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Neutrino magnetic moment*

J. E. Kim
Department of Physics, Brown University, Providence, Rhode Island 02912
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The neutrino magnetic moment $f^{\nu\nu'}$ is calculated in the SU(2)$\otimes$U(1) gauge model with the doublets, $a(\nu, L^-)_L$, $b(\nu', L^-)_R$. The order of magnitude of $f^{\nu\nu'}$ is barely within the upper bound for f^{ν_μ} obtained from $\bar{\nu}_\mu$-e elastic scattering data.

A neutrino, which is massless and electrically neutral, can have electromagnetic properties through its weak interactions with charged particles. In the past, an estimate for these properties was obtained indirectly from astrophysical data.[1] Recent neutral-current experiments, however, give valuable information[2] on the upper bounds of muonic-neutrino charge radius (r) and magnetic moment (f), viz. $r \leq 10^{-15}$ cm and $f \leq 10^{-8}$. It is of some interest to see whether the values appearing in the right-hand sides of these inequalities can be reproduced by existing weak-interaction theories. We note that these values are not obtainable in the gauge theories[3] without right-handed currents. In the Weinberg-Salam (WS) model[3] without right-handed currents the charge radius is of the order of 10^{-16}–10^{-17} cm while the magnetic moment is zero.[4] (The reason why the neutrino magnetic moment is zero is that the neutrino is only left-handed in this theory.)

Recently, there have been great efforts to explore the right-handed current,[5] opening up the possibility for the right-handed neutrinos. In this article I will point out that the *neutrino magnetic moment* is significantly large if a heavy lepton L^- is coupled to two different chirality states of the neutrino. It should be noted that though the large magnetic moment of the neutrino alone may not contribute to the neutral-current experiments, its calculation is important for the following reasons. If the neutrino magnetic moment (mixed or unmixed) turns out not to be negligible compared to the Z_μ-boson-exchange diagrams in certain neutral-current interactions, the phenomenological WS neutral-current structure is not yet specified, leaving room for a better data fitting. Also a sufficiently large neutrino magnetic moment may reduce significantly the solar-neutrino detection rate[6] for three separate reasons: (a) The electron neutrino loses energy,[7] (b) The flux is reduced to $\frac{1}{2}$ its former values, if the neutrino has a mixed magnetic moment, after its multiple scattering inside the sun, and (c) It may escape the earth due to the complex terrestrial electromagnetic field. Further, the constraints on the magnetic moment of the neutrino will give valuable information on the heavy-lepton mass in a specified model.

For a specific calculational purpose, I assume two weak doublets with neutrinos ν' (they can be identical or distinct neutrinos) in the WS theory:

$$a\begin{pmatrix}\nu\\ L^-\end{pmatrix}_L, \quad b\begin{pmatrix}\nu'\\ L^-\end{pmatrix}_R, \tag{1}$$

which can be a substructure of

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L, \quad \begin{pmatrix}\nu_E\\ E^-\end{pmatrix}_L, \quad \begin{pmatrix}\nu_M\\ M^-\end{pmatrix}_L,$$

$$\begin{pmatrix}\nu_e\\ E^-\end{pmatrix}_R, \quad \begin{pmatrix}\nu_\mu\\ M^-\end{pmatrix}_R, \quad \begin{pmatrix}E^0\\ e^-\end{pmatrix}_R, \quad \begin{pmatrix}M^0\\ \mu^-\end{pmatrix}_R,$$

with $a=b=1$, or a substructure of

$$\begin{pmatrix}\nu_l\\ l^-\cos\alpha + L^-\sin\alpha\end{pmatrix}_L, \quad \begin{pmatrix}L^0\\ -l^-\sin\alpha + L^-\cos\alpha\end{pmatrix}_L,$$

$$\begin{pmatrix}\nu_l\\ L^-\end{pmatrix}_R, \quad \begin{pmatrix}L^0\\ l^-\end{pmatrix}_R,$$

with $a=\sin\alpha$, $b=1$, and $l=e$ or μ. Generally, $|a|$, $|b| \leq 1$. In the latter example, one can ingeniously draw the relation $G_\beta = \cos\theta_c G_\mu$ by assuming suitable mixed quark states in quark doublets. In the above, the subscripts L and R refer to left- and right-handed chiralities and E^0-, M^0-, L^0- are heavy leptons. Here ν and ν' are assumed to be massless. This does not necessarily imply that its magnetic moment is zero, which is a consequence of two helicity states of the neutrino.

The matrix element of the electromagnetic current $J_\mu^{\rm em}(0)$ between ν and ν' states is

$$\begin{aligned}\langle \nu'(k')|J_\mu^{\rm em}(0)|\nu(k)\rangle &= \bar{\nu}'(k')\{\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu}q^\nu F_2(q^2)\\ &\quad + \gamma_\mu\gamma_5 G_1(q^2) - i\gamma_5\sigma_{\mu\nu}q^\nu G_2(q^2)\}\nu(k),\end{aligned} \tag{2}$$

where $q=k-k'$, and F_1, F_2, G_1, and G_2 are (unmixed or mixed) charge, magnetic, anapole, and dipole form factors[8] (depending on the neutrino identity). These form factors at $q^2=0$ are physically observable quantities and hence are gauge-invariant. I am interested only in the charge and magnetic form factors. Since the neutrino charge is zero,

$$F_1(0)=0\ . \tag{3}$$

The magnetic form factor $F_2(0)$ is defined in terms of the electron Bohr magneton,

$$F_2(0)=\begin{cases}\dfrac{ef}{2m_e}\ \text{for identical } \nu \text{ and } \nu'\ ,\\[2ex] \dfrac{ef'}{2m_e}\ \text{for distinct } \nu \text{ and } \nu'\ ,\end{cases} \tag{4}$$

where m_e is the electron mass.

I calculate $F_2(0)$ from the six triangle Feynman diagrams of Fig. 1. The weak correction to the muon magnetic moment has been calculated by several authors.[9] Note that the neutrino gets additional contributions from Figs. 1(e) and 1(f). It is checked that $F_2(0)$ is gauge-invariant for two separate groups, Figs. 1(a)–1(d) and Figs. 1(e) and 1(f).

In $\xi=1$ gauge, the contributions of the individual diagrams are proportional to

$$\text{(a)}\ I_a=\frac{3}{(1-y)^2}\left[\frac{1}{2}-\frac{3}{2}y-\frac{y^2\ln y}{(1-y)}\right]\ ,$$

$$\text{(bc)}\ I_{bc}=\frac{1}{(1-y)^2}\left(\frac{1}{2}-\frac{3}{2}y-\frac{y^2\ln y}{1-y}\right)\ ,$$

$$\text{(d)}\ I_d=\frac{y}{(1-y)^2}\left(\frac{1}{2}+\frac{y}{2}+\frac{y\ln y}{1-y}\right)\ ;$$

$$I_{abcd}=\frac{1}{2}+\frac{3}{(1-y)^3}\left(\tfrac{1}{2}-2y+\tfrac{3}{2}y^2-y^2\ln y\right)\ ; \tag{5}$$

$$\text{(e)}\ I_e=\frac{4}{(1-y)^2}\left(\frac{1}{2}+\frac{y}{2}+\frac{y\ln y}{1-y}\right)\ ,$$

$$\text{(f)}\ I_f=\frac{y}{(1-y)^2}\left(-\frac{3}{2}+\frac{y}{2}-\frac{\ln y}{1-y}\right)\ ;$$

$$I_{ef}=\frac{1}{2}+\frac{3}{(1-y)^3}\left(\tfrac{1}{2}-\tfrac{1}{2}y^2+y\ln y\right)\ . \tag{6}$$

The sum of (5) and (6) is

$$\begin{aligned}I&\equiv I_{abcd}+I_{ef}\\ &=1+\frac{3}{1-y}+\frac{3y\ln y}{(1-y)^2}\ ,\end{aligned} \tag{7}$$

where $y=m_L^2/M_W^2$, m_L is the mass of the heavy lepton L^- and M_W is the W-boson mass satisfying $G_F/\sqrt{2}=g^2/8M_W^2$. The magnetic-moment coefficient f or f' is

$$f \text{ or } f'=\frac{G_F m_L m_e abI}{2\sqrt{2}\pi^2}\left(\tfrac{1}{2}+\tfrac{1}{2}\delta_{\nu\nu'}\right)\ , \tag{8}$$

where the Kronecker $\delta_{\nu\nu'}$ distinguishes the identical and the distinct neutrinos ($\delta_{\nu\nu'}=1$ for the identical

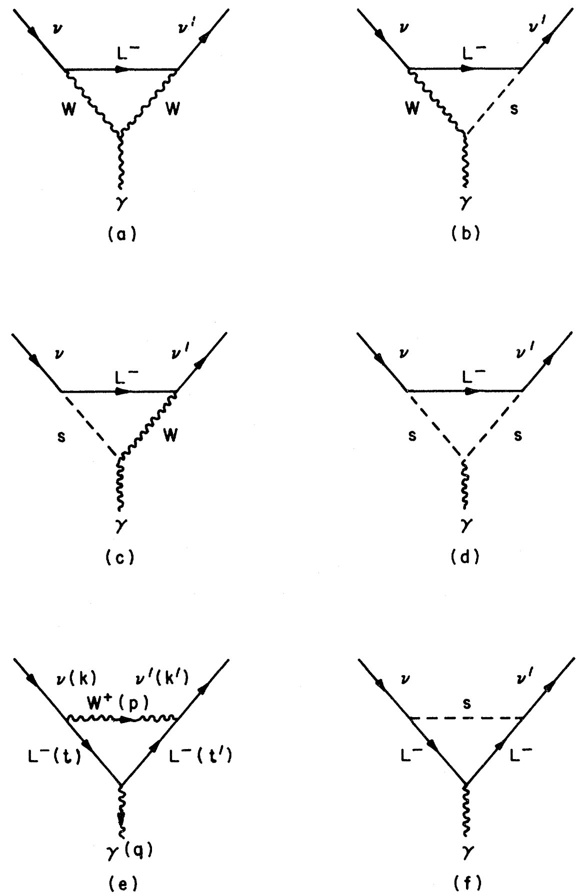

FIG. 1. Six Feynman diagrams which contribute to the neutrino magnetic moment in the WS model. W, γ, and s are W-boson, photon and scalar particle, respectively. In Fig. 1(e) the particle momenta are defined for a sample calculation.

ν, ν' and $\delta_{\nu\nu'}=0$ for the distinct ν, ν'), and a, b and I are defined in (1) and (7).

I illustrate a sample calculation for Fig. 1(e). Consider the matrix element for the distinct neutrino case (Feynman rules are given in the first paper of Ref. 9),

$$\begin{aligned}&\int\frac{d^4p}{(2\pi)^4}\bar{u}(k')\,\frac{1-\gamma_5}{2}\left(\frac{-ibg}{\sqrt{2}}\right)\gamma_\beta i(\not{t}'+m_L)(-ie\gamma_\mu)\\ &\qquad\times i(\not{t}+m_L)\left(\frac{-iag}{\sqrt{2}}\right)\gamma_\alpha\,\frac{1-\gamma_5}{2}u(k)(-ig^{\alpha\beta})/D\ ,\end{aligned}$$

where $D=(t^2-m_L{}^2)(t'^2-m_L{}^2)(p^2-M_W{}^2)$ with momenta defined in Fig. 1(e). The relevant term for the $F_2(0)$ calculation is

$$\frac{-m_Lg^2}{2}eab\int\frac{d^4p}{(2\pi)^4}\frac{1}{D}\bar{u}(k')\,\frac{1-\gamma_5}{2}\gamma_\alpha(\gamma_\mu\not{t}+\not{t}'\gamma_\mu)\gamma^\alpha u(k)\ ,$$

which contains a term proportional to $\bar{u}(k')(-i\sigma_{\mu\nu}q^\nu)u(k)$,

$$\frac{-im_L g^2 e}{16\pi^2} ab \int_0^1 \int_0^x dx\, dy \frac{x-y}{m_L{}^2(1-x+y)+M_W{}^2(x-y)} \overline{u}(k')(-i\sigma_{\mu\nu}q^\nu)u(k)$$

$$= \frac{-ie}{2m_e} \frac{g^2 m_e m_L}{8\pi^2 M_W{}^2} \frac{ab}{(1-y)^2}\left(\frac{1}{2}+\frac{y}{2}+\frac{y\ln y}{1-y}\right)\overline{u}(k')(-i\sigma_{\mu\nu}q^\nu)u(k).$$

Hence, I obtain I_e in Eq. (5) multiplied by a common factor $g^2 m_e m_L/32$.

Returning to (7) and (8), one can estimate that

$$f \text{ or } f' \underset{y\to 0}{\approx} 1.6\times(10^{-9}-10^{-8})\times ab(\tfrac{1}{2}+\tfrac{1}{2}\delta_{\nu\nu'}) \qquad (9)$$

for $m_L \approx 2-20$ GeV. This value is very close to the upper bound for the muon-neutrino magnetic moment 8×10^{-9} from $\overline{\nu}_\mu$-e elastic scattering.[1]

Using the value given in (9), a few conclusions can be drawn:

(i) With the specific structure of doublets (1), the heavy-lepton mass m_L cannot exceed 20 GeV. For $2 < m_L < 20$ GeV, the WS neutrino neutral current J_μ^Z together with the neutrino electromagnetic current J_μ^{em} contributes to the neutrino-induced neutral-current phenomena such that the interaction takes the form $-\mathcal{H}_{\text{int}}^0 = J_\mu^Z \mathcal{J}_Z^\mu + J_\mu^{\text{em}}\mathcal{J}_{\text{em}}^\mu/q^2$, with suitable coupling constants included in the currents where $\mathcal{J}$'s are the appropriate leptonic plus hadronic currents.

(ii) Some have conjectured a large electron-neutrino magnetic moment to explain the solar-neutrino nondetection,[7] but the gauge-theory calculation does not give such a large moment as order of 10^{-4}.

In this paper, I have shown that the neutrino magnetic moment arises even for the massless neutrinos if one introduces two neutrino helicity states coupled to the same heavy lepton, and it is very close to the presently available upper bound. Of course, if one assumes a small mass of the neutrino, one can always obtain the magnetic moment proportional to the neutrino mass without the assumption of two neutrino helicities.

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