CP violation in the six-quark model*

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(Received 29 September 1975)

We construct a Weinberg-Salam-type gauge theory of a weak interaction with CP violation based on the six-quark model. Under the assumption of the validity of the Zweig-Iizuka rule and (quark mass/W-meson mass)$^2 \ll 1$ this leads to the superweak theory of CP violation for both uncharmed and charmed hadrons. We also propose a new assignment for the J and other ψ particles, which predicts the existence of a 3.5-GeV 0^- meson using the 2.85-GeV 0^- state as input.

Recently the six-quark model was considered by many people[1] as a candidate to accommodate the newly discovered narrow resonances. The model was also much discussed in connection with the possible existence of $V+A$ currents in the weak interaction, although its *raison d'être* is still very uncertain. In this note we restrict ourselves to the traditional $V-A$ currents for weak interactions and look for a possibility of constructing an acceptable CP-violating Hamiltonian. We also propose a new classification scheme for the J and other ψ particles.

SIX-QUARK MODEL

We assume the existence of six quarks $\mathcal{P}$, $\mathcal{N}$, λ, $\mathcal{P}'$, $\mathcal{N}'$, and $\mathcal{P}''$ with electric charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, $\frac{2}{3}$, $-\frac{1}{3}$, and $\frac{2}{3}$, respectively. The highest symmetry group that we consider [aside from the color SU(3)] is SU(6) with the generators λ_i, h_α, and $h_\alpha\lambda_i$, where λ_i are Gell-Mann matrices and h_α are Pauli spin matrices. The ordinary quarks $\mathcal{P}$, $\mathcal{N}$, and λ are assumed to have $h_3=-\frac{1}{2}$, and $\mathcal{P}'$, $\mathcal{N}'$, and $\mathcal{P}''$ have $h_3=\frac{1}{2}$. Ordinary SU(3) corresponds to $\lambda_i^{(-)}=(\frac{1}{2}-h_3)\lambda_i$. SU(3) transformations among $\mathcal{P}'$, $\mathcal{N}'$, and $\mathcal{P}''$ are caused by $\lambda_i^{(+)}=(\frac{1}{2}+h_3)\lambda_i$. The Gell-Mann–Nishijima formula takes the form

$$Q=\tfrac{1}{2}Y^{(-)}+I_3^{(-)}-\tfrac{1}{2}Y^{(+)}+I_3^{(+)}+\tfrac{1}{3}h_3+\tfrac{1}{2}B\,. \qquad (1)$$

WEAK-INTERACTION MODEL

A few years ago Kobayashi and Maskawa[2] pointed out that CP violation can be incorporated into the standard $V-A$ Weinberg-Salam[3] model if we increase the number of quarks from four to six. Let us repeat their argument. The charged weak current in the Weinberg-Salam model with the Glashow-Iliopoulos-Maiani (GIM) mechanism[4] can be written in the following way:

$$J_\mu=(\overline{\mathfrak{P}}\gamma_\mu(1+\gamma_5)U\mathfrak{N})_\mu\,, \qquad (2)$$

where $\mathfrak{P}$ stands for $(\mathcal{P},\mathcal{P}')$ and $\mathfrak{N}$ for $(\mathcal{N},\lambda)$ in the case of SU(4) and $\mathfrak{P}$ stands for $(\mathcal{P},\mathcal{P}',\mathcal{P}'')$ and $\mathfrak{N}$ for $(\mathcal{N},\lambda,\mathcal{N}')$ in the case of SU(6). U is a unitary matrix in general. In case of four quarks U has four parameters but three of them can be absorbed into the phase of the wave functions $(\mathcal{P},\mathcal{P}')$ and $(\mathcal{N},\lambda)$ [note that the overall phase transformation of left quarks $(\mathcal{N},\lambda)$ is equivalent to that of the right quarks $(\mathcal{P},\mathcal{P}')$]. We are thus left with only one parameter (Cabibbo angle) and U becomes orthogonal. In case of six quarks, however, we are left with four parameters and we cannot make U orthogonal in general. On this basis Kobayashi and Maskawa[2] propose the following form for U:

$$U=\begin{pmatrix}\cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3\\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta}\\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\cos\theta_3 e^{i\delta}\end{pmatrix}. \qquad (3)$$

SUPERWEAK THEORY OF CP VIOLATION

We now want to prove that the model leads to an approximate superweak theory of CP violation in the case of ordinary particles. For this purpose we first observe that the first-order effect coming from the diagram shown in Fig. 1(a) is strongly suppressed owing to the Zweig-Iizuka rule. In Fig. 1(a) $\mathcal{P}'$ or $\mathcal{P}''$ must annihilate to give a state of ordinary particles leading to the Zweig-Iizuka suppression. We also note that the $\mathcal{P}'$ and $\mathcal{P}''$ contributions cancel each other in the $m_{\mathcal{P}'}=m_{\mathcal{P}''}$

limit. Thus the ratio of the CP-violating amplitude to the CP-conserving amplitude is proportional to

$$\alpha\beta \cos\theta_1 \sin\theta_2 \cos\theta_2 \tan\theta_3 \sin\delta\ , \tag{4}$$

where α is a constant less than 1 and proportional to $(m_{\mathcal{P}'} - m_{\mathcal{P}''})/m_p$, and β gives the Zweig-Iizuka suppression factor. Besides the factors $\sin\theta_2 \cos\theta_2$ $\tan\theta_3 \sin\delta$ which appear in any CP-violating term we have the extra factor β. As long as $\beta \ll 1$ we can neglect this term. The only other first-order CP violation comes from the diagram shown in Fig. 1(b).

We explicitly calculated this diagram in the form of the effective Lagrangian up to $(m_q/m_W)^2 \ll 1$ in the limit of $m_q = m_\lambda = m_n$.

$$\mathcal{L}_{\text{eff}} = -\frac{G_F m_\lambda}{(2\pi)^2\sqrt{2}} \times (\text{angle factors}) \times \bar{\lambda}\tilde{D}(1-\gamma_5)D\mathcal{N}\ , \tag{5}$$

where D stands for Dirac operators. Angle factors are

$$\sin\theta_1 \cos\theta_2 (\cos\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 e^{-i\delta}) \quad \text{for } \mathcal{P}'$$

and

$$\sin\theta_1 \sin\theta_2 (\cos\theta_1 \sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3 e^{-i\delta}) \quad \text{for } \mathcal{P}''\ .$$

The crucial point is that this does not depend on the mass of the $m_{\mathcal{P}'}$ or the $m_{\mathcal{P}''}$, in contrast with the case of $V-A$ and $V+A$.[5] The contribution of $\mathcal{P}'$ and $\mathcal{P}''$ to the CP-violating part (imaginary part) exactly cancels each other. Instead of the factor β we have the factor $(m_q/m_W)^2 \approx 10^{-3}$ in this case in Eq. (4).

Having established the smallness of the first-order effect of the CP violation we can now calculate the ϵ parameter in K decay. We calculate the mass matrix in the quark model as shown in Fig. 1(c). The decay matrix is calculated using the 2π dominance, as usual. We obtain

$$2\sqrt{2}\ |\epsilon| \cong \frac{m_{\mathcal{P}'} - m_{\mathcal{P}''}}{m_{\mathcal{P}'}} \frac{1}{\cos\theta_1} \sin(2\theta_2) \tan\theta_3 \sin\delta \tag{6}$$

and the usual unitarity relation

$$\tan\theta_\epsilon = 2\Delta m/\Gamma_S\ , \tag{7}$$

where Δm is the K_S-K_L mass difference and Γ_S is the K_S width. We also obtain an expression for Δm using the vacuum insertion, following Gaillard and Lee.[6] The only difference is that we have $\sin^2\theta_1 \cos^2\theta_1 \cos^2\theta_3$ instead of $\sin^2\theta_C \cos^2\theta_C$ as an angle factor.

The fact that CP violation in the loop diagram [Fig. 1(b)] is suppressed also holds in the case of charmed-particle decays. A choice of angles that is consistent with all known facts is possible. For example: Take $\theta_3 \lesssim 0.1$ so as not to violate the fit[7] to Cabibbo current; then $\theta_2 \gtrsim 80°$ and $30° \lesssim \delta \lesssim 45°$ give an acceptable value for $|\epsilon|$. Such a choice of angles also makes $\mathcal{P}' \to \lambda$ transition purely imaginary and helps in the K/π problem by making $\mathcal{P}' \to \mathcal{N}/\mathcal{P}' \to \lambda$ larger than $\sin\theta_C$ by a factor of $\sqrt{2}$ to 2. *With this choice of angles our model predicts approximate superweak theory for dominant weak decays of charmed mesons.*[8]

ASSIGNMENTS FOR THE J AND OTHER ψ PARTICLES

Our assignments of the new resonances[9] are the following:

$$J/\psi(3.1) = (\bar{\mathcal{P}}'\mathcal{P}' - \bar{\mathcal{N}}'\mathcal{N}')/\sqrt{2}\ ,$$
$$\psi(3.7) = (\bar{\mathcal{P}}'\mathcal{P}' + \bar{\mathcal{N}}'\mathcal{N}' - 2\bar{\mathcal{P}}''\mathcal{P}'')/\sqrt{6}\ , \tag{8}$$

and

$$\psi(4.1) = (\bar{\mathcal{P}}'\mathcal{P}' + \bar{\mathcal{N}}'\mathcal{N}' + \bar{\mathcal{P}}''\mathcal{P}'')/\sqrt{3}\ .$$

This assignment is similar to that of Harari,[1] but there is an essential difference. $\psi(3.1)$ and $\psi(3.7)$ belong to an octet and $\psi(4.1)$ to a singlet. The assignment is more akin to that of Barnett,[10] with the charges of the quarks being different. Also, we do not assume low-lying radial excitations.[11] The widths of $\psi(3.1)$ and $\psi(3.7)$ are doubly sup-

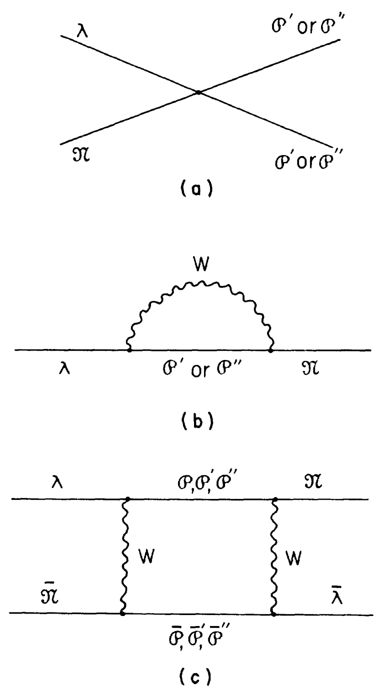

FIG. 1. (a) First-order diagram suppressed by the Zweig-Iizuka rule. (b) λ-$\mathcal{N}$ transition induced in first order. (c) Second-order diagram for ϵ.

pressed because of the Zweig-Iizuka rule and the $SU(3)^{(+)}$ symmetry, whereas the $\psi(4.1)$ is suppresed only the Zweig-Iizuka rule. $\psi(4.1)$ also decays into charmed-meson pairs with no suppression. The leptonic decay widths have the ratio

$$\Gamma_{3.1}^{\bar{\mu}\mu} : \Gamma_{3.7}^{\bar{\mu}\mu} : \Gamma_{4.1}^{\bar{\mu}\mu} = 3:1:2\,, \tag{9}$$

which agrees quite well with the experiment.

MASS FORMULA

For the vector mesons we postulate the following mass formula which makes the above assignment possible:

$$M^2 = M_0{}^2 + \kappa h_3 + a^{(-)} T_8^{(-)} + \xi \lambda_i^{(+)} \lambda_i^{(+)} + a^{(+)} T_8^{(+)} + b^{(+)} T_8'^{(+)} + c^{(+)} T_3''^{(+)}\,, \tag{10}$$

where h_3 and T_i are SU(6) tensors which transform like the generators indicated by each index and suffix. We assume that $[T_8^{(+)}, C_i^{(+)}] = 0$, where $C_i^{(+)}$ are the Casimir operators for $SU(3)^{(+)}$, and $a^{(+)} >> b^{(+)}, c^{(+)}$. The first assumption is necessary to suppress the octet-singlet mixing and the second is necessary to guarantee the small widths for $\psi(3.1)$ and $\psi(3.7)$. The masses and the quark contents of all the other 1^- mesons are summarized in Table I. To obtain these masses $b^{(+)}$ and $c^{(+)}$ terms are neglected and 3.1 GeV, 3.7 GeV, and 4.1 GeV are used as input. The pseudoscalar masses are estimated by simply adding a term $\propto J(J+1)$ to the vector mass formula and by using the 2.85-GeV state as input.

R FACTOR

As one can see from the table the lightest meson which contains a $\mathcal{P}''$ quark is at approximately 3 GeV. Taking into account one heavy lepton, which is needed to make the weak interaction renormalizable, we get the following prediction:

$R = 2$ (below $E \cong 3.5$ GeV) (assuming that the heavy-lepton mass ~ 1.7 GeV),

$R = \frac{14}{3}$ (3.5 GeV $\lesssim E \lesssim$ 7 GeV),

and

$R = 6$ ($E \gtrsim 7$ GeV).

CP-VIOLATING WEAK DECAYS

The only CP-violating decays besides K_S and K_L are those of $(\bar{\mathcal{P}}\mathcal{P}' \pm \bar{\mathcal{P}}'\mathcal{P})/\sqrt{2}$, $(\bar{\mathcal{N}}\mathcal{N}' + \bar{\mathcal{N}}'\mathcal{N})/\sqrt{2}$, $(\bar{\lambda}\mathcal{N}' \pm \bar{\mathcal{N}}'\lambda)/\sqrt{2}$, $(\bar{\mathcal{P}}\mathcal{P}'' \pm \bar{\mathcal{P}}''\mathcal{P})/\sqrt{2}$, and $\bar{\mathcal{P}}'\mathcal{P}'' \pm \bar{\mathcal{P}}''\mathcal{P}')/\sqrt{2}$ mesons with spin parity 0^-. They satisfy the superweak conditions. The last one, which is the only heavy-heavy meson with CP-violating decay amplitudes, decays dominantly into either $\bar{\mathcal{N}}'\lambda(\bar{\lambda}\mathcal{N}')$ + ordinary hadrons or $\bar{\lambda}\mathcal{P}'(\bar{\mathcal{P}}'\lambda)$ + leptons. We have, therefore, a cascade decay of the heavy-heavy and the heavy-light mesons in this case. This can be checked experimentally by an emulsion analysis.

TABLE I. Classification of new mesons. The underlined masses are used as input.

	$SU(3)^{(-)} \otimes SU(3)^{(+)}$	Symbol for 1^-	1^- mass (GeV)	0^- mass (GeV)	
Heavy-heavy (mesons) ($h_3 = 0$)	(1, 8)	J/ψ	$\underline{3.1}$	$\underline{2.85}$	$(\bar{\mathcal{P}}'\mathcal{P}' - \bar{\mathcal{N}}'\mathcal{N}')/\sqrt{2}$
		$J^\pm$	3.1	2.85	$\bar{\mathcal{P}}'\mathcal{N}'$ and $\bar{\mathcal{N}}'\mathcal{P}'$
			3.56	3.34	$\bar{\mathcal{P}}''\mathcal{P}'$, $\bar{\mathcal{P}}''\mathcal{N}'$ $\bar{\mathcal{P}}'\mathcal{P}''$, $\bar{\mathcal{N}}'\mathcal{P}''$
		ψ'	$\underline{3.7}$	3.49	$(\bar{\mathcal{P}}'\mathcal{P}' + \bar{\mathcal{N}}'\mathcal{N}' - 2\bar{\mathcal{P}}''\mathcal{P}'')/\sqrt{6}$
	(1, 1)	ψ''	$\underline{4.1}$	3.91	$(\bar{\mathcal{P}}'\mathcal{P}' + \bar{\mathcal{N}}'\mathcal{N}' + \bar{\mathcal{P}}''\mathcal{P}'')/\sqrt{3}$
Light-heavy mesons ($h_3 = \pm 1$)	(3, 3*) or (3*, 3)		2.14	1.78	$\bar{\mathcal{P}}\mathcal{P}'$, $\bar{\mathcal{P}}'\mathcal{N}'$, $\bar{\mathcal{P}}'\mathcal{P}$, $\bar{\mathcal{N}}'\mathcal{P}'$, $\bar{\mathcal{N}}\mathcal{P}'$, $\bar{\mathcal{N}}\mathcal{N}'$, $\bar{\mathcal{P}}'\mathcal{N}$, $\bar{\mathcal{N}}'\mathcal{N}$
			2.24	1.88	$\bar{\lambda}\mathcal{P}'$, $\bar{\lambda}\mathcal{N}'$, $\bar{\mathcal{P}}'\lambda$, $\bar{\mathcal{N}}'\lambda$
			3.27	3.03	$\bar{\mathcal{P}}\mathcal{P}''$, $\bar{\mathcal{N}}\mathcal{P}''$, $\bar{\mathcal{P}}''\mathcal{P}$, $\bar{\mathcal{P}}''\mathcal{N}$
			3.34	3.11	$\bar{\lambda}\mathcal{P}''$, $\bar{\mathcal{P}}''\lambda$

ACKNOWLEDGMENTS

One of the authors (Hirotaka Sugawara) would like to thank Dr. J. Arafune and Dr. Y. Iwasaki for discussions. He is also grateful to the High Energy Theory Group at the University of Hawaii for its hospitality.

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*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. AT(04-3)-511.

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