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Higher e^+e^- resonances

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Possible e^+e^- resonances are considered as exotic mesons consisting of multiple quark-antiquark pairs in both the SU(4) or charm model and the Han-Nambu SU(3)$'\times$SU(3)$''$ or color model. Both models lead to qualitatively similar conclusions, and indicate the possibility of a sequence of narrow e^+e^- resonances at about 3, 6, 9,... GeV.

Recent measurement[1] with proton-induced reactions at Fermilab showed the $\psi(3.1)$ but not the $\psi(3.7)$ in the production of muon pairs; it also suggested the presence of another narrow state around 6 GeV, which decays into muon pairs. A search with e^+e^- collisions in this region did not reveal[2] any such state, but the present variability of production mechanisms between even the $\psi(3.1)$ and $\psi(3.7)$ is not well understood. It may therefore be worthwhile to speculate on possible structures for such a $\psi(6)$ state. The basic mechanism invoked is the formation of exotic states[3] consisting of multiple quark-antiquark pairs. In Ref. 3 it was concluded that the most likely models for the e^+e^- resonances were SU(4), or "charm", and SU(3)$'$ $\times$SU(3),$''$ or "color." Accordingly, we limit discussion to these models, which lead to qualitatively very similar conclusions.

I. MASS LEVELS

The relevant states must be fairly loose combinations of subunits comprising quarks and antiquarks, so that quark exchange between these units is relatively weak; otherwise the exotic meson will be unstable against rapid decay into a less exotic state. We classify the exotic states as n-fold according to the number of their $(q\bar{q})$ pairs: $(q\bar{q})^n$, with $n=1$ representing nonexotic mesons.

A corollary of loose binding is that the mass of an n-fold exotic state will be roughly the sum of the masses of its $(q\bar{q})$ subunits. This additivity implies that the n-fold systems are widely spread in mass according to their degree of charm or color. Let n_c be the number of charmed or colored subunits, and $n_0=n-n_c$. Then the minimum mass of an n-fold exotic state is

$$m_0(n_0,n_c)\simeq n_0\, m_0(1,0)+n_c\, m_0(0,1)\,,\qquad m_0(1,0)\simeq 0.6\text{ GeV},\quad m_0(0,1)\simeq 3.1\text{ GeV}. \tag{1}$$

Only for $n_0=0$ (and $n_0=1$ in the case of charm) will the exotic state be stable against rapid decay with ordinary hadron emission; the narrow exotic states therefore have minimum masses

$$m_0(n_c)\simeq 3n_c\text{ GeV}. \tag{2}$$

It may make the existence of such states a little more plausible to use a crude model fixed by the Okubo-Zweig-Iizuka (OZI) rule. If one regards the $\phi(1020)$ wave function as having a $K\bar{K}$ component, then the OZI rule is broken through annihilation of the $\bar{\lambda}$ and λ from the K and $\bar{K}$, respectively. The corresponding 3π decay rate is inhibited by a factor of order 10^2 relative to $K\bar{K}$ emission. Let the quark-antiquark pair in a K meson have a Gaussian distribution with a mean squared radius $r_K^{\,2}$, and similarly a mean squared radius R^2 for the $K\bar{K}$ in the ϕ meson. The $\lambda\bar{\lambda}$ overlap is then proportional to $F=(1+R^2/r_K^{\,2})^{-2}$; if we take $r_K\simeq m_K^{-1}$, corresponding to the Compton wavelength of the K, and take $R\simeq 1$ F because of the loose $K\bar{K}$ binding, then $F_\phi\simeq 2\times 10^{-2}$. This corresponds directly to the breaking of the OZI rule. For the ψ particles in the charm scheme the corresponding $D\bar{D}$ pair has $r_D\simeq\frac{1}{4}r_K$; again taking $R\simeq 1$ F since the ψ are near the $D\bar{D}$ threshold, we have $F_\pi\simeq 10^{-4}$. This is the right order of magnitude for reducing $\Gamma\simeq 200$ MeV to $\Gamma\simeq 50$ keV as observed. To extend this to the $\xi=\psi(6)$, as composed of a $\psi\bar{\psi}$ pair, the r_ψ $\simeq\frac{2}{3}r_D$, so $F_\xi\simeq 2\times 10^{-5}$. This may overestimate the reduction: R is probably no longer so large as 1 F, and the Gaussian model falls off too rapidly when $r\ll R$.

One can imagine going to higher complexes with $n_c=3,4,\ldots$ subunits of $(q\bar{q})$ type: $\eta=\psi(9)$, ζ $=\psi(12),\ldots$. These should rapidly become broader. There are $\frac{1}{2}n_c(n_c-1)$ annihilation pairings possible, and the subunits should be bound closer together with increasing n. If we take $1/R\simeq\frac{1}{2}n_c(n_c-1)$, then $F_\eta\simeq 10^{-3}$, $F_\zeta\simeq 10^{-2},\ldots$, so that widths $\Gamma\simeq 1$ MeV are expected and $\Gamma_{\mu\mu}/\Gamma\simeq 10^{-3}$, which would be hard to detect. However, if the observation at 6 GeV should be confirmed, it might be of interest to look for states with higher n: $n_0=0$ and $n_c=3,4\ldots$ at 9, 12, ... GeV, or $n_0=1$, $n_c=2$ at around 7 GeV, which is the ξ' analog of the ψ'

$=\psi(3.7)$. This latter state should have the less favorable $F \simeq 10^{-2}$ or 10^{-3} characteristic of $n=3$.

II. COLORED EXOTIC STATES

The Han-Nambu model[4] anticipates the existence of exotic mesons like $q\bar{q}q\bar{q}$. We denote[3] by $\psi_V^{(2)}$ and $\psi_P^{(2)}$ the vector and pseudoscalar states of the simplest $n_c=2$ system. The corresonding $n_c=1$ states $\psi_V^{(1)}$ are assigned as previously[5] to the observed ψ mesons around 3–5 GeV. We assume the existence of corresponding colored pseudoscalars $\psi_P^{(1)}(i,j)$ in the same mass range, and for a simple concrete picture take the $\psi_V^{(2)}(i,j)$ $(i=\omega,\phi;\ j=\rho^0,\phi)$ as an SU(3)″ octet state of $\psi_P^{(1)}(i,\rho^+)\psi_P^{(1)}(i,\rho^-)$ and $\psi_P^{(1)}(i,K)\psi_P^{(1)}(i,\bar{K})$ $(i=\omega,\phi)$ loosely bound in a relative $l=1$ state. The SU(3)″ coupling is taken to be

$$\underline{8})''_{q\bar{q}} \times (\underline{8})''_{q\bar{q}} \to \psi_V^{(2)}(\underline{8})'' . \tag{3}$$

The $\psi_V^{(2)}$ cannot be $(\underline{1})''$ because it would mix strongly with ordinary bosons: The $(\underline{10})''$, $(\overline{\underline{10}})''$, and $(\underline{27})''$ are presumably at higher energies and do not couple directly to e^+e^- since the electromagnetic current has only octet components in SU(3)′ and SU(3)″.

Denoting the SU(3)″ octet index by $j=1-8$, we expect the $\psi_P^{(1)}$ to resemble the $\psi_V^{(1)}$ in having broad and relatively unstable $j=8$ components because of color singlet-octet mixing.[5,6] A narrow $\psi_V^{(2)}$ can be constructed only by avoiding $j=8$ components from $\psi_P^{(1)}$, namely,

$$\psi_V^{(2)} \sim (\underline{8})''_a, \quad I''=0,1 . \tag{4}$$

This is achieved through the antisymmetric coupling of the constituent $\psi_P^{(1)}\bar{\psi}_P^{(1)}$ in an $l=1$ state.

The arguments here can be extended to higher n, e.g., $\psi_V^{(4)}$ is made up of an exotic pair $\psi_P^{(2)}\bar{\psi}_P^{(2)}$, etc. At each state we take the antisymmetric combination $(\underline{8})\times(\underline{8})\to(\underline{8}_a)$.

The relative lepton widths $\Gamma_{e^+e^-}$ for these resonances can be determined[7] for $\psi_V^{(1)}$ and are in satisfactory agreement with present observation. The ratios for $\Gamma_V^{(n)}$ $(n=2,3,4,\ldots)$ should follow exactly the same rule, because the coupling constants for $(\underline{8})\times(\underline{8})\to(\underline{8})_a$ are the fundamental ones that also apply to $(\underline{3})\times(\overline{\underline{3}})\to(\underline{8})$. Thus

$$\Gamma^{(n)}(\omega,\rho^0):\Gamma^{(n)}(\phi,\rho^0):\Gamma^{(n)}(\omega,\phi):\Gamma^{(n)}(\phi,\phi)=6:3:2:1 . \tag{5}$$

From the above considerations we again obtain an indication that the sequence cannot be infinite because $\psi_P^{(2)}(i,K)$ contains a component of $\psi^{(1)}(i,8)$ in its makeup. Hence $\psi_V^{(4)}$ constructed according to the above should be unstable and hence broadened. This broadening will increase with n_c. It should be noted, finally, that the ratio $\Gamma^{(m)}(i,j)/\Gamma^{(n)}(i,j)$ does not follow from the simple model we have used.

III. CHARMED EXOTIC STATES

It has been pointed out[8] that the $\psi(3.1)$ is an excellent candidate for the meson $(c\bar{c})$ with "hidden charm", expected on the SU(4) basis.[9] Although the $\psi(3.7)$ and $\psi(4.1)$ have most often been interpreted as orbital or radial excitations of $(c\bar{c})$, it has also been suggested[2,3] that they be considered as exotic states; here we use the same type of $(q\bar{q})$ constructuion as in the preceding section.

In this model n_c is the number of $(c\bar{c})$ pairs in a state. These pairs are all SU(3) singlets, so that SU(3) quantum numbers must come from adjoined uncharmed $(q\bar{q})$ pairs. The observed e^+e^- resonances around 4 GeV can be accomodated by a combination of states $\psi_V(0;1)$, $\psi_V(\rho^0;1)$, $\psi_V(\omega;1)$, $\psi_V(\phi;1)$. We use the notation $\psi(i,n_c)$ where "i" denotes the SU(3) configuration of the ordinary quarks; $i=0$ corresponds to $n_0=0$, and $i=\phi,\omega,\rho^0$ corresponds to $n_0=1$. The narrowness of any $n_c=1$ components in these combinations is argued from the OZI rule. Exact assignments of these states to the observed resonances, and their corresponding $\Gamma_{e^+e^-}$ values, are a matter of some uncertainty at present. However, this uncertainty does not prevent us from being able to sketch a pattern for the higher resonances very similar to that in the preceding section.

The next set of exotic states to provide candidates for narrow e^+e^- resonances is

$$\psi_V(i;2)=\psi_P(i';1)\times\bar{\psi}_P(i'';1) , \tag{6}$$

where $i=0,\rho^0,\omega,\phi$ and i' and/or $i''=0$. As in the preceding section the lowest ψ_V in Eq. (6), presumably $\psi_V(0;2)$, may be observable as a narrow e^+e^- resonance with $\Gamma_{e^+e^-}$ comparable to that for lower ψ. The higher states of Eq. (6) type may already be unstable against rapid decay into $\psi(i;1)$ of various types and hence may appear only as broad resonances.

Higher $\psi_V(i,n_c)$ with $n_c=3,4\ldots$ can be compounded as in Eq. (6). There does not seem to be any specific mechanism like that discussed after Eq. (5) which will make these higher charmed states more unstable than their lower counterparts. However, the general arguments of Sec. I should still be valid, and the observable sequence will be finite.

The general picture of higher e^+e^- resonances that emerges here is similar to that for color and hence does not provide a ready means for distinguishing between charm and color.

IV. DIQUARK MODEL

The ambiguities present in alternative coupling schemes for higher values of n_c lead to the consideration of another one, which is extremely systematic: a description of $2, 3, \ldots, n$-fold exotic states in terms of symmetric di-, tri-, ..., n-quark combinations,[10] loosely bound to the corresponding anti-n-quark. In the case of color, for example, the octet $(q\bar{q})$ and sextet (qq) have practically the same interaction on a simple SU(3)″ coupling[11] and may therefore be equally suitable subunits for loosely bound exotic states, and loosely bound triquarks would be just colored N-$\bar{N}$ combinations; there is already the suggestion[12] that such states have been observed for ordinary $N\bar{N}$.

On the color model, di-, tri-, ..., n-quarks symmetric in the SU(3)″ indices only would have representations $\underline{(6)}''$, $\underline{(10)}'', \ldots, \underline{(S_n)}''$, where $S_n = \frac{1}{2}(n+1)(n+2)$. There is always just one $\underline{(8)}''$ contained in

$$\psi_V^{(n)}(i,j) = \psi_P(i', S_n) \times \bar{\psi}_P(i'', S_n)\,, \tag{7}$$

and the SU(3)″ coupling coefficients for this state are just those to reproduce Eq. (5) for $\Gamma^{(n)}_{e^+e^-}$. This model gives one further hint about the relative $\Gamma^{(n)}_{e^+e^-}$ for different values of n. The SU(3)″ factors in $\Gamma^{(n)}_{e^+e^-}$ increase asymptotically at least as rapidly as n^2; if the configuration-space effects do not reduce too greatly the probability amplitude for the simple particle-antiparticle picture given in Eq. (7), we can expect an observable increase of $\Gamma^{(n)}_{e^+e^-}$ with n.

For SU(4) the symmetric n-quark combination is $[\underline{S_n}]$, where $S_n = \frac{1}{6}(n+1)(n+2)(n+3)$. The combination

$$\psi_P(S_n) \times \bar{\psi}_P(S_n) \tag{8}$$

always contains just one $\psi_V(0;n)$ and one each of $\psi_V(i;n-1)$, where $i = \rho^0, \omega, \phi$. We thus obtain the same pattern as before, but again this n-quark model predicts that $\Gamma_{e^+e^-}$ increases asymptotically as n^2.

This increase of $\Gamma_{e^+e^-}$ with n is characteristic of the n-quark model because the charges of the n quarks add coherently. This does not occur for the $(q\bar{q})$ models of the preceding section, so that a possible observational test of the extreme n-quark model is suggested. As before, however, this does not seem to permit a clear distinction between charm and color.

Since this paper was written, independent suggestions of multiquark systems have appeared.[13]

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