Electromagnetic N-N^* transition form factors

R. C. E. Devenish, T. S. Eisenschitz,* and J. G. Körner
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany
(Received 17 February 1976)

We define constraint-free electromagnetic N-N_J^* transition form factors for general abnormal- and normal-parity transitions and relate them to multipole and helicity transition form factors. Possible parametrizations of the constraint-free form factors are discussed and various theoretically motivated simplifications are considered which are then compared to the available transition-form-factor data. If analyzed in terms of the constraint-free form factors a very simple picture is obtained for the three leading resonances P_{33}, D_{13}, and F_{15}. Namely, the qualitative features of the multipole structure of each resonance are governed by one coupling ratio each. The results of recent multipole analyses indicate that these coupling ratios are approximately the same in all three cases. Our analysis supports the contention made by some authors that the N-Δ form factor falls with one power faster asymptotically than that corresponding to canonical dipole behavior.

I. INTRODUCTION

The notion of the universality of elastic and inelastic transition form factors is frequently alluded to in a naive manner, although it is very difficult to make this notion theoretically precise. For asymptotically large timelike or spacelike values of q^2 one may define such a universality by appealing to the Drell-Yan relation, which relates the spin-averaged contribution of elastic and inelastic form factors to the universal threshold behavior of certain structure functions. Because of the appearance in both spacelike and timelike regions of kinematical factors of the order $|q^2|^{2(J'-J)}$ for the contributions of $J\to J'$ transition form factors to cross sections, the Drell-Yan relation could be valid only if there were a dynamical damping mechanism in the form factors for these kinematical factors. On a more fundamental level the necessity of such dynamical compensations is implied by the fact that an increase in the production of a given final state must be damped if unitarity is not to be violated.

One would suppose that the dynamics which provides for such a correlation between the spin of an electro-excited resonance and the dynamic q^2 dependence of its form factor should be rather of a global nature. Attempts at obtaining such global correlations have been made, for example, by Dürr and Pilkuhn extending a nonrelativistic model of potential scattering,[1] and also by Fujimura, Kobayashi, and Namiki in the context of the quark model, from Lorentz contraction factors.[2] There is also the dual current model of Sugawara, Ohba, Ademollo, and Del Guidice where such a correlation arises from analytic continuation in the J plane in the presence of variable current masses.[3]

Apart from the quite general theoretical interest in the q^2 dependence of the transition form factor at lower values of q^2 as witnessed by a wealth of papers on this subject in the last few years, one would like to find out if the above-mentioned global correlations leading to asymptotic universality can already be seen at lower values of q^2. For example, the recurring question of whether the $N-\Delta$ transition form factor as measured up to $-q^2\sim$ 5–6 GeV^2 shows canonical dipole behavior can only be answered after an attempt has been made to solve this question. Unfortunately this problem cannot even be posed in a very well-defined way, if only for the fact that the question of what set of form factors should show universal or even regular behavior cannot be answered without appealing to a detailed, as yet nonexistent, dynamics.

We take the point of view that the most likely candidates for a possible universal or global behavior should be sought among the constraint-free form-factor invariants and not among the physical helicity or multipole form factors since the latter have a known underlying kinematic q^2 structure resulting from constraints they have to obey at threshold and pseudothreshold. Even among the constraint-free form factors there is a plethora of possible choices. Among these we select a certain set for their simplicity and define a criterion of minimality which such a set has to obey. The aim of this paper is to find out whether the present preliminary transition-form-factor data show any evidence for such a universal or global behavior of suitably chosen constraint-free form factors.

In Sec. II we deal with the kinematics of transition form factors, where we have tried to remain as brief as possible, since the material is either known or derivable in a straightforward manner from two excellent previous articles on the subject.[4,5] The reader who is not so interested in the kinematical details can skip the entire Sec. II and move to Sec. III without loss of understanding. In Sec. II A we write down covariant projections on constraint-free form factors, multipole form factors, and helicity form factors for general $J \geq \frac{3}{2}$ abnormal- and normal-parity transitions. These lead to constraint relations for the multipole and helicity form factors which are given explicitly. In Sec. II B we treat the two exceptional cases of transitions to $J=\frac{1}{2}$ isobars of positive and negative parity. In Sec. II C we write down the contributions of the multipole and helicity form factors to the corresponding multipole and helicity amplitudes of single-pion electroproduction. In Sec. II D we give the cross sections.

In Sec. III we discuss the problem of finding appropriate parametrizations for the form-factor data and arrive at a suitable general form in Sec. III A. Possible simplifications of this general form are then proposed in Sec. III B and tested with some of the available form-factor data in Sec. IV. In Sec. V we summarize our results and give our conclusions.

II. KINEMATICS

A. Definitions of the vertex

Following Jones and Scadron[5] we write

$$\langle N_J^*| j_\mu(0)|N\rangle = e\,\bar u^{\beta_1\cdots\beta_l}(p^*)\Gamma_{\beta_1\cdots\beta_l\mu}u(p), \quad (2.1)$$

where $u^{\beta_1\cdots\beta_l}$ is the generalized Rarita-Schwinger spinor (see e.g. Ref. 4) for a fermion of spin $J = l+\frac{1}{2}$ $(J \geq \frac{3}{2})$ and where the momenta are defined according to Fig. 1.

Without loss of generality the matrix element Eq. (2.1) can be written as

$$\langle N_J^*| j_\mu(0)|N\rangle = e\,\bar u^{\beta_1\cdots\beta_{l-1}\beta}(p^*)q_{\beta_1}\cdots q_{\beta_{l-1}} \times \Gamma^{(l)}_{\beta\mu}(q^2)u(p). \quad (2.2)$$

The general problem of defining a set of constraint-free and gauge-invariant form-factor invariants may be approached in a variety of ways.[6-8] In our specific case we solve this problem by direct construction following the work of Bardeen and Tung[9] and Tarrach.[10] First we expand $\Gamma^{(l)}_{\beta\mu}(q^2)$ along a minimal set of non-gauge-invariant covariants. For abnormal-parity transitions $\frac{1}{2}^+ \to \frac{3}{2}^+, \frac{5}{2}^-, \ldots$ these are given by

$$\Gamma_{\beta\mu} = B_1 g_{\beta\mu}\gamma_5 + B_2 q_\beta\gamma_\mu\gamma_5 + B_3 q_\beta p^*_\mu\gamma_5 + B_4 q_\beta q_\mu\gamma_5. \quad (2.3)$$

We follow the conventions of Ref. 5. Thus we use spacelike metric and covariant normalization. γ matrices are defined by $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ with $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ so that $\gamma_5^2 = -1$. We have dropped explicit reference to the dependence on l and q^2 in Eq. (2.3) in order to avoid cluttering the invariants with indices. Whenever this does not lead to confusion we shall also in the following omit explicit reference to these dependencies.

The gauge-invariance condition $q_\mu\Gamma_{\beta\mu}=0$ imposes one constraint on the four invariants B_i which reads

$$B_1 + (M+m)B_2 + \tfrac{1}{2}(M^2 - m^2 + q^2)B_3 + q^2 B_4 = 0, \quad (2.4)$$

where m and M denote the nucleon and isobar masses.

Using the gauge projector method of Ref. 9 one can then construct a set of gauge-invariant covariants $\mathcal{K}^i_{\beta\mu}$ for which we take a set that is simply related to the set used in Ref. 5:

$$\begin{aligned}\mathcal{K}^1_{\beta\mu} &= (q_\beta\gamma_\mu - \not{q}\, g_{\beta\mu})\gamma_5,\\ \mathcal{K}^2_{\beta\mu} &= (q_\beta p^*_\mu - p^*\cdot q\, g_{\beta\mu})\gamma_5,\\ \mathcal{K}^3_{\beta\mu} &= (q_\beta q_\mu - q^2 g_{\beta\mu})\gamma_5.\end{aligned} \quad (2.5)$$

The set (2.5) has the advantage that the leading q^2 contribution to the scalar amplitudes comes from the third invariant only [see Eq. (2.11)].

The set of form factors G_i defined by expanding $\Gamma_{\beta\mu}$ along the gauge-invariant covariants (2.5)

$$\Gamma_{\beta\mu} = G_1\mathcal{K}^1_{\beta\mu} + G_2\mathcal{K}^2_{\beta\mu} + G_3\mathcal{K}^3_{\beta\mu} \quad (2.6)$$

can then be shown to be free of kinematical singularities and constraints since from comparing (2.3) and (2.6) one has

$$\begin{aligned}G_1 &= B_2,\\ G_2 &= B_3,\\ G_3 &= B_4.\end{aligned} \quad (2.7)$$

Since Eq. (2.4) shows that there is no constraint on the three invariants B_2, B_3, and B_4, there is also no constraint on the three form factors G_1, G_2, and G_3. Further, Eq. (2.7) shows that the G_i are kinematic-singularity-free since the B_i are kinematic-singularity-free.

The three constraint-free form factors G_i are advantageous from a theoretical point of view since they are independent for all values of q^2. However, since they do not describe transitions

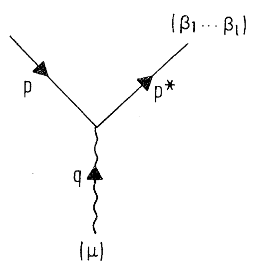

FIG. 1. The γNN_J^* vertex. $q = p^* - p$ and $P = \frac{1}{2}(p^* + p)$.

between physical states they obviously do not enter diagonally into cross-section formulas such as e.g. the generalized Rosenbluth formula (see Sec. II D). For the experimental analysis it is desirable to use form-factor invariants which describe physical transitions. These would e.g. correspond to definite multipole or helicity transitions in a given reference frame. We shall in the following refer to these sets of form factors as the physical form factors.

Covariant couplings that induce definite helicity transitions in the isobar rest frame have already been written down for the general case of normal- and abnormal-parity transitions by Bjorken and Walecka.[4] The covariants that induce definite multipole transitions can be written down by generalizing the analysis of Jones and Scadron.[5]

For the abnormal-parity transitions the multipole covariants that induce magnetic, electric, and Coulombic multipole transitions are given by

$$\begin{aligned}\mathcal{K}^M_{\beta\mu} &= -[3(M+m)/2mQ^+]\epsilon_{\beta\mu}(Pq),\\ \mathcal{K}^E_{\beta\mu} &= -\mathcal{K}^M_{\beta\mu} - \tfrac{1}{2}(l+1)[3(M+m)/2m\Delta]\\ &\qquad\times 4\epsilon_{\beta\sigma}(Pq)\epsilon_{\mu\sigma}(p^*q)\gamma_5,\\ \mathcal{K}^C_{\beta\mu} &= -[3(M+m)/2m\Delta]2q_\beta(q^2P_\mu - q\cdot Pq_\mu)\gamma_5,\end{aligned} \tag{2.8}$$

which define multipole form factors G_M, G_E, and G_C via the expansion

$$\Gamma_{\beta\mu} = G_M\mathcal{K}^M_{\beta\mu} + G_E\mathcal{K}^E_{\beta\mu} + G_C\mathcal{K}^C_{\beta\mu}. \tag{2.9}$$

Note that the magnetic and Coulombic multipole covariants are not l-dependent, whereas the electric multipole covariant involves an l-dependent term. In Eq. (2.8) we have introduced functions $Q^+(q^2)$ and $\Delta(q^2)$ defined by

$$Q^\pm(q^2) = [(M\pm m)^2 - q^2]$$

and (2.10)

$$\Delta(q^2) = Q^+Q^- \equiv 4M^2q_c^2,$$

where $q_c \equiv |\vec{q}_c|$ is the momentum of the virtual photon in the isobar rest frame.

In order to relate the multipole form factors G_M, G_E, and G_C to the constraint-free form factors G_i, one expresses the multipole covariants (2.8) by the covariants (2.5) using standard identities involving $\epsilon_{\alpha\beta\gamma\delta}$ tensors (see e.g. Ref. 11), and after inverting these expressions one obtains

$$\begin{pmatrix}\tfrac{1}{2}(l+1)G_M\\ \tfrac{1}{2}(l+1)G_E\\ G_C\end{pmatrix} = \frac{m}{3(M+m)}\begin{pmatrix}\sigma+q^2+(l+1)Q^+ & \sigma+q^2 & q^2\\ \sigma+q^2 & \sigma+q^2 & q^2\\ 4M^2 & 4M^2 & \sigma+q^2\end{pmatrix}\begin{pmatrix}G_1/M\\ G_2\\ 2G_3\end{pmatrix}, \tag{2.11}$$

where we define $\sigma = M^2 - m^2$.

We have written Eq. (2.11) in such a way that it is immediately apparent that, apart from the l-dependent normalization of G_M and G_E, the constraint equations between G_M, G_E, and G_C implied by (2.11) at threshold $Q^+=0$ and pseudothreshold $Q^-=0$ are l-independent. We have the following:

(i) At threshold $Q^+=0$,

$$\tfrac{1}{2}(l+1)G_M = \tfrac{1}{2}(l+1)G_E = G_C(M+m)/2M. \tag{2.12}$$

(ii) At pseudothreshold $Q^-=0$,

$$\tfrac{1}{2}(l+1)G_E = G_C(M-m)/2M. \tag{2.13}$$

For the normal-parity transitions $\frac{1}{2}^+ \to \frac{3}{2}^-, \frac{5}{2}^+ \ldots$ we define covariants and invariants in complete analogy to Eqs. (2.2), (2.5), and (2.6) by the replacement

$$\bar{u}^{\beta_1\cdots\beta_{l-1}\beta}(p^*) \to \bar{u}^{\beta_1\cdots\beta_{l-1}\beta}(p^*)\gamma_5, \tag{2.14}$$

implying a multiplication of the covariants (2.5) by γ_5 from the left. The respective covariants and invariants are denoted by primes. For example, one has

$$\begin{aligned}\Gamma_{\beta\mu} &= G_1'\gamma_5\mathcal{K}^1_{\beta\mu} + \cdots\\ &= G_1'\mathcal{K}^{1\prime}_{\beta\mu} + \cdots,\ \text{etc.}\end{aligned} \tag{2.15}$$

The multipole covariants for the normal-parity case are given by

$$\begin{aligned}\mathcal{K}'^M_{\beta\mu} &= -\frac{3(M-m)}{2m}\frac{1}{Q^-}\Big[-(l+1)\gamma_5\epsilon_{\beta\mu}(Pq)\\ &\qquad - \frac{2}{Q^+}\epsilon_{\beta\sigma}(Pq)\epsilon_{\mu\sigma}(Pq)\Big],\\ \mathcal{K}'^E_{\beta\mu} &= -\frac{3(M-m)}{2m}\frac{2}{\Delta}\epsilon_{\beta\sigma}(Pq)\epsilon_{\mu\sigma}(Pq),\\ \mathcal{K}'^C_{\beta\mu} &= \frac{3(M-m)}{2m}\frac{2}{\Delta}q_\beta(q^2P_\mu - P\cdot qq_\mu).\end{aligned} \tag{2.16}$$

The corresponding multipole form factors defined by

$$\Gamma_{\beta\mu} = G_M'\mathcal{K}'^M_{\beta\mu} + G_E'\mathcal{K}'^E_{\beta\mu} + G_C'\mathcal{K}'^C_{\beta\mu} \tag{2.17}$$

are related to the constraint-free form factors by

$$\begin{pmatrix} \frac{1}{2}(l+1)G'_M \\ \frac{1}{2}(l+1)G'_E \\ G'_C \end{pmatrix} = \frac{m}{3(M-m)} \begin{pmatrix} -Q^- & 0 & 0 \\ -Q^- - (l+1)(\sigma+q^2) & -(l+1)(\sigma+q^2) & -(l+1)q^2 \\ 4M^2 & 4M^2 & \sigma+q^2 \end{pmatrix} \begin{pmatrix} -G'_1/M \\ G'_2 \\ 2G'_3 \end{pmatrix}. \tag{2.18}$$

The constraint equations for G'_M, G'_E, and G'_C can be read off from Eq. (2.18). One has the following:

(i) At threshold ($Q^+=0$),

$$\frac{M+m}{M} G'_C = G'_M - G'_E. \tag{2.19}$$

(ii) At pseudothreshold ($Q^-=0$)

$$G'_M = 0, \qquad -\frac{M-m}{M} G'_C = G'_E. \tag{2.20}$$

Helicity form factors may be introduced in analogy to Ref. 4 according to the decomposition

$$\Gamma_{\beta\mu} = h_1 \Delta^{-1} q_\beta (p \cdot q q_\mu - q^2 p_\mu)\gamma_5 + h_2 \Delta^{-1}[2\epsilon_{\beta\sigma}(qp)\epsilon_{\mu\sigma}(qp)\gamma_5 + \not{p}^* q_\beta \epsilon_\mu(qp\gamma)] + h_3 \Delta^{-1} \not{p}^* q_\beta \epsilon_\mu(qp\gamma), \tag{2.21}$$

where h_1, h_2, and h_3 induce scalar, transverse $\frac{3}{2}$, and transverse $\frac{1}{2}$ transitions. Our set h_i differs from the set g_i introduced in Ref. 4 in that we have removed explicit kinematic singularities at threshold and pseudothreshold in the set g_i by writing

$$h_i = g_i \Delta(q^2). \tag{2.22}$$

We have also replaced M by $\not{p}^*$ in the transverse helicity covariants for the sake of convenience in discussing the necessary changes going from the abnormal-parity transitions to the normal-parity transitions.

The helicity form factors are related to the constraint-free form factors by

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 4M^2 & 4M^2 & \sigma+q^2 \\ -2M(M+m) & -(\sigma+q^2) & -q^2 \\ 2q^2 - 2m(M+m) & (\sigma+q^2) & q^2 \end{pmatrix} \begin{pmatrix} G_1/M \\ G_2 \\ 2G_3 \end{pmatrix}. \tag{2.23}$$

As usual the constraint equations for the h_i can be read off from Eq. (2.23). One has the following:

(i) At threshold ($Q^+=0$),

$$-\frac{M+m}{2M} h_1 = h_2 = -h_3. \tag{2.24}$$

(ii) At pseudothreshold ($Q^-=0$),

$$-\frac{M-m}{M} h_1 = h_2 - h_3. \tag{2.25}$$

In the case of the helicity form factors the change to the case of normal-parity transitions is particularly simple: After the replacement of Eq. (2.14) is made, the relations Eq. (2.23) and the constraints (2.24) and (2.25) remain unchanged for the corresponding primed objects if one makes the substitution $M \leftrightarrow -M$. Of course the latter substitution implies an exchange of the role of threshold and pseudothreshold, since $M \leftrightarrow -M$ results in $Q^+ \leftrightarrow Q^-$.

We complete this subsection by giving the relation between the multipole and helicity form factors. For abnormal-parity transitions one has

$$\begin{aligned} \tfrac{1}{2}(l+1)G_M &= -\frac{m}{6(M+m)}[(l+2)h_2 + l h_3], \\ \tfrac{1}{2}(l+1)G_E &= -\frac{m}{6(M+m)}(h_2 - h_3), \\ G_C &= \frac{m}{3(M+m)} h_1, \end{aligned} \tag{2.26}$$

and for the normal-parity transitions

$$\begin{aligned} \tfrac{1}{2}(l+1)G'_M &= \frac{m}{6(M-m)}(h'_2 + h'_3), \\ \tfrac{1}{2}(l+1)G'_E &= \frac{m}{6(M-m)}[(l+2)h'_2 - l h'_3], \\ G'_C &= \frac{m}{3(M-m)} h'_1. \end{aligned} \tag{2.27}$$

B. The exceptional cases

The abnormal-parity transitions $\frac{1}{2}^+ \to \frac{1}{2}^-$ and the normal-parity transitions $\frac{1}{2}^+ \to \frac{1}{2}^+$ involve only two independent form factors each and therefore have to be discussed separately.

For the abnormal-parity case $\frac{1}{2}^+ \to \frac{1}{2}^-$ one defines gauge-invariant and constraint-free form factors via the decomposition

$$\Gamma_\mu = G_1(q^2\gamma_\mu - \not{q} q_\mu)\gamma_5 + G_2(P\cdot q\gamma_\mu - P_\mu \not{q})\gamma_5. \tag{2.28}$$

[The second covariant in Eq. (2.28) can be rewritten in the familiar form $(P\cdot q\gamma_\mu - P_\mu\not{q})\gamma_5 = -\frac{1}{2}(M+m)\sigma_{\mu\nu}q_\nu\gamma_5$.] The absence of constraints for the set of form factors G_1 and G_2 can be demonstrated similarly to the case discussed in Sec. II A by expanding Γ_μ first along a set of minimal non-gauge-invariant covariants:

$$\Gamma_\mu = B_1\gamma_\mu\gamma_5 + B_2P_\mu\gamma_5 + B_3q_\mu\gamma_5. \tag{2.29}$$

Gauge invariance, $q_\mu\Gamma_\mu = 0$, implies the constraint

$$B_1(M+m) + \tfrac{1}{2}(M^2 - m^2)B_2 + q^2B_3 = 0. \tag{2.30}$$

Since

$$G_1 = -\frac{1}{M+m}B_3$$

and (2.31)

$$G_2 = -\frac{1}{M+m}B_2,$$

one concludes from Eq. (2.31) that the set G_1 and G_2 is constraint-free.

The physical form factors h_i are defined by the decomposition

$$\Gamma_\mu = h_1\frac{1}{Q^-}(p\cdot q q_\mu - q^2p_\mu)\gamma_5 + h_3\frac{1}{Q^-}[\not{p}^*\epsilon_\mu(qp\gamma)], \tag{2.32}$$

where h_1 denotes the longitudinal and h_3 the transverse transition form factors. Compared to the definition of Ref. 4 we have introduced the extra factor $(Q^-)^{-1}$ in Eq. (2.32) in order to avoid the kinematic singularity at pseudothreshold present in the helicity form factors g_1 and g_3 of Ref. 4. In terms of the constraint-free form factors one obtains

$$\begin{aligned} h_1 &= -[2(M-m)G_1 + (M+m)G_2], \\ h_3 &= -\frac{1}{M}[2q^2G_1 + (M^2 - m^2)G_2], \end{aligned} \tag{2.33}$$

implying the pseudothreshold constraint

$$\frac{M-m}{M}h_1 = h_3. \tag{2.34}$$

The corresponding relations for the normal-parity transitions $\frac{1}{2}^+ \to \frac{1}{2}^+$ can be obtained from (2.28)–(2.34) as usual by replacing $\bar{u}(p^*) \to \bar{u}(p^*)\gamma_5$ and $M \leftrightarrow -M$. The corresponding covariants and form factors will be denoted by primes in the following.

C. Four-point function multipoles

As far as the q^2 structure is concerned it makes no difference whether one discusses the q^2 behavior of the three-point vertices of Secs. II A and II B or of their contributions to the multipole or helicity expansions of the four-point function $\gamma_V + N \to N + \pi$ (see Fig. 2). Since most of the data of the q^2 behavior of resonance form factors has been obtained from measurements of the four-point function it has become customary to exhibit the q^2 structure of N-N^*_J transitions in terms of their multipole or helicity contributions to the four-point-function partial-wave expansions.

We shall denote the residues of the Dennery amplitudes A_1, $(t-\mu^2)A_2$, $(t-\mu^2)(A_3 - A_4)$, $A_3 + A_4$, $(t-\mu^2)A_5$, and A_6 in terms of the coefficient of the leading power of $\cos\theta_s$ by r_1, r_2, r_{3-4}, r_{3+4}, r_5, and r_6. The full angular structure of the residues can be easily written down in terms of derivatives of the Legendre polynomials (see e.g., Ref. 12).

For the abnormal-parity case one has

$$\begin{aligned} r_1 &= (-G_1 + mG_2)g^*(p_cp_c')^l \\ r_2 &= [-2G_1 - (M-m)G_2]g^*(p_cp_c')^l, \\ r_5 &= [-G_1 - \tfrac{3}{2}(M-m)G_2 - 2(M-m)G_3]g^*(p_cp_c')^l, \end{aligned} \tag{2.35}$$

and the remaining three residues are given by the parity constraints

$$\begin{aligned} (M-m)r_{3-4} &= -(\sigma - \tfrac{1}{2}q^2)r_2 - q^2r_5, \\ (M+m)r_{3+4} &= 2r_1 - r_2, \\ (M-m)r_6 &= -\tfrac{1}{4}r_2 + \tfrac{1}{2}r_5. \end{aligned} \tag{2.36}$$

The strong coupling constant g^* is defined by

$$\langle N|j_\pi(0)|N^*_J\rangle = g^*\bar{u}(p')p'_{\beta_1}\cdots p'_{\beta_l}u^{\beta_1\cdots\beta_l}(p^*), \tag{2.37}$$

and p_c and p_c' are the magnitude of incoming and outgoing nucleon momenta in the c.m. frame. The

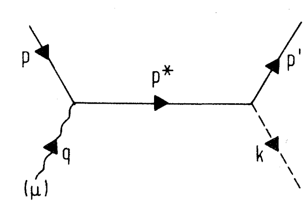

FIG. 2. Isobar contribution to resonant photo- and electroproduction.

partial $N\pi$ width corresponding to the $N^*N\pi$ coupling Eq. (2.37) is given by

$$\Gamma_{N_J^*\to N\pi}=\frac{g^{*2}}{4\pi}\,\frac{E'+m}{M}\,\frac{1}{(l+1)\tau_{l+1}}\,p_c'^{2l+1}, \qquad (2.38)$$

where E' is the energy of the nucleon in the c.m. frame and τ_l is defined in (2.41).

The appropriate multipole contributions can be calculated from (2.35) by a standard projection formula (see e.g., Ref. 13) and one obtains

$$\begin{aligned} lM_{l+}&=\eta_l G_M\,,\\ E_{l+}&=-\eta_l G_E\,,\\ S_{l+}/q_c&=-\eta_l\frac{2}{l+1}G_C/2M\,, \end{aligned} \qquad (2.39)$$

where

$$\eta_l=\frac{1}{(l+1)\tau_{l+1}}\,\frac{3(M+m)}{2m}M\left[\frac{(M+m)^2-m_\pi{}^2}{(M+m)^2-q^2}\right]^{1/2}\times(p_cp_c')^l g^*\frac{1}{s-M^2+iM\Gamma}\,, \qquad (2.40)$$

and where Γ is the total width of the isobar. The leading-power coefficient τ_l of the Legendre polynomial $P_l(x)$ appearing in Eq. (2.40) is given by

$$\tau_l=\frac{(2l)!}{2^l(l!)^2}\,. \qquad (2.41)$$

The multipoles in Eq. (2.39) exhibit the appropriate threshold and pseudothreshold power behavior $q_c^l(Q^+)^{-1/2}$ in addition to the following three threshold and pseudothreshold constraints:

(i) At threshold $Q^+=0$,

$$\begin{aligned} &E_{l+}+lM_{l+}=0,\\ &\frac{S_{l+}}{q_c}+lM_{l+}\frac{1}{M+m}=0. \end{aligned} \qquad (2.42)$$

(ii) At pseudothreshold $Q^-=0$,

$$\frac{S_{l+}}{q_c}-\frac{1}{M-m}E_{l+}=0. \qquad (2.43)$$

These follow from the corresponding constraints for G_M, G_E, and G_C, Eq. (2.12) and Eq. (2.13).

One can calculate the helicity amplitudes A_{l+}, B_{l+}, and C_{l+} in terms of the helicity form factors h_1, h_2, and h_3 from Eq. (2.39) by using the relation between the helicity and multipole amplitudes given in the Appendix and the corresponding relation between the multipole and helicity form factors written down in Eq. (2.26).

For the normal-parity case one has

$$\begin{aligned} M_{l+1,-}&=-\eta_l'G_M'\,,\\ lE_{l+1,-}&=-\eta_l'G_E'\,,\\ S_{l+1,-}/q_c&=-\eta_l'\frac{2}{l+1}G_c'/2M\,, \end{aligned} \qquad (2.44)$$

where η_l' is given by

$$\eta_l'=\frac{1}{(l+1)\tau_{l+1}}\,\frac{3(M-m)}{2m}M\left[\frac{(M-m)^2-m_\pi{}^2}{(M-m)^2-q^2}\right]^{1/2}\times(p_cp_c')^l g^*\frac{1}{s-M^2+iM\Gamma}\,. \qquad (2.45)$$

The relations for the residues of the Dennery amplitudes, Eqs. (2.35) and (2.36), are replaced by the corresponding equations with $G_i\to G_i'$ and $M\to -M$.

The common threshold and pseudothreshold power behavior of $M_{l+1,-}$, $E_{l+1,-}$, and $S_{l+1,-}/q_c$ is given by $q_c^l(Q^-)^{-1/2}$ and in addition one has the following constraints:

(i) At threshold $Q^+=0$,

$$\frac{S_{l+1,-}}{q_c}+\frac{1}{l+1}\,\frac{1}{(M+m)}(lE_{l+1,-}-M_{l+1,-})=0. \qquad (2.46)$$

(ii) At pseudothreshold $Q^-=0$,

$$\begin{aligned} &M_{l+1,-}=0,\\ &\frac{S_{l+1,-}}{q_c}+\frac{l}{l+1}\,\frac{1}{(M-m)}E_{l+1,-}=0. \end{aligned} \qquad (2.47)$$

The helicity amplitudes $A_{l+1,-}$, $B_{l+1,-}$, and $C_{l+1,-}$ can be obtained from Eq. (2.44) in the same manner as discussed in the abnormal-parity case.

For the exceptional case $\frac{1}{2}^+\to\frac{1}{2}^-$ one obtains for the residues of the Dennery amplitudes

$$\begin{aligned} r_1&=[-q^2G_1-\tfrac{1}{2}(M+m)^2G_2]g^*\,,\\ r_2&=-2q^2G_1g^*\,, \end{aligned} \qquad (2.48)$$

and

$$r_5=-\frac{1}{q^2}(\sigma-\tfrac{1}{2}q^2)r_2\,, \qquad (2.49)$$

where the strong coupling constant g^* is defined by

$$\langle N|j_\pi(0)|N^*_{1/2}\rangle=g^*\bar{u}(p')u(p^*)\,. \qquad (2.50)$$

The $N\pi$ width corresponding to the coupling Eq. (2.50) is again given by Eq. (2.38). The remaining residues can be calculated with the help of the parity constraints (2.36).

For the multipole amplitudes we have

$$\begin{aligned} E_{0+}&=-\frac{M}{2}[(M+m)^2-m_\pi{}^2]^{1/2}(Q^+)^{1/2}\times h_3\frac{g^*}{s-M^2+iM\Gamma}\,,\\ S_{0+}/q_c&=-\tfrac{1}{2}[(M+m)^2-m_\pi{}^2]^{1/2}(Q^+)^{1/2}\times h_1\frac{g^*}{s-M^2+iM\Gamma}\,, \end{aligned} \qquad (2.51)$$

so that one has at pseudothreshold ($Q^-=0$) the constraint

$$E_{0+} = (M - m)S_{0+}/q_c\,. \tag{2.52}$$

Similarly one obtains in the case of normal-parity transitions $\frac{1}{2}^+ \to \frac{1}{2}^+$

$$M_{1-} = -\frac{M}{2}[(M - m)^2 - m_\pi^2]^{1/2}(Q^-)^{1/2}h_3'\frac{g^*}{s - M^2 + iM\Gamma}, \tag{2.53}$$

$$S_{1-}/q_c = -\tfrac{1}{2}[(M - m)^2 - m_\pi^2]^{1/2}(Q^-)^{1/2}h_1'\frac{g^*}{s - M^2 + iM\Gamma},$$

implying the threshold constraint

$$M_{1-} = (M + m)S_{1-}/q_c\,. \tag{2.54}$$

D. Cross sections

The generalized Rosenbluth formula gives the differential cross section $d\sigma/d\omega_L$ in terms of the transverse and longitudinal three-point couplings defined in Secs. II A and II B. ω_L is the electron solid angle in the lab frame. For abnormal-parity transition one obtains

$$\frac{d\sigma}{d\omega_L} = \tfrac{3}{2}\sigma_{NS}\frac{(M + m)^2}{Q^+}\frac{(-q^2)}{4m^2}\frac{3}{2\tau_{l+1}}q_c^{2(l-1)}\epsilon^{-1}\left(\frac{(l + 1)}{2l}|G_M|^2 + \frac{(l + 1)(l + 2)}{2}|G_E|^2 - \epsilon\frac{q^2}{M^2}|G_c|^2\right), \tag{2.55}$$

and for normal-parity transitions

$$\frac{d\sigma}{d\omega_L} = \frac{3}{2}\sigma_{NS}\frac{(M - m)^2}{Q^-}\frac{(-q^2)}{4m^2}\frac{3}{2\tau_{l+1}}q_c^{2(l-1)}\epsilon^{-1}\left(\frac{(l + 1)(l + 2)}{2}|G_M'|^2 + \frac{(l + 1)}{2l}|G_E'|^2 - \epsilon\frac{q^2}{M^2}|G_c'|^2\right), \tag{2.56}$$

where

$$\epsilon^{-1} = 1 + 2\tan^2\tfrac{1}{2}\psi_B = 1 - 2\frac{M^2}{m^2}\frac{q_c^2}{q^2}\tan^2\tfrac{1}{2}\psi_L, \tag{2.57}$$

and

$$\sigma_{NS} = \frac{\alpha^2}{4\epsilon_L^2}\frac{\cos^2\frac{1}{2}\psi_L}{\sin^4\frac{1}{2}\psi_L}\frac{1}{1 + 2(\epsilon_L/m)\sin^2\frac{1}{2}\psi_L}. \tag{2.58}$$

ψ_B and ψ_L are the electron scattering angles in the Breit and lab frames, and ϵ_L the incident electron energy in the lab frame. In comparing our formula (2.55) with Eq. (40) of Ref. 5 for the $l = 1$ case, one should remember to omit the factor $\frac{2}{3}$ arising from an explicit Clebsch-Gordan factor used in Ref. 5. [There is also a misprint in the longitudinal contribution in Eq. (40) of Ref. 5.] One should also keep in mind that in our convention $J = l + \frac{1}{2}$ always. The equivalent expressions in terms of the helicity couplings can easily be obtained by substitution.

The partial width $N_J^* \to N\gamma$ is given by

$$\Gamma_\gamma = \frac{\alpha}{(l + 1)\tau_{l+1}}\frac{9}{4m^2}q_c^{2l+1}\left(\frac{(l + 1)}{2l}|G_M|^2 + \frac{(l + 1)(l + 2)}{2}|G_E|^2\right)$$

for the abnormal-parity decays and, for the normal-parity decays, by

$$\Gamma_\gamma = \frac{\alpha}{(l + 1)\tau_{l+1}}\frac{9}{4m^2}q_c^{2l+1}\left(\frac{(l + 1)(l + 2)}{2}|G_M'|^2 + \frac{(l + 1)}{2l}|G_E'|^2\right). \tag{2.60}$$

For the exceptional cases one has

$$\Gamma_\gamma = \frac{\alpha}{4}(M + m)^2 q_c |h_3|^2, \tag{2.59$'$}$$

and

$$\Gamma_\gamma = \frac{\alpha}{4}(M - m)^2 q_c |h_3'|^2. \tag{2.60$'$}$$

In the case of the four-point function the double differential cross section $d^2\sigma/d\omega_L d\epsilon_L'$ is as usual written in the form

$$\frac{d^2\sigma}{d\omega_L d\epsilon_L'} = \Gamma_t(\sigma_T + \epsilon\sigma_L), \tag{2.61}$$

where

$$\Gamma_t = \frac{\alpha}{4\pi^2} \frac{W^2 - m^2}{2m} \frac{1}{(-q^2)} \frac{\epsilon_L'}{\epsilon_L}(2 + \cot g^2 \tfrac{1}{2}\psi_B), \tag{2.62}$$

and where ϵ_L' is the laboratory energy of the scattered electron. The transverse and longitudinal cross sections σ_T and σ_L are given by

$$\sigma_T = \frac{\alpha p_c'}{8\pi m W} \frac{2m}{(W^2 - m^2)} \sum_{l=0}^{\infty} [l(l+1)|M_{l+}|^2 + (l+1)^2(l+2)|M_{l+1,-}|^2 + (l+1)^2(l+2)|E_{l+}|^2 + l(l+1)^2|E_{l+1,-}|^2], \tag{2.63}$$

$$\sigma_L = \frac{\alpha p_c'}{8\pi m W} \frac{2m}{(W^2 - m^2)} \sum_{l=0}^{\infty} 2 \frac{(-q^2)}{q_c^2}(l+1)^3(|S_{l+}|^2 + |S_{l+1,-}|^2). \tag{2.64}$$

We shall also define helicity $\frac{1}{2}$ and helicity $\frac{3}{2}$ cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ by

$$\sigma_{1/2} = \frac{\alpha p_c'}{8\pi m W} \frac{2m}{(W^2 - m^2)} \sum_{l=0}^{\infty} 2(l+1)(|A_{l+}|^2 + |A_{l+1,-}|^2),$$
$$\sigma_{3/2} = \frac{\alpha p_c'}{8\pi m W} \frac{2m}{(W^2 - m^2)} \sum_{l=0}^{\infty} \tfrac{1}{2}l(l+1)(l+2)(|B_{l+}|^2 + |B_{l+1,-}|^2), \tag{2.65}$$

so that

$$\sigma_T = \sigma_{1/2} + \sigma_{3/2}. \tag{2.66}$$

If one integrates the four-point cross section Eq. (2.61) for a particular resonance N_J^* with regard to $d\epsilon_L'$ using the narrow-resonance approximation one obtains the fraction $(\Gamma_{N^* \to N\pi}/\Gamma_{N^* \to \text{all}})$ of the three-point cross sections Eqs. (2.55) or (2.56).

III. PARAMETRIZATION OF FORM-FACTOR DATA

A. General remarks

When one is attempting to parametrize transition-form-factor data one needs a representation of the multipole amplitudes that automatically incorporates the necessary kinematic-constraint structure. One of many possibilities is to choose the representation of multipoles in terms of the constraint-free form factors G_i as derived in Sec. II.

After combining Eq. (2.11) with Eq. (2.39) one obtains for the abnormal-parity transitions

$$\begin{bmatrix} lM_{l+} \\ E_{l+} \\ S_{l+}/q_c \end{bmatrix} \propto \frac{q_c^l}{(Q^+)^{1/2}} \begin{bmatrix} \sigma + q^2 + (l+1)Q^+ & \sigma + q^2 & q^2 \\ -(\sigma + q^2) & -(\sigma + q^2) & -q^2 \\ -2M & -2M & -\frac{1}{2M}(\sigma + q^2) \end{bmatrix} \begin{bmatrix} G_1/M \\ G_2 \\ 2G_3 \end{bmatrix}, \tag{3.1}$$

where we have omitted all q^2-independent factors. For the normal-parity transitions one obtains

$$\begin{bmatrix} M_{l+1,-} \\ lE_{l+1,-} \\ S_{l+1,-}/q_c \end{bmatrix} \propto \frac{q_c^l}{(Q^-)^{1/2}} \begin{bmatrix} Q^- & 0 & 0 \\ Q^- + (l+1)(\sigma + q^2) & (l+1)(\sigma + q^2) & (l+1)q^2 \\ -2M & -2M & -\frac{1}{2M}(\sigma + q^2) \end{bmatrix} \begin{bmatrix} -G_1'/M \\ G_2' \\ 2G_3' \end{bmatrix}. \tag{3.2}$$

For the sake of completeness we shall also list the corresponding representation of the helicity amplitudes, which can be obtained from (3.2) using the relations between helicity and multipole amplitudes (A2):

$$\begin{bmatrix} \frac{2}{l+1}A_{l+1,-} \\ \frac{l}{l+1}B_{l+1,-} \\ S_{l+1,-}/q_c \end{bmatrix} \propto \frac{q_c^l}{(Q^-)^{1/2}} \begin{bmatrix} Q^- - (\sigma + q^2) & -(\sigma + q^2) & -q^2 \\ Q^- + (\sigma + q^2) & \sigma + q^2 & q^2 \\ -2M & -2M & -\frac{1}{2M}(\sigma + q^2) \end{bmatrix} \begin{bmatrix} -G_1'/M \\ G_2' \\ 2G_3' \end{bmatrix}. \tag{3.3}$$

The helicity-amplitude representation for the abnornal-parity transitions can be obtained as usual from (3.3) by making the replacements $M \to -M$,

$$A_{l+1,-} \to -A_{l+}, \ B_{l+1,-} \to B_{l+}, \ S_{l+1,-} \to S_{l+}, \text{ and } G_i' \to G_i.$$

For the exceptional cases one has

$$\begin{bmatrix} E_{0+} \\ S_{0+}/q_c \end{bmatrix} \propto (Q^+)^{1/2} \begin{bmatrix} 2q^2 & M^2-m^2 \\ 2(M-m) & M+m \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \tag{3.4}$$

for the abnornal-parity case $\frac{1}{2}^+ \to \frac{1}{2}^-$, and

$$\begin{bmatrix} M_{1-} \\ S_{1-}/q_c \end{bmatrix} \propto (Q^-)^{1/2} \begin{bmatrix} 2q^2 & M^2-m^2 \\ 2(M+m) & M-m \end{bmatrix} \begin{bmatrix} G_1' \\ G_2' \end{bmatrix}. \tag{3.5}$$

for the normal-parity case $\frac{1}{2}^+ \to \frac{1}{2}^+$.

As one can easily convince oneself, the constraint structure of the multipole representations in Eqs. (3.1)–(3.5) is identical to the constraint structure of the corresponding multipole representations given in Ref. 14. Of course, if an effective description of form-factor data is attempted in terms of constraint-free form factors by using a definite type of parametrization, say by a number of poles in the timelike region, then the choice of representation may in general affect the resulting fit. The falling-type form factors in Ref. 14 would in general not lead to the same behavior for the constraint-free form factors used in Eqs. (3.1)–(3.5).

This point is very well illustrated by considering the set of form factors $F_i(q^2)$ introduced in Ref. 15, which are related to the $G_i(q^2)$ in the following manner:

$$\begin{aligned} F_1 &= \frac{1}{M+m}(\tfrac{1}{2}G_2 + G_3), \\ F_2 &= \tfrac{1}{2}(M-m)G_2, \\ F_3 &= G_1 + \frac{1}{2(M+m)}(\sigma+q^2)G_2 + \frac{1}{M+m}q^2 G_3. \end{aligned} \tag{3.6}$$

Since the determinant of the transformation connecting the two sets is q^2-independent the set $F_i(q^2)$ is constraint-free as well and should be *a priori* as good a choice as the set $G_i(q^2)$. From Eq. (3.6) it is obvious, though, that a falling type of behavior for $G_i(q^2)$ would not in general lead to a falling-type behavior of the $F_i(q^2)$.

If a sufficiently flexible falling-type parametrization of different choices of sets of constraint-free form factor is used, one may hope that one or the other choice will not significantly prejudice the final form of the q^2 dependence of the multipoles that one is attempting to fit. As we hope to demonstrate in Sec. IV, though, the choice of constraint-free form factors used in Eqs. (3.1)–(3.5) may be a preferred one. Thus we shall propose that the representations of Eqs. (3.1)–(3.5) should be used for a description of the q^2 dependence of the multipole form factors, with an effective parametrization of the constraint-free form factors G_i, G_i' in (3.1)–(3.5) in terms of a number of poles in the timelike region. In order to obtain an asymptotic suppression of the scalar contribution one would require G_3 and G_3' to fall off one power faster than G_1 and G_2 and G_1' and G_2', respectively (see Sec. II D). For the exceptional cases the scalar contribution is automatically suppressed if the same power behavior is used for both constraint-free form factors. The number of poles required in the G_i is determined by the spin J of the isobar and should be chosen such that the transverse parts E and M fall at least as fast as the canonical dipole form, i.e., $E, M \sim (q^2)^{-3/2}$.

B. Possible simplifications

The parametrization of physical form factors as proposed in Sec. III A will eventually have to be tested for its suitability as an effective description of the form-factor data, especially with regard to extrapolations from the spacelike to the timelike regions. Here we propose some further simplifications starting from the assumptions made in Sec. III A which allow a direct test to be made with the form-factor data now available:

(i) The constraint-free form factors $G_i(q^2)$ have a common q^2 dependence, i.e., $G_i(q^2) = g_i F_c(q^2)$ [we normalize $F_c(0)=1$].

(ii) Asymptotic dominance of the transverse form factors, which implies, through proposal (i), the relation $g_3=0$ and $g_3'=0$. In the exceptional cases transverse dominance is automatic and no relation is obtained.

(iii) The common form factor has the form

$$\begin{aligned} F_c^{(l)}(q^2) &= \prod_{n=0}^{l+c-1}\left(1-\frac{q^2}{m_\rho^2+ns_0}\right)^{-1} \quad (l\neq 0), \\ F_c^{(0)}(q^2) &= \prod_{n=0}^{c}\left(1-\frac{q^2}{m_\rho^2+ns_0}\right)^{-1} \quad (l=0), \end{aligned} \tag{3.7}$$

where s_0 is the inverse universal Regge slope ($s_0 = 1/\alpha'$) and c controls the asymptotic q^2 dependence of the spin-averaged cross section (see Sec. III D) $\sigma \sim (q^{-2})^{2c-1}$ ($c=2$ corresponds to canonical "dipole" behavior). At fixed W, the form factors (3.7) lead to a threshold behavior $\nu W_2 \to (\omega-1)^{2c-1}$ for the deep-inelastic structure function νW_2.

Note that proposal (i) puts all sets of form factors that are related to the set G_i by a q^2-independent transformation on the same footing, and, even if it is a very strong assumption, eliminates some of the ambiguity in selecting a set of constraint-free form factors. The remaining ambiguity can be completely removed if one further imposes the requirement that the constraint-free form factors be minimal in the following sense: The determinant of the transformation matrix relating the physical to the constraint-free form factors must be a cubic equation in q^2 with zeros at threshold and pseudothreshold. A minimal way to attain this is to allow the coefficients of the transformation matrix to be of first order in q^2 only. The set F_i in Eq. (3.6) is not minimal in this sense. The set G_i or any set related to it by a q^2-independent transformation are minimal and, if proposal (i) holds true, are all equivalent.

A second motivation for proposal (i) can be given by noting that a common q^2 behavior of the Dennery electroproduction amplitudes A_1, A_2, and A_5 leads, via factorization, to a common q^2 behavior of the set G_i [see Eq. (2.35)]. Such a common q^2 dependence of the Dennery amplitudes with the additional constraints

$$A_5 = \tfrac{1}{2}A_2, \qquad A_6 = \tfrac{1}{2}(A_3 - A_4) \tag{3.8}$$

has been postulated by Cho and Sakurai[16] as a convenient basis for the formulation of the vector-dominance model in terms of s-channel helicity amplitudes. The starting point of Ref. 16 was to demand common q^2 behavior of the non-gauge-invariant Ball amplitudes, which then through the gauge-invariance constraints, leads to a common q^2 behavior of the Dennery amplitudes with the constraints (3.8). As can be seen from Eq. (2.35), the constraints (3.8) lead via factorization to the condition

$$G_2 + 2G_3 = 0 , \tag{3.9}$$

which is, at the three-point level, equivalent to demanding a common q^2 behavior for the non-gauge-invariant minimal set B_i as can be seen from Eqs. (2.4) and (2.7). Together with proposal (ii) the above constraint leads to

$$g_2 = g_3 = 0 , \tag{3.10}$$

which we shall refer to as the Cho-Sakurai condition. One should note also that the Cho-Sakurai constraints Eq. (3.8) leading to $G_2 = -2G_3$ are necessary for the above factorization arguments to work in a consistent manner as is apparent from the parity constraints Eq. (2.36).

The Cho-Sakurai condition in the form (3.9) can be seen to lead to asymptotic dominance of the helicity-$\frac{1}{2}$ excitation for both normal- and abnormal-parity transitions [see Eq. (2.65) and (3.3)], i.e.,

$$A = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \xrightarrow[-q^2\to\infty]{} 1 , \tag{3.11}$$

which, via Bloom-Gilman duality, would lead to large positive asymmetries in the deep-inelastic scattering region, which seems to be favored by quark-parton-model considerations.[17,18]

For the exceptional cases the Cho-Sakurai hypothesis leads to

$$G_1 = 0 . \tag{3.12}$$

As one can see from Eq. (3.4) the vanishing of G_1 would be in contradiction to proposal (ii), namely if $G_1 = 0$ holds then the longitudinal coupling would dominate asymptotically. It remains to be seen, which, if either, of the two hypotheses is operative.

Concerning proposal (iii) we note that the form-factor behavior Eq. (3.7) obtains for the leading resonances via factorization from dual current models describing pion electroproduction[19,20] or forward Compton scattering[21] involving dual B_5 and B_6 functions of the type originally proposed by Sugawara, Ohba, Ademollo, and Del Guidice.[3] The dual current B_6 representation of forward Compton scattering has been shown to satisfy Bloom-Gilman duality, and thus the above-mentioned correlation between the spin of an excited resonance and the asymptotic power of its form factor is a necessary consequence of such models. The form-factor behavior arises through a Veneziano spectrum of vector-meson recurrences $\rho, \rho', \rho'', \ldots$ coupling with alternating sign of residues that terminate at the $(l + c - 1)$th recurrence, which then leads to the form-factor behavior Eq. (3.7). Note that such a picture is in agreement with the experimental observation that higher vector-meson recurrences tend to decouple from low-spin mesons. In Ref. 19 it was shown that the elastic nucleon form factors can be quite well described by form factors of the form (3.7).

The set of proposals (i), (ii), and (iii) may turn out to be far too restrictive to account for detailed aspects of future transition-form-factor data. We have written down these proposals in the spirit of hypotheses about the gross features of form-factor data which will be easy to test due to the small number of parameters that are involved. It is gratifying that the data obtained through the analysis of Ref. 14 seem to be amenable to such simple parametrization, at least for the leading resonances, as will be shown in the next section.

IV. COMPARISON WITH DATA

We will first compare the simple parametrization of the nonexceptional cases discussed in Sec.

III B for the resonances P_{33}, D_{13}, F_{15}, and F_{37} excited off protons using the results of a recent analysis of resonance electroproduction by Devenish and Lyth (DL) (see Ref. 14).

Given the function $F_c^{(1)}(q^2)$ [Eq. (3.7)] the two remaining parameters g_1 and g_2 of Sec. III B can be fixed in terms of the values of the transverse multipoles E and M at the photoproduction point $q^2=0$. Throughout this section the results have been normalized to the channel $\gamma p \to \pi^0 p$ using the results of the multipole analysis of Devenish, Lyth, and Rankin.[22]

The q^2 dependence of the P_{33} form factor is well known through an extensive series of total-cross-section and coincidence measurements over the last few years. The situation has been nicely summarized by Gayler.[23] If the canonical choice is made for the form factor $F_c^{(1)}(q^2)$ with $c=2$, $m_\rho^2=0.593\ \text{GeV}^2$, and $\alpha'=1.0\ \text{GeV}^{-2}$, then the decrease of M_{1+} with increasing q^2 is too slow com-

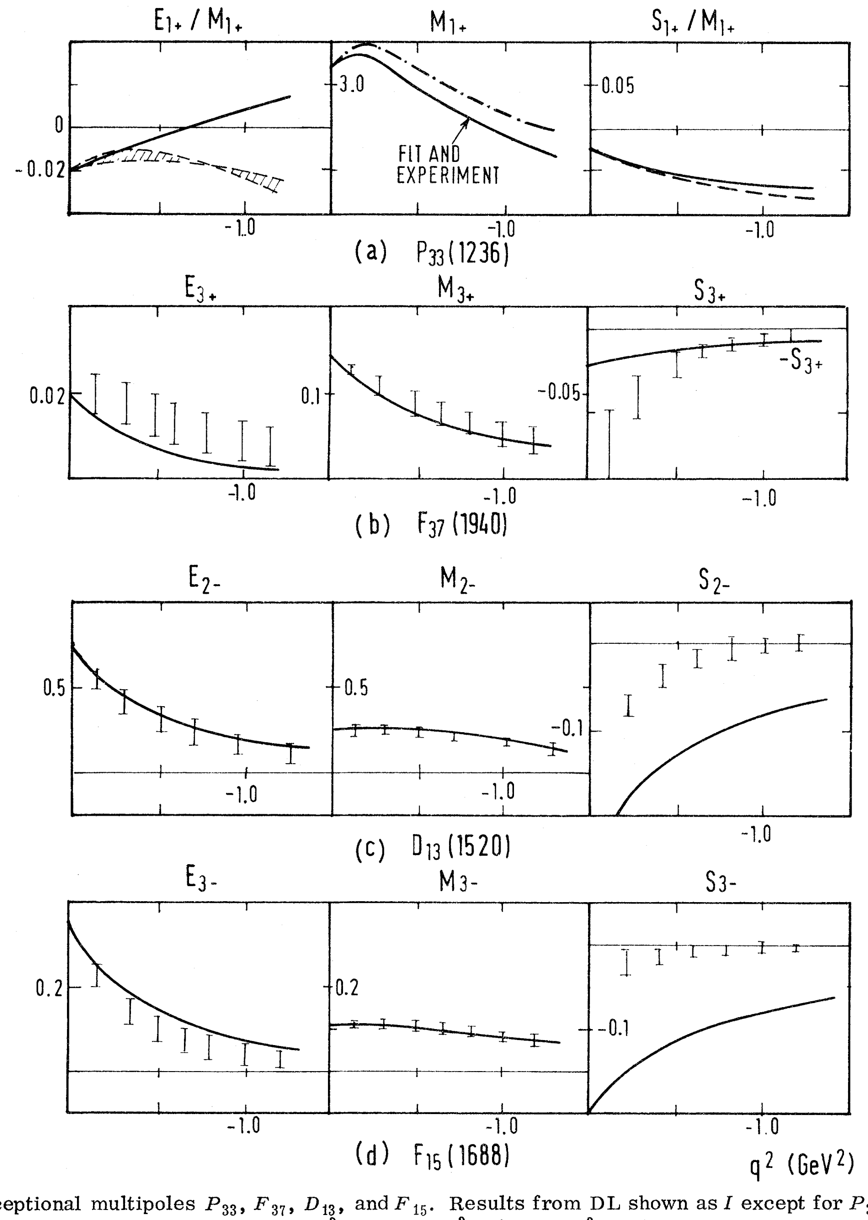

FIG. 3. Nonexceptional multipoles P_{33}, F_{37}, D_{13}, and F_{15}. Results from DL shown as I except for P_{33} (dashed line). Results of our fit are given by full line, using $m_\rho^2=0.593\ \text{GeV}^2$, $\alpha'=1\ \text{GeV}^2$, and $c=3$ for P_{33} and F_{37}, $c=2$ for D_{13} and F_{15}. For M_{1+} dash-dotted line gives our result for $c=2$. For $c=3$ our predictions are not discernible from the DL fit. For S_{3+} (F_{37}) the full line shows $-S_{3+}$. The prediction has opposite sign to that found in the fit.

pared to the data [see Fig. 3(a)]. In Fig. 4 we have plotted the ratio

$$R(q^2) = G_M^{(1)}(q^2)/G_M^{(1)}(0)G_D(q^2), \qquad (3.13)$$

where $G_D(q^2)$ is the usual dipole form factor with dipole mass 0.71 GeV^2. The experimental band given by the shaded area is taken from Gayler's review Ref. 23. Our prediction can be calculated according to Eq. (2.11) with the same normalization of E_{1+}/M_{1+} at $q^2=0$ as in Fig. 3(a) (dashed line) and using $(c; m_\rho{}^2; \alpha') = (3; 0.593; 1)$. The calculated curve falls nicely into the experimental band. A very simple formula for the above ratio (3.13) can be obtained if one makes the approximation $E_{1+}(0)/M_{1+}(0) = 0$. In this case one obtains

$$R(q^2) = [1 - q^2(M+m)^{-2}][1 - q^2(0.71)^{-1}]^2 \times F_c^{(1)}(q^2). \qquad (3.14)$$

The small deviation from the exact $R(q^2)$ can be seen in Fig. 4 where we have drawn also $R(q^2)$ according to Eq. (3.14) with the same set of parameter values (3; 0.593; 1). The other two curves with parameter values (2; 0.593; 1.9) and (2; 0.5; 1) have also been drawn according to the simplified formula Eq. (3.14).

Again a better fit to the data is obtained with $c=3$. For the canonical value $c=2$ a fit within the experimental band can only be obtained for a rather large effective Regge slope of $\alpha' \approx 1.9$. Changing the effective ρ mass to smaller values increases the negative slope of R at the origin but is less effective in bringing R down at larger values of q^2. If the experimental ρ mass and the canonical Regge slope $\alpha' \sim 1$ is used with $c=2$, the fit is not acceptable. Since we were not able to fit this curve on the graph, we plotted the corresponding curve with a reduced ρ mass.

An extra pole in the form factor compared to the "dipole" form $c=2$ will mean that the P_{33} contribution to the total cross section will fall like q^{-10} rather than q^{-6}. That the P_{33} has a faster falloff with q^2 than the other peaks seen in the e-p total cross section has been remarked on by a number of authors, for example the work of Bloom and Gilman.[24]

Recently this observation has led some authors to speculate on the possibility that such a behavior indicates the presence of non-negligible SU(6)-breaking effects in the quark-parton model approach. This has led to some interesting theoretical results for inclusive and exclusive electroproduction[18,25] and for fixed-angle scattering cross sections.[26] DL (Ref. 14) also found that the extra pole gave a better fit to the first-resonance-region data, even though the mass parameters in the function $F_c^{(1)}(q^2)$ were allowed to vary.

The q^2 dependence of the ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} are also shown in Fig. 3(a). The scalar

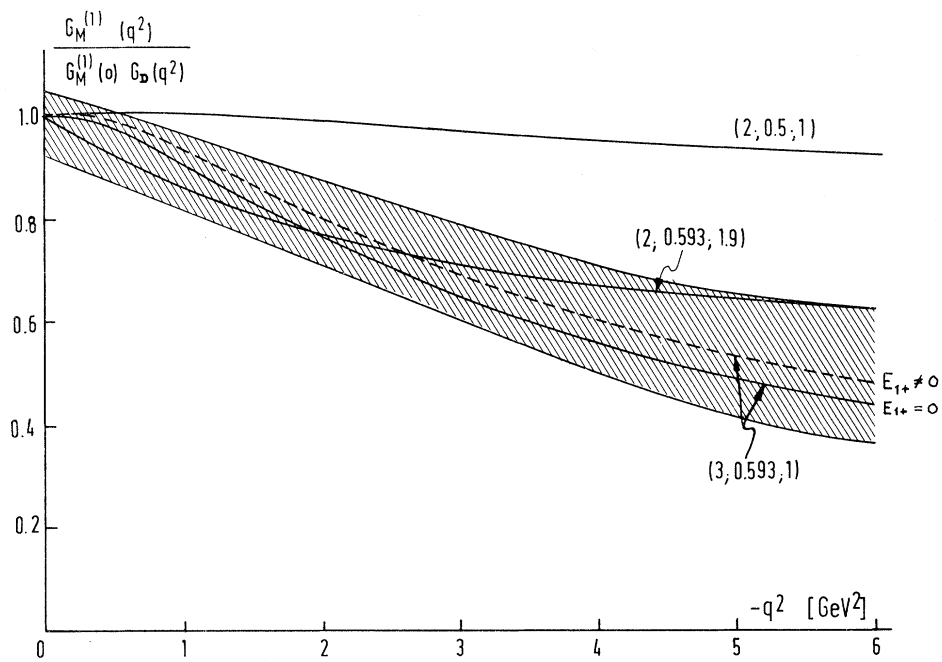

FIG. 4. Normalized ratio $G_M^{(1)}(q^2)/G_M^{(1)}(0)\, G_D(q^2)$. Experimental band taken from review of Gayler (see Ref. 23). Full line: Our results for various sets of values $(c; m_\rho{}^2; \alpha')$ using approximation $E_{1+}=0$. Dashed line: Our prediction for (3; 0.593; 1).

contribution is quite well reproduced. For the E_{1+} we predict a zero at $q^2=-(M^2-m^2)=-0.65\ \mathrm{GeV}^2$ which does not appear in the analysis of DL. The E_{1+} is, however, experimentally so small that the data analysis does not allow one to exclude this possibility.

The comparisons for the D_{13} and F_{15} form factors using the canonical $F_2^{(1)}(q^2)$ and $F_2^{(2)}(q^2)$ in each case are shown in Figs. 3(c) and 3(d). The agreement for the transverse multipoles is very good. [Note that the absence of error bars for $q^2=0$ in Fig. 3 is a result of the fit procedure used by DL (Ref. 14); the $q^2=0$ multipoles were fixed at the best values of Ref. 22 and only the $q^2\neq 0$ values were allowed to vary.] In both cases, however, the scalar multipoles, though having the correct sign, are somewhat too large. At this point it must be remarked that there are no direct measurements of the scalar contribution to pion electroproduction outside the first resonance region. [In a contribution to the SLAC Conference, 1975, the DESY electroproduction group[27] have been able to separate the scalar/transverse interference term in the region of interest. It is small and in agreement with the results of the analysis of DL which used their preliminary unseparated data.] Note also that the small size of the scalar contribution arises from a near cancellation of the G_1' and G_2' contributions. This cancellation is quite sensitive to the exact input ratio $E_{l+1,-}/M_{l+1,-}$ at $q^2=0$. The effect of varying the input ratio $E_{l+1,-}/M_{l+1,-}$ on the size of $S_{l+1,-}$ can be seen by noting that $S_{l+1,-}$ is proportional to $(1+r')$, where r' is the coupling ratio defined in (3.15). This coupling ratio is in turn determined by $E_{l+1,-}/M_{l+1,-}$ at $q^2=0$. In Eq. (3.16) we have calculated r' from numbers quoted in various photoproduction fits (see Refs. 22, 28, and 29). In particular in the case of the F_{15} the value of $(1+r')$ and thereby S_{3-} may go down by a factor of 3, depending on what fit is taken. Further analysis on this point is needed.

Although the qualitative behaviors of the multipoles in the abnormal- and normal-parity cases are quite different and seem unrelated, there is a close similarity between the two cases if analyzed in terms of the constraint-free couplings.

We define the relevant ratio of coupling strengths as

$$r=Mg_2/g_1$$

and (3.15)

$$r'=-Mg_2'/g_1',$$

and calculate these for the three best established resonances using the results of three recent $q^2=0$ multipole analyses.[22,28,29] One obtains

$$r'_{(D_{13})}=\begin{cases}-0.79\pm0.11 & \text{MW (Ref. 28)},\\ -0.84\pm0.36 & \text{KMORR (Ref. 29)},\\ -0.80\pm0.28 & \text{DLR (Ref. 22)},\end{cases}\tag{3.16}$$

$$r'_{(F_{15})}=\begin{cases}-0.76\pm0.21 & \text{MW (Ref. 28)},\\ -0.87\pm0.17 & \text{KMORR (Ref. 29)},\\ -0.54\pm0.36 & \text{DLR (Ref. 22)},\end{cases}\tag{3.17}$$

and

$$r_{(P_{33})}=\begin{cases}-0.83\pm0.26 & \text{MW (Ref. 28)},\\ -0.86\pm0.10 & \text{KMORR (Ref. 29)},\\ -0.74\pm0.22 & \text{DLR (Refs. 22, 30)}.\end{cases}\tag{3.18}$$

With the large theoretical uncertainty going into a multipole analysis one should avoid a literal interpretation of the quoted errors and rather take the mean of the above 3 respective values of r and r' as a basis of estimate.

The coupling ratios are close to -1 in all three cases resulting in the near vanishing of E_{1+}, S_{1+}, and S_{2-}. In fact the qualitative behavior of the multipoles can be easily reconstructed from Eqs. (3.1) and (3.2) with the coupling ratios (3.16)–(3.18) (after setting $G_3=G_3'=0$).

From the similarity of the coupling ratios (3.16)–(3.18) one expects also a similarity in the helicity $\frac{1}{2}-\frac{3}{2}$ asymmetry A [see Eq. (3.11)] for large q^2 values, since

$$\frac{\sigma_{1/2}}{\sigma_{3/2}}\xrightarrow[-q^2\to\infty]{}\frac{l}{l+2}\left(\frac{r+2}{r}\right)^2\tag{3.19}$$

for both abnormal- and normal-parity transitions (replacing $r\to r'$).

In Fig. 5 we have plotted our predictions for the asymmetry of P_{33}, D_{13}, and F_{15} using the DLR (Ref. 22) values as input. In the abnormal-parity case the q^2 dependence is much smoother and the asymptotic value approached slower because the scaling mass is $(M+m)^2$ instead of the $(M-m)^2$ in the normal-parity case. The asymptotic values of P_{33} and D_{13} differ significantly from the values

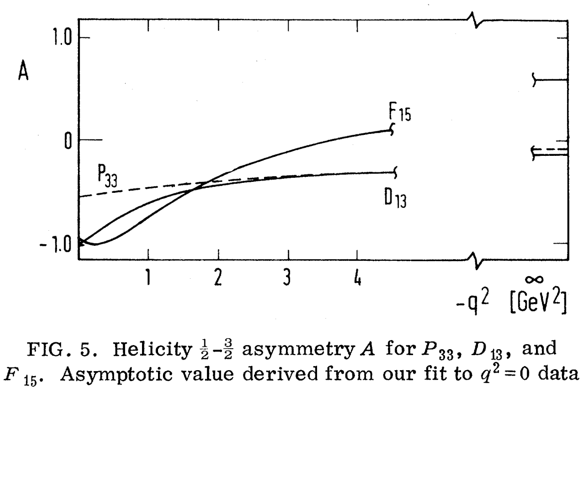

FIG. 5. Helicity $\frac{1}{2}$-$\frac{3}{2}$ asymmetry A for P_{33}, D_{13}, and F_{15}. Asymptotic value derived from our fit to $q^2=0$ data.

$$A = \mp \frac{2}{2J+1}, \quad (3.20)$$

valid for magnetic dominance [Eq. (3.20) is good for all values of q^2 for the abnormal-parity case, but only for asymptotic q^2 in the normal-parity case], whereas for F_{15} the asymptotic asymmetry ($A \sim 0.5$) is slightly above the magnetic dominance value Eq. (3.20).

Of course the sensitivity of $r(r')$ on the $q^2=0$ input values of the ill-determined ratios E/M or A/B renders the extrapolation of the asymmetry A to large q^2 values unreliable at present. However, if these input ratios and thereby r could be estimated more reliably, the resulting asymmetries could lead to some interesting comparisons with quark-parton-model results for the corresponding asymmetry in the deep-inelastic scattering region using Bloom-Gilman duality (see e.g. Refs. 17 and 18).

The above coupling ratios r also give a good measure of how well the Cho-Sakurai hypothesis is realized at the three-point level. As discussed in Sec. III B, the Cho-Sakurai hypothesis together with the assumption of an asymptotic suppression of the scalar contribution predicts zero for the ratios Eq. (3.15). Unless the assumption of scalar suppression is dropped, one must conclude from the nonzero values of r in Eqs. (3.16)–(3.18) that the Cho-Sakurai hypothesis is not realized at the three-point level. This can also be immediately appreciated from the asymptotic values of the asymmetry A which are approximately zero compared to $A=1$ resulting from the Cho-Sakurai hypothesis [see Eq. (3.11)].

The results for the F_{37} are shown in Fig. 3(b). Since the F_{37} is a Regge recurrence of the P_{33} we have again used a form factor $F_3^{(3)}(q^2)$ falling one power faster than that corresponding to the canonical dipole form. Again the results for the transverse multipoles are in agreement with the data, but the scalar multipole has the opposite sign. However, the details of the analysis in the fourth resonance region cannot be considered reliable as the data are very limited and only gross trends should be noted. In particular the relative sign of E_{3+} and S_{3+} is fixed in our simple parametrization, and thus an error in the determination of the sign of the small amplitude E_{3+} would propagate to give a wrong sign for the scalar amplitude S_{3+}. It would be quite surprising if the relative phases of E_{3+} and S_{3+} compared to M_{3+} would be different from those of the P_{33}, since the F_{37} is a Regge recurrence of the P_{33}.

For the two exceptional cases S_{11} and P_{11} there is again some information in the analysis of DL. We are now faced with a problem, namely that photoproduction can only supply one number for each case but two parameters are still required even for the simple parametrization. A very crude estimate of the behavior has been obtained by determining the parameters from the values of the transverse multipoles at two values of $-q^2$ (0 and 1 GeV^2). From this it is clear that the simple parametrization of the exceptional cases will not be in agreement with the results of DL, if the conventional α' and the experimental m_ρ^2 are used in $F_c^{(0)}(q^2)$.

We will first discuss the S_{11}. Measurements of the total $ep \to ep\eta$ cross section near threshold[31] show that the S_{11} form factor must fall rather slowly with q^2. From these experiments it is not possible to separate the contributions of σ_S and σ_T. In DL the scalar contribution was found to be quite large for small q^2. As can be seen from Eq. (3.4) if the parameters are adjusted to give a slow decrease with q^2 (i.e., g_1/g_2 large and negative), then in the expression for the scalar term the two contributions will tend to cancel out. This problem could be overcome to some extent if the mass parameters m_ρ^2 and $(\alpha')^{-1}$ in $F_c^{(0)}(q^2)$ would be increased.

The P_{11} shows the opposite behavior. There is good evidence for its presence in photoproduction but it seems to be effectively absent in electroproduction. With the standard $F_c^{(0)}(q^2)$, to accommodate such a rapid change, it is necessary to choose g_1'/g_2' large and positive. Now the terms in the scalar multipole will tend to add in contradistinction to the above result. Again this would be allowed for by changing the mass parameters in $F_c^{(0)}(q^2)$.

In particular in the case of the exceptional transitions the parametrization used in DL (Ref. 14) may turn out not to be sufficiently flexible in that the transverse multipoles have been parametrized as decreasing functions, whereas Eqs. (3.4) and (3.5) show that this choice is not natural. The rather flat behavior of E_{0+} for the $S_{11}(1505)$ in the fit of DL (Ref. 14) indicates that the parametrization may have prejudiced the results in this case. Further analysis on this problem is needed.

V. SUMMARY AND CONCLUSION

As a basis for understanding the phenomenology of transition form factors we have proposed a parametrization scheme that incorporates two necessary minimal theoretical requirements: (i) the correct threshold and pseudothreshold constraint structure, and (ii) dynamic damping of J-dependent q^2 powers arising from kinematics.

We further proposed that for a given isobar excitation, certain suitably chosen constraint-free form factors show a common q^2 dependence and that the scalar contributions are suppressed asymptotically. This led to a representation of the three form factors involved in each $J \geq \frac{3}{2}$ transition in terms of two parameters only.

For the three leading resonances P_{33}, D_{13}, and F_{15} such a simple parametrization was shown to account quite well for the q^2 dependence of the transverse multipole form factors with some possible difficulties remaining for the scalar contributions in the case of the D_{13} and F_{15} which need further analysis. The present evidence for the q^2 dependence of the multipole form factors S_{11} and P_{11} indicates that these may have a more complicated q^2 dependence.

We have demonstrated the advantages of using a description of the form-factor data for the P_{33}, D_{13}, and F_{15} in terms of constraint-free form factors. Using only one coupling ratio for each resonance the qualitative behavior of the three multipoles as regarding their relative size and phases and the presence or absence of zeros can be easily reconstructed. The results of three different multipole analyses suggests a near equality of these three coupling ratios. In this case one would expect also a near equality of their $\frac{1}{2}$-$\frac{3}{2}$ asymmetries for large q^2 values.

The success in fitting the P_{33}, D_{13}, and F_{15} form-factor data gives some support to the underlying theoretical ideas. Namely, the constraint-free form factors seem to exhibit a global universal q^2-behavior, which is of the form predicted by the dual current model with form factors of the generalized-vector-dominance-model type[32]. On the basis of this evidence we concluded that the N-Δ form factor falls quicker than expected from a canonical dipole behavior.

The phenomenological evidence for structural simplicity of leading-resonance excitation in terms of relativistic three-point invariants warrants further theoretical study using fully relativistic models of the nucleon as a bound state. In particular one would hope that the assumption of the asymptotic suppression or the vanishing of the invariant coupling G_3 could be put on a firmer theoretical basis. Also one would want to understand the approximate equality of coupling ratios MG_2/G_1 and $-MG_2'/G_1'$ for abnormal- and normal-parity transitions.

Compared to the quark-model prediction for the q^2 dependence of the ratio of helicity-$\frac{1}{2}$ and -$\frac{3}{2}$ couplings of the D_{13} and F_{15} the analysis of DL indicates a much less rapid change than predicted by the quark model. Our analysis indicates that the observed slow change is to a large extent due to the underlying relativistic kinematic-constraint structure. This would suggest that the essentially nonrelativistic quark-model results could be much improved if care is taken to incorporate the correct relativistic constraint structure.

We have not discussed the question of the natural scale in the coupling strength of different isobars lying on the same exchange-degenerate trajectory, as e.g. D_{13} and F_{15}, or P_{33} and F_{37}. This has to be discussed in explicit dual models, as for example in Refs. 13 and 19.

More and better data at possibly higher q^2 values and more extensive analyses of these data expected in the near future will show whether the ideas expressed in this paper and the preliminary evidence presented for them will hold up. In particular this applies to the timelike region, where the proposed dual-current-model form-factor behavior is also expected to hold. Future e^+e^- experiments will be able to test the proposed structure over a much wider range of q^2 values.

ACKNOWLEDGMENTS

One of the authors (J.G.K.) would like to thank Dr. F. Gutbrod for several helpful discussions. (T.S.E.) wants to express her gratitude to Professor H. Schopper, Professor G. Weber, Professor K. Symanzik, and Professor H. Joos for their hospitality at DESY.

APPENDIX

The relation between helicity and multipole amplitudes is given by

$$A_{l+} = \tfrac{1}{2}lM_{l+} + \tfrac{1}{2}(l+2)E_{l+}, \quad B_{l+} = -M_{l+} + E_{l+}, \tag{A1}$$

$$A_{l+1,-} = \tfrac{1}{2}(l+2)M_{l+1,-} - \tfrac{1}{2}lE_{l+1,-}, \quad B_{l+1,-} = M_{l+1,-} + E_{l+1,-}, \tag{A2}$$

and the inverse

$$M_{l+} = \frac{1}{2(l+1)}[2A_{l+} - (l+2)B_{l+}], \quad E_{l+} = \frac{1}{2(l+1)}(2A_{l+} + lB_{l+}), \tag{A3}$$

$$M_{l+1,-} = \frac{1}{2(l+1)}(2A_{l+1,-} + lB_{l+1,-}), \quad E_{l+1,-} = \frac{1}{2(l+1)}[-2A_{l+1,-} + (l+2)B_{l+1,-}]. \tag{A4}$$

One defines scalar amplitudes C_{l+} and $C_{l+1,-}$ by

$$C_{l+} = \sqrt{-q^2}(l+1)S_{l+}/q_c, \quad C_{l+1,-} = -\sqrt{-q^2}\,(l+1)S_{l+1,-}/q_c. \tag{A5}$$

Current conservation implies

$$q_{c0}S_{l+} = q_cL_{l+}$$

and (A6)

$$q_{c0}S_{l+1,-}=q_c L_{l+1,-}$$

Under the replacement $M \leftrightarrow -M$ McDowell symmetry implies

$$A_{l+} \leftrightarrow -A_{l+1,-},$$
$$B_{l+} \leftrightarrow B_{l+1,-}, \quad \text{(A7)}$$
$$C_{l+} \leftrightarrow -C_{l+1,-},$$

which gives for the multipoles

$$M_{l+} \leftrightarrow \frac{1}{l+1}[-(l+2)M_{l+1,-}-E_{l+1,-}],$$
$$E_{l+} \leftrightarrow \frac{1}{l+1}(-M_{l+1,-}+lE_{l+1,-}), \quad \text{(A8)}$$

$$M_{l+1,-} \leftrightarrow \frac{1}{l+1}(-lM_{l+}-E_{l+}),$$
$$E_{l+1,-} \leftrightarrow \frac{1}{l+1}[-M_{l+}+(l+2)E_{l+}]. \quad \text{(A9)}$$

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*Work supported by the Royal Society European Fellowship Program.

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