nu_1\Gamma_\mu\nu, \\ l_\mu^a &\equiv \tfrac{1}{2}(a^*-a)\bar\nu\Gamma_\mu\nu - \tfrac{1}{2}b\bar\nu\Gamma_\mu\nu_1 + \tfrac{1}{2}b^*\bar\nu_1\Gamma_\mu\nu. \end{aligned} \tag{7}$$

Let us consider now a semileptonic neutral-current reaction, for example, neutrino-nucleon scattering. The effective Lagrangian reads

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\frac{1}{2}\frac{G'}{\sqrt{2}}(l_\mu J_\mu^{Z\dagger} + \text{H.c.}), \tag{8}$$

where J_μ^Z is the hadronic neutral current.

From Eqs. (7) and (8) we conclude that the hadronic neutral weak current may consist of a Hermitian part J_μ^{Zh} and an anti-Hermitian part J_μ^{Za},

$$J_\mu^Z = J_\mu^{Zh} + J_\mu^{Za}. \tag{9}$$

We mean in Eq. (9) that the anti-Hermitian part J_μ^{Za} *contributes* to reaction (2) if l_μ^a is nonzero; that is, ν and ν' are not identical. In the case of vanishing l_μ^a, any anti-Hermitian part of J_μ^Z does not contribute to reaction (2), as it cancels out in the effective Lagrangian.

Let us summarize: *If ν and ν' are not identical, then the leptonic neutral current contains an anti-Hermitian component and the hadronic neutral current need not be Hermitian.*

Now we turn to the problem of the second-class charged current. Experimentally the existence of second-class currents has been neither excluded nor confirmed.[6] For our purpose, however, we assume that the charged strangeness-conserving $I=1$ (isospin) current $J_\mu^\dagger$ contains a small admixture of the second-class current:

$$J_\mu^\dagger = J_{1\mu}^\dagger + J_{2\mu}^\dagger , \tag{10}$$

where $J_{1\mu}^\dagger$ ($J_{2\mu}^\dagger$) is a first-class (second-class) charged current. A way of classifying the weak currents into first and second classes is by the Hermiticity or anti-Hermiticity of their isospin third component[7]:

$$(J_{1\mu}^3)^{\text{H.c.}} = +J_{1\mu}^3 , \quad \text{first class}$$
$$(J_{2\mu}^3)^{\text{H.c.}} = -J_{2\mu}^3 , \quad \text{second class} \tag{11}$$

with

$$J_{i\mu}^3 \equiv -\tfrac{1}{2}[T^-, J_{i\mu}^\dagger], \quad i=1,2$$
$$(J_{i\mu}^3)^{\text{H.c.}} = \text{Hermitian conjugate of } J_{i\mu}^3 .$$

T^+, T^3, and T^- are the infinitesimal generators of the isospin group. Thus the third component of $J_\mu^\dagger$, $J_\mu^3 \equiv -\frac{1}{2}[T^-, T_\mu^\dagger]$ is not Hermitian, but contains an anti-Hermitian component:

$$J_\mu^3 = J_{1\mu}^3 + J_{2\mu}^3 . \tag{12}$$

Before the discovery of neutral-current events, one generally believed that weak neutral currents did not exist and considered J_μ^3 to be unphysical. Now the situation has changed both experimentally and theoretically. In fact, most of the gauge models suggest that the neutral part of charged currents is at least a part of the neutral weak current. Hence it is clear that second-class currents put some constraints on gauge models.[8] In the present note we only accept the fact that charged and neutral weak currents (or a part of them) are in the same isomultiplet. With this assumption, the neutral hadronic weak current has the form

$$J_\mu^Z = J_{1\mu}^3 + J_{2\mu}^3 + J_\mu^0 , \tag{13}$$

where J_μ^0 represents the part of J_μ^Z which does not belong to the same isomultiplet as $J_\mu^\dagger$. (J_μ^0 may or may not be Hermitian.) Thus J_μ^Z is not Hermitian. As mentioned already, the anti-Hermitian $J_{2\mu}^3$ can become operative in reaction (2) only if the leptonic neutral current l_μ is also non-Hermitian. To see this we rewrite the Lagrangian (8) in terms of the currents of Eqs. (7) and (13):

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\frac{G'}{\sqrt{2}}(l_\mu^h J_{1\mu}^3 - l_\mu^a J_{2\mu}^3) - \frac{1}{2}\frac{G'}{\sqrt{2}}(l_\mu J_\mu^{0\dagger} + l_\mu^\dagger J_\mu^0). \tag{14}$$

Now we see that $J_{2\mu}^3$ contributes to reaction (2) only if l_μ^a does not vanish.

Thus one is led to the following conclusion: *If a second-class current exists and its neutral component contributes to neutral weak reactions, then ν and ν' are not identical*:

$$\langle \nu' | \nu \rangle = a \quad |a| < 1. \tag{15}$$

This means that in Eq. (5) b does not vanish and ν' contains a component ν_1 which is orthogonal to ν.

Unfortunately, we cannot make any definite quantitative statement about a and b except the following general remark: If the existence of a second-class current is the only reason for nonidentity, then it is a partial nonidentity; that is, both a and b in (5) are nonvanishing.

ν_1 might be a new neutrino, a neutral heavy lepton, or even ν_e (if $\nu = \nu_\mu$). In the last case the separate lepton-number conservation is violated. The above conclusion applies also for the electron and the muon, as far as neutral weak processes are concerned.

Finally, we remark that nonidentity of ν and ν' does not necessarily imply the existence of second-class currents.

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