Neutral currents in elastic and inelastic neutrino scattering*

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The neutral-current phenomenology of several $SU(2) \times U(1)$ models of quarks and leptons is considered in the light of recent data. Elastic and deep-inelastic neutrino-nucleon scattering and elastic neutrino-electron scattering are examined. Four models are shown to be reasonably consistent with these data, while two models are in some conflict with the data.

I. INTRODUCTION

There have been several quark-lepton models[1-12] proposed within the general framework of the Weinberg-Salam $SU(2) \times U(1)$ gauge theory[1] of weak and electromagnetic interactions. These models contain specific weak charged and neutral currents whose consequences can be tested experimentally. In these models, there are only vector and axial-vector currents. Interesting terms in these charged currents may be suppressed at present energies by the large mass of some quarks.[13-17] However, the consequences of the neutral currents differ considerably from model to model at all energies. Several of the models considered have right-handed as well as the usual left-handed neutral (and charged) currents.

The neutral currents in deep-inelastic neutrino scattering have been widely discussed elsewhere[18] and will be used here only to place limits on the values of $\sin^2\theta_W$ and the mass of Z^0 (the neutral intermediate vector boson). The elastic neutrino scattering ($\nu p \to \nu p$) provides an independent test of neutral currents and does not require the parton-model assumptions used in deep-inelastic scattering. Most attention here will be devoted to the elastic neutrino-nucleon scattering. The values of $\sin^2\theta_W$ and m_{Z^0} found in neutrino-nucleon scattering must also be consistent with their allowed values in elastic neutrino-electron scattering. The calculation of ν-e scattering involves the fewest theoretical assumptions, but the experiments are very difficult.

It is found that four models are reasonably consistent with the data, but two models have some difficulty with the data.

In Sec. II a detailed description of the formalism for elastic and quasielastic neutrino-proton scattering is presented. A very brief description of deep-inelastic neutrino-nucleon scattering for the case of neutral currents is given in Sec. III. The charged and neutral currents of a number of $SU(2) \times U(1)$ models are shown and briefly discussed in Sec. IV. In Sec. V a comparison is made of the predictions of these models and of the existing data. Some conclusions can already be reached although more data are needed. A short discussion of neutrino-electron scattering is given in Sec. VI along with some conclusions.

II. FORMALISM FOR ELASTIC AND QUASIELASTIC SCATTERING

For charged currents, the usual first-class transformation properties under G conjugation are assumed here. For neutral currents, the usual properties of vector and axial-vector currents under charge conjugation are assumed (as is found in models considered here); charge symmetry is not assumed and is not found in all models. If the muon mass is taken to be zero, then the general matrix elements of charged and neutral currents have the form[19]

$$\langle N' | J_\mu | N\rangle = \bar{u}_{N'}\left[\gamma_\mu F_1(q^2) + i\,\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2) + \gamma_\mu\gamma_5 F_A(q^2)\right]u_N\,, \tag{2.1}$$

where $q^2 = (p'_\mu - p_\mu)^2$ and p_μ (p'_μ) is the momentum of the incoming (outgoing) nucleon N (N'). F_1 and F_2 are the vector and F_A the axial-vector form factors.

The form factors used by Lee and Yang[20] for charged currents and by Weinberg[21] for neutral currents are obtained by use of the Gordon decomposition

$$\bar{u}(p')\gamma_\mu u(p) = \bar{u}(p')\left(\frac{p_\mu + p'_\mu}{2m} + i\,\frac{\sigma_{\mu\nu}q^\nu}{2m}\right)u(p)\,, \tag{2.2}$$

$$\langle N' | J_\mu | N\rangle = \bar{u}_{N'}[\gamma_\mu F'_1(q^2) - (p_\mu + p'_\mu)F'_2(q^2) + \gamma_\mu\gamma_5 F_A(q^2)]u_N\,, \tag{2.3}$$

where

$$F'_1(q^2) = F_1(q^2) + F_2(q^2)\,, \tag{2.4}$$

$$F'_2(q^2) = F_2(q^2)/2m_N\,. \tag{2.5}$$

However, it is the Sachs form factors[22] G_E and

G_M which experimentalists give in terms of which ($Q^2 \equiv -q^2$)

$$F_1^{(i)}(q^2) = \frac{G_E^{(i)} + (Q^2/4m_N^2)G_M^{(i)}}{1+Q^2/4m_N^2}, \tag{2.6}$$

$$F_2^{(i)}(q^2) = \frac{G_M^{(i)} - G_E^{(i)}}{1+Q^2/4m_N^2}, \tag{2.7}$$

where we now distinguish $i=c$ (charged currents) or 0 (neutral currents).

The conserved-vector-current (CVC) hypothesis identifies the weak hadronic vector current with the isospin current. CVC allows one (with an isospin rotation) to use the proton and neutron electromagnetic form factors (G_E^p, G_M^p, G_E^n, and G_M^n) in finding the weak vector form factors for the charged current ($\nu n \to \mu^- p$):

$$G_E^{(c)} = \cos\theta_C(G_E^p - G_E^n)\,,\quad G_M^{(c)} = \cos\theta_C(G_M^p - G_M^n)\,. \tag{2.8}$$

The factor $\cos\theta_C$ is present because the charged current is

$$J_\mu^{(c)} = \cos\theta_C \bar{u}\gamma_\mu(1+\gamma_5)d\,. \tag{2.9}$$

In the $SU(2)\times U(1)$ models considered, the neutral current is

$$J_\mu^{(0)} = J_\mu^w - 2\sin^2\theta_W J_\mu^{em}, \tag{2.10}$$

where J_μ^w can be found by an isospin rotation from the complete charged currents (discussed later) and where J_μ^{em} is the electromagnetic current. In this model J_μ^w has the general form (when quarks other than u and d are ignored)

$$J_\mu^w = J_{V\mu}^w + J_{A\mu}^w, \tag{2.11}$$

$$J_{V\mu}^w = g_V^{I=1}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + g_V^{I=0}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)\,, \tag{2.12}$$

$$J_{A\mu}^w = g_A^{I=1}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) + g_A^{I=0}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\,. \tag{2.13}$$

Since in the neutral currents quarks other than u and d are ignored here, the relations Eq. (2.8) can, with use of quark field theory, be extended to the neutral currents where there are both isovector and isoscalar terms so that

$$G_E^{(0)} = \pm g_V^{I=1}(G_E^p - G_E^n) + 3g_V^{I=0}(G_E^p + G_E^n) - 2\sin^2\theta_W G_E^{p,n} \tag{2.14}$$

(and the same for $G_M^{(0)}$ where + (−) on the first term and p (n) on the last term refer to scattering off protons (neutrons). The factors ± and 3 are obtained in Appendix A. $g_V^{I=1}$ and $g_V^{I=0}$ are the couplings in the current [Eq. (2.12)] of a given model.

G_E^n is experimentally[23] (from electron-proton scattering) much smaller than G_E^p for relevant (small) Q^2, and here it is assumed that

$$G_E^n = 0\,. \tag{2.15}$$

Also measured is

$$G_E^p \approx \frac{G_M^p}{1+\mu_p} \approx \frac{G_M^n}{\mu_n} \approx \left(1+\frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2} \tag{2.16}$$

($\mu_p = 1.79$ and $\mu_n = -1.91$). For $Q^2 < 1.0$ the first equality is good to 10% accuracy, and the second equality seems good to 10% accuracy for Q^2 up to 45 GeV^2/c^2.

The axial-vector form factor is determined from charged-current neutrino scattering[24] to be approximately

$$F_A^{(c)}(q^2) \equiv \cos\theta_C F_A(q^2) \approx +1.23\cos\theta_C\left(1+\frac{Q^2}{m_A^2}\right)^{-2}, \tag{2.17}$$

where $m_A^2 \approx 0.79\ \mathrm{GeV}^2$ (see further discussion in Sec. V).

Again the axial-vector neutral currents contain both isovector and isoscalar parts (in general) although there is no identification with the electromagnetic current, of course. As before, only u and d quarks are kept in the neutral current. The axial-vector form factor for the neutral currents is then

$$F_A^{(0)}(q^2) = \pm g_A^{I=1}F_A(q^2) + \tfrac{3}{5}g_A^{I=0}F_A(q^2)\,, \tag{2.18}$$

where + (−) refers to scattering off protons (neutrons) and the factor of $\frac{3}{5}$ is discussed in Appendix A. $g_A^{I=1}$ and $g_A^{I=0}$ are the coupling appearing in the current, Eq. (2.13). It has not, of course, been shown experimentally that the isoscalar axial-vector form factor has the same Q^2 dependence as the isovector part, but it is probably a reasonable approximation.

To find $g_V^{I=0,1}$ and $g_A^{I=0,1}$, recall that J_μ^w [see Eq. (2.10)] can be found by an isospin rotation from the charged current $J_\mu^{(c)}$. If one writes[2]

$$J_\mu^{(c)} = \bar{q}C\gamma_\mu(1+\gamma_5)q$$

and (2.19)

$$J_\mu^w = \bar{q}C^0\gamma_\mu(1+\gamma_5)q\,,$$

where q is the vector (u, c, d, s) (or the equivalent in other models) and C is the four-by-four matrix (or equivalent) giving the appropriate charged currents of a given $SU(2)\times U(1)$ model; then C^0 describing the neutral currents is

$$C^0 = [C, C^\dagger]\,. \tag{2.20}$$

Following this procedure, one finds for these models [see Eq. (2.10)] the general form of the neutral currents (where quarks other than u and d are neglected):

$$J_\mu = \tfrac{1}{2}\left[(\bar{u}\gamma_\mu u)_L - (\bar{d}\gamma_\mu d)_L + \alpha(\bar{u}\gamma_\mu u)_R - \beta(\bar{d}\gamma_\mu d)_R\right] - 2\sin^2\theta_W J_\mu^{\text{em}}, \qquad (2.21)$$

where the subscript L or R (left or right handed) indicates $(1+\gamma_5)$ or $(1-\gamma_5)$, and $\alpha = 2\tau_3^u$ and $\beta = -2\tau_3^d$. τ_3 is the weak isospin of the right-handed charged-currents. $|\alpha|$ and $|\beta|$ are ≤ 1. The factor of one half in Eq. (2.21) is discussed in Appendix A.

J_μ can be rewritten as the sum of isovector and isoscalar parts and separated into vector and axial-vector terms:

$$J_\mu^V = \tfrac{1}{4}(2+\alpha+\beta)(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \tfrac{1}{4}(\alpha-\beta)(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) - 2\sin^2\theta_W J_\mu^{\text{em}}, \qquad (2.22)$$

$$J_\mu^A = \tfrac{1}{4}(2-\alpha-\beta)(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) + \tfrac{1}{4}(-\alpha+\beta)(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d). \qquad (2.23)$$

Recalling Eqs. (2.12) and (2.13), it follows that

$$g_{V,A}(I=1) = \tfrac{1}{2}(1 \pm \tfrac{1}{2}\alpha \pm \tfrac{1}{2}\beta), \qquad (2.24)$$

$$g_{V,A}(I=0) = \tfrac{1}{2}(\pm\tfrac{1}{2}\alpha \mp \tfrac{1}{2}\beta). \qquad (2.25)$$

Given Eq. (2.1), one can write the cross section for elastic charged-current and neutral-current νN scattering as[19-21]

$$\frac{d\sigma}{dQ^2} = \frac{G^2 m_N^2 \kappa^{-4}}{8\pi E_\nu^2}\left[A \pm B\frac{s-u}{m_N^2} + C\frac{(s-u)^2}{m_N^4}\right], \qquad (2.26)$$

where (+) refers to neutrinos and (−) to antineutrinos, $G^2 = (10^{-5}/m_N^2)^2\ \text{GeV}^{-4} = 5.02\times 10^{-38}\ \text{cm}^2/\text{GeV}^2$, $(s-u) = 4m_N E_\nu - Q^2$, and E_ν is the lab energy of the incoming neutrino. κ is equal to one for charged currents and is defined in Eq. (2.33) for neutral currents. Setting $m_\mu^2 = 0$ and with the assumptions given before Eq. (2.1), one has (for charged and neutral currents)

$$A = \frac{Q^2}{m_N^2}\left[F_A^2\left(1+\frac{Q^2}{4m_N^2}\right) - F_1^2\left(1-\frac{Q^2}{4m_N^2}\right) + \frac{Q^2}{4m_N^2}\left(1-\frac{Q^2}{4m_N^2}\right)F_2^2 + \frac{Q^2}{m_N^2}F_1F_2\right], \qquad (2.27)$$

$$B = \frac{Q^2}{m_N^2}F_A(F_1+F_2), \qquad (2.28)$$

$$C = \frac{1}{4}\left(F_A^2 + F_1^2 + \frac{Q^2}{4m_N^2}F_2^2\right), \qquad (2.29)$$

where F_1, F_2, and F_A are defined in Eqs. (2.6)–(2.8), (2.14)–(2.18), (2.24), (2.25).

G^2 may be written in terms of the mass of the intermediate vector bosons, $W^\pm$ and Z^0, as follows: for charged currents

$$G^2 = \frac{g^4}{32 m_W^4} \qquad (2.30)$$

and for neutral currents

$$G^2 = \frac{(g^4/\cos^4\theta_W)}{32 m_Z^4}. \qquad (2.31)$$

However, in the usual Weinberg-Salam model[1]

$$m_{Z^0}^2 = m_W^2/\cos^2\theta_W \qquad (2.32)$$

so that the G^2 is the same. But in other models these relations may differ, and one can define κ by

$$m_{Z^0}^2 = \kappa^2 m_W^2/\cos^2\theta_W \qquad (2.33)$$

and κ^{-4} therefore appears in Eq. (2.26).

III. NEUTRAL CURRENTS IN DEEP-INELASTIC SCATTERING

The deep-inelastic neutrino-scattering experiments considered here have been done off isoscalar targets (with equal numbers of neutrons and protons). In the discussion which follows, quarks other than u and d will be ignored. The neutral currents in the SU(2) × U(1) models considered are assumed to have the form of Eq. (2.21). In the quark-parton model, the left-handed currents have a constant y dependence for neutrinos and a $(1-y)^2$ dependence for antineutrinos, while right-handed currents have the opposite y dependences. Using the fact that for left-handed currents the weak isospin $\tau_3 = \frac{1}{2}$ for u quarks, and $-\frac{1}{2}$ for d quarks, one obtains[12,25,26]

$$\frac{d^2\sigma^\nu}{dx\,dy} = \frac{G^2 m_N E \kappa^{-4}}{\pi}F(x) \times \left[(a_L^2 + b_L^2) + (a_R^2 + b_R^2)(1-y)^2\right], \qquad (3.1)$$

$$\frac{d^2\sigma^{\bar\nu}}{dx\,dy} = \frac{G^2 m_N E \kappa^{-4}}{\pi}F(x) \times \left[(a_L^2 + b_L^2)(1-y)^2 + (a_R^2 + b_R^2)\right], \qquad (3.2)$$

where

$$a_L = \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W, \quad a_R = \tfrac{1}{2}\alpha - \tfrac{2}{3}\sin^2\theta_W, \qquad (3.3)$$

$$b_L = -\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W, \quad b_R = -\tfrac{1}{2}\beta + \tfrac{1}{3}\sin^2\theta_W. \qquad (3.4)$$

$x = Q^2/2p\cdot q$, $y = (E-E')/E$, κ was defined in Eq. (2.33), and $\alpha = 2\tau_3^u$ and $\beta = -2\tau_3^d$ for the right-handed part of the currents. Most experimental results are given in an integrated (over y) form in which case $(1-y)^2$ in Eqs. (3.1) and (3.2) is replaced with $\frac{1}{3}$. The neutral-current cross sections are often given relative to the charged-current cross sections. In theoretical calculations, this ratio commonly (and here also) is expressed in terms of the naive charged-current cross sections expected at low energies:

$$\frac{d^2\sigma}{dx\,dy} = \frac{G^2 m_N E}{\pi}F(x) \times \begin{cases} 1, & \text{for } \nu\text{'s} \\ (1-y)^2, & \text{for } \bar\nu\text{'s}. \end{cases} \qquad (3.5)$$

However, in many of these models[3-9] these are not the cross sections expected,[13-17] nor is it the cross section found in the antineutrino data.[27,28]

If one defines

$$R^{\bar\nu} \equiv \sigma(\bar\nu N \to \bar\nu + X)/\sigma^{\text{naive}}(\bar\nu N \to \mu^+ + X) \tag{3.6}$$

(and similarly for neutrinos), then the number to be compared with experiment is $R^{\bar\nu}$ divided by an energy-dependent factor[27] (which is about 1.35 at $E_{\bar\nu} \approx 40$ GeV, but approximately equal to 1 for neutrinos at all present energies). Another frequently used ratio is

$$R_N \equiv \sigma(\bar\nu N \to \bar\nu + X)/\sigma(\nu N \to \nu + X)\,. \tag{3.7}$$

IV. QUARK MODELS

Only those quark models proposed within the general framework of the Weinberg-Salam $SU(2) \times U(1)$ gauge theory[1] are considered here (leptons are discussed only in Sec. VI). These place the quarks either in weak "isodoublets" or "isosinglets" for left-handed and right-handed currents. The standard Weinberg-Salam (W-S) and Glashow-Iliopoulos-Maiani (GIM) model[1,2] has four quarks all in left-handed doublets and right-handed singlets; therefore in Eq. (2.21) $\alpha = \beta = 0$. The doublets are (neglecting Cabibbo angles)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L . \tag{4.1}$$

The Caltech-Harvard-Hawaii-Princeton (CHHP) vector model[5-8] has six quarks all in both left-handed and right-handed doublets ($\alpha = 1,\ \beta = 1$):

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L ; \begin{pmatrix} u \\ b \end{pmatrix}_R, \begin{pmatrix} c \\ s \end{pmatrix}_R, \begin{pmatrix} t \\ d \end{pmatrix}_R . \tag{4.2}$$

The Harvard-Yale-Maryland-Caltech (HYMC) model[3,4,9] has six quarks (not the same charges as CHHP) with a mixture of doublets and singlets ($\alpha = 1,\ \beta = 0$). Its doublets are

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L ; \begin{pmatrix} u \\ b \end{pmatrix}_R, \begin{pmatrix} c \\ g \end{pmatrix}_R . \tag{4.3}$$

Several variations of the fourth doublet are possible which do not alter the nature of the model significantly; of course, there is also the freedom to mix b and g.

An interesting possibility which is consistent with the charged-current data is another model which is a mixture of doublets and singlets. This model has six quarks, but two have charge $-\frac{4}{3}$ because $\tau_3^d = +\frac{1}{2}$ for the right-handed d ($\alpha = 0,\ \beta = -1$). The mass of the $-\frac{4}{3}$ charged quark would be greater than 4 GeV. The doublets are

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L ; \begin{pmatrix} d \\ a \end{pmatrix}_R, \begin{pmatrix} s \\ v \end{pmatrix}_R . \tag{4.4}$$

There are other models including an axial-vector model which has $-\frac{4}{3}$ and $+\frac{5}{3}$ quarks ($\alpha = -1,\ \beta = -1$) and a seven-quark model[29] which was constructed to maintain the desirable features of the vector model, but to try to do better in fitting the observed neutral currents ($\alpha \approx 0.75,\ \beta = 1$). All other combinations of integer α and β were tried.

V. CONCLUSIONS FROM νN SCATTERING

The data for $\nu p \to \nu p$ scattering considered here is that of the Harvard-Pennsylvania-Wisconsin (HPW) and Columbia-Illinois-Rockefeller (CIR) collaborations at Brookhaven National Laboratory (BNL). The HPW data has $0.3 < Q^2 < 0.9$ GeV$^2/c^2$, and the CIR data has similar cuts. There is a spectrum of neutrino energies centered around 1 GeV. The calculations reported here make the same Q^2 cut and are integrated over the ν energy spectrum of these experiments.[30] Since the value of $m_A{}^2$ in Eq. (2.17) is not well determined a variety of such values were tried. Generally the results are relatively insensitive (10% change) for reasonable $m_A{}^2$, but in some instances changing $m_A{}^2$ from 0.79 to 1.2 GeV^2 can have a noticeable effect (in the W-S-GIM model, $R_{\bar\nu}$ increases by 50%). Here $m_A{}^2$ is chosen as 0.79 GeV^2 since this is the best value from weak interactions.[24]

The values of R_ν and $R_{\bar\nu}$ for deep-inelastic scattering [Eq. (3.6)] have been calculated previously[4,6,8,9,12,25,26] for many models and are shown in Fig. 1. This figure shows (for three models) these ratios as a function of the Weinberg angle ($\sin^2\theta_W$) which is a free parameter. The vector and HYMC models shown have $\kappa = 1$, but that is not necessary. Also, it should be recalled (see Sec. III) that $R_{\bar\nu}$ for theory has not accounted for the rising value of $\sigma(\bar\nu N \to \mu^+ + X)/E$; an easy, visual means of accounting for this effect is to multiply the Harvard-Pennsylvania-Wisconsin-Fermilab (HPWF) data[27] by about 1.35 and the Caltech-Fermilab (CF) data[28] by about 1.50. Two values are shown for the HPWF point, because a model-dependent extrapolation[27] to low hadron energies is needed to calculate $R_{\bar\nu}$. The two points show are the extremes from this extrapolation for the models considered here. The CF data are raw data with no extrapolation performed; for the models considered here, $R_{\bar\nu}$ would probably decrease slightly.

For elastic scattering R_ν and $R_{\bar\nu}$ can be defined similarly to Eq. (3.6), but it is important to notice that unlike those for deep-inelastic scattering, these ratios are energy dependent and here are defined for $0.3 < Q^2 < 0.9$ GeV$^2/c^2$ only. These ra-

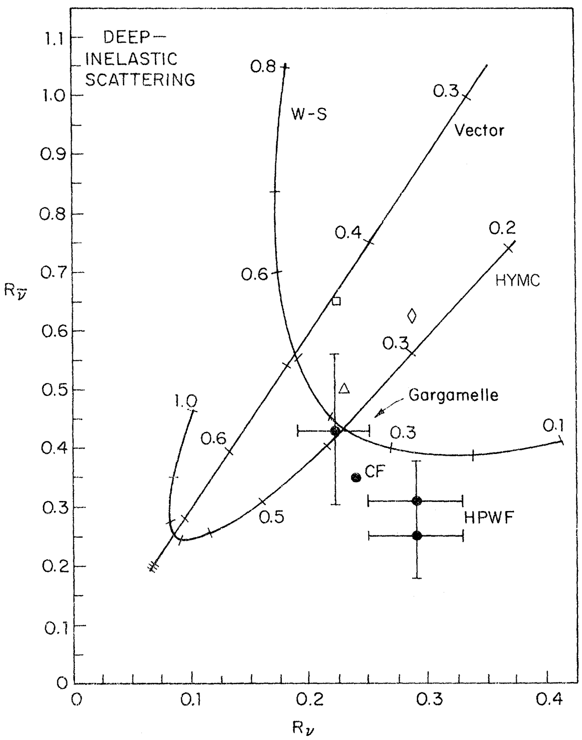

FIG. 1. The ratios in deep-inelastic scattering of neutral-current to charged-current cross sectins for antineutrinos vs neutrinos. The theoretical values as a function of $\sin^2\theta_W$ (for $\kappa=1$) and using naive charged-current cross sections are shown for the W-S-GIM model, the CHHP vector model and the HYMC model. Other models shown (with $\sin^2\theta_W=0$ and different κ) have $\alpha=0$, $\beta=-1$ and $\alpha=-1$, $\beta=0$ (triangle); $\alpha=-1$, $\beta=-1$ (square); and $\alpha=0$, $\beta=1$ (diamond). The data are from Refs. 27 and 28 and J. G. Morfin, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 537.

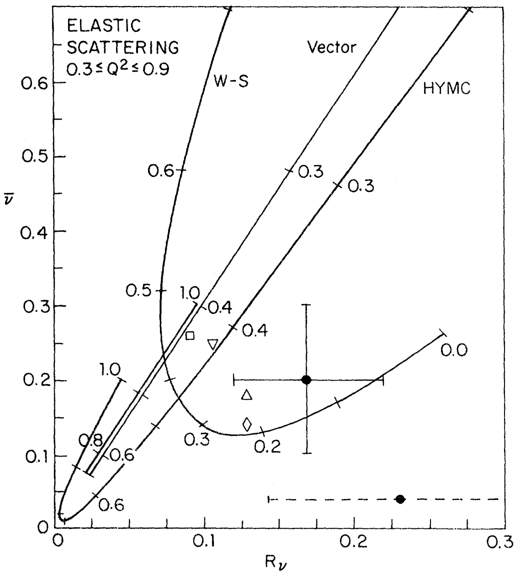

FIG. 2. The ratios in elastic scattering of neutral-current to charged-current cross sections for antineutrinos vs neutrinos (where $0.3<Q^2<0.9$ GeV$^2/c^2$ and σ are averaged over E_ν). The theoretical values as a function of $\sin^2\theta_W$ (for $\kappa=1$) are shown for the W-S-GIM model, the CHHP vector model, and the HYMC model. Other models shown (with $\sin^2\theta_W=0$ and different κ) have $\alpha=0$, $\beta=-1$ (triangle pointing up); $\alpha=-1$, $\beta=0$ (triangle pointing down); $\alpha=\beta=-1$ (square); and $\alpha=0$, $\beta=1$ (diamond). The data are from Ref. 31. The point shown at the bottom with dashed error bars is that of CIR for neutrino scattering; CIR do not report antineutrino data.

tios are shown in Fig. 2, again as a function of $\sin^2\theta_W$.

The ratios, R_N, of $\bar{\nu}$ to ν neutral-current cross sections for both elastic and deep-inelastic scattering are shown in Table I for the best overall values of $\sin^2\theta_W$. The HPWF data[27] for deep-inelastic scattering are again subject to the extrapolation to low energies, but for the models considered here R_N lies between $R_N=0.40\pm0.17$ and $R_N=0.48\pm0.20$. From the CF data[28] (for deep-inelastic scattering) without any extrapolation, one can find $R_N=0.73$ (CF do not show their error bars). For elastic scattering, HPW[31] give $R_N=0.40\pm0.20$; however, there is a systematic uncertainty comparable to the statistical uncertainty shown (the background for $\bar{\nu}p\to\mu^+n$ is different than the background for $\nu n\to\mu^-p$, and both ν and $\bar{\nu}$ neutral-current cross sections are normalized to charged-current cross sections).

The W-S-GIM model [Eq. (4.1)] is consistent with all neutral-current data, but it should be noticed that the best fits in Figs. 1 and 2 are *not* for the same Weinberg angle in elastic as in deep-inelastic scattering. Choosing $\sin^2\theta_W\approx0.3$ gives a reasonable fit to both sets of data, and also gives good agreement with the ratios, R_N in Table I.

The CHHP vector model, Eq. (4.2) (and also the axial-vector model, $\alpha=\beta=-1$) appears to be somewhat inconsistent with both sets of data. This inconsistency is clearest in the ratios, R_N, in Table I, but it is probably wise to wait for further confirmation of these results.

The HYMC model [Eq. (4.3)] is consistent with the data although it does not do quite as well as the W-S-GIM model. If $\alpha=1$ were reduced to $\alpha=0.75$, this model is in better agreement with the data, but this mixing of weak isodoublets and isosinglets for right-handed currents might be theoretically unappealing.

Three other models including that of Eq. (4.4) ($\alpha=0$, $\beta=-1$) are consistent with the data if κ [see Eq. (2.33)] is increased to 1.26 (for $\alpha=0$,

TABLE I. The ratio, R_N, of antineutrino to neutrino neutral-current cross sections for elastic (BNL energies) and deep-inelastic scattering for given values of $\sin^2\theta_W$.

Model	$\sin^2\theta_W$	$R_N^{\text{deep inelastic}}$	R_N^{elastic} $[(0.3<Q^2<0.9)(\text{GeV}/c)^2]$
W-S-GIM	0.3	0.50	0.49
CHHP Vector	independent	1.0	1.0
HYMC	0.4	0.63	0.76
$\alpha=0,\ \beta=-1$	0.0	0.71	0.48
Axial vector	0.0	1.0	1.0
$\alpha=-1,\ \beta=0$	0.0	0.71	0.75
$\alpha=0,\ \beta=1$	0.0	0.71	0.34

$\beta=-1$ and $\alpha=-1$, $\beta=0$ models) or to 1.19 (for $\alpha=0$, $\beta=1$ model). In Figs. 1 and 2 these values of κ and $\sin^2\theta_W=0.0$ have been used.

Only the CHHP, HYMC, and Eq. (4.4) models have a mechanism for explaining the rise of $\sigma(\bar{\nu}N\to\mu^+ + X)/E$, and can therefore be completely consistent with the charged-current data.

The Q^2 dependence with present error bars is not capable of clearly distinguishing among models considered here. The results are shown in Figs. 3 and 4. The CHHP vector model has a rather poorer fit to this data than other models. All of the other above models give more or less the same fits.

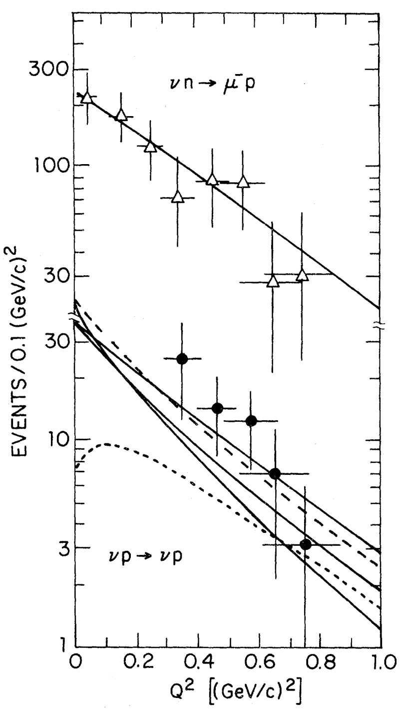

FIG. 3. The elastic neutrino-nucleon cross sections as a function of Q^2. The charged-current reaction is the usual $V-A$ result. The neutral-current cross sections are for the W-S-GIM model (solid curves; on right side, from top to bottom $\sin^2\theta_W=0.2, 0.3, 0.4$), the CHHP vector model (dotted curve; $\kappa^{-4}=2$ and $\sin^2\theta_W=0.6$), and the HYMC model (dashed curve; $\sin^2\theta_W=0.4$ and $\kappa=1.0$). The data are from Ref. 31.

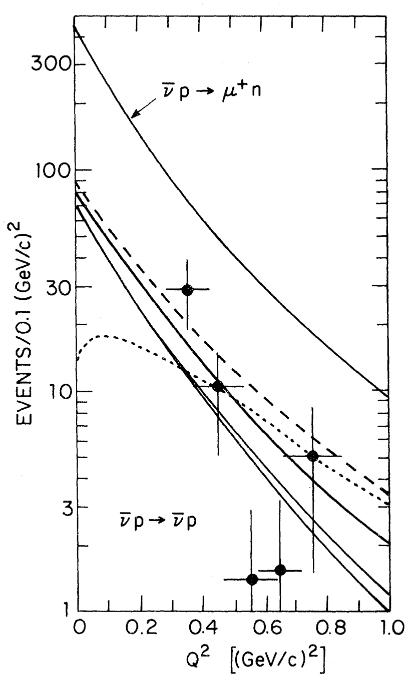

FIG. 4. The elastic antineutrino-nucleon cross sections as a function of Q^2. The solid curve at the top is the theoretical calculation of charged-current scattering ($V-A$). The neutral-current cross sections are for the W-S-GIM model (solid curves; on right side, from top to bottom $\sin^2\theta_W=0.4, 0.3, 0.2$), the CHHP vector model (dotted curve; $\kappa^{-4}=2$ and $\sin^2\theta_W=0.6$), and the HYMC model (dashed curve; $\sin^2\theta_W=0.4$ and $\kappa=1.0$). The data are from Ref. 31.

VI. NEUTRINO-ELECTRON SCATTERING

The values of $\sin^2\theta_W$ and κ obtained from neutrino-nucleon scattering must be consistent with the allowed domains of those parameters in neutrino-electron scattering. Figure 5 (taken from Ref. 6) shows the allowed values (for current experimental results) of g_A and g_V (see Refs. 6 and 32) in neutrino-electron scattering where

$$g_V = \kappa^{-2}(-\tfrac{1}{2} + \tau_e + 2\sin^2\theta_W)\,, \quad g_A = \kappa^{-2}(-\tfrac{1}{2} - \tau_e) \tag{6.1}$$

(τ_e is the weak isospin of the right-handed electron where left-handed $\tau_e = -\frac{1}{2}$). Clearly, changing κ moves any point toward or away from the origin in Fig. 5. It has been assumed that the couplings of ν_e and ν_μ in the neutral currents are the same.

The W-S-GIM model[1,2] (which has $\tau_e = 0$ and $\kappa = 1$) is consistent with this data for $\sin^2\theta_W \approx 0.3$ as found from neutrino-nucleon data. The CHHP vector model[5-8] (with $\tau_e = -\frac{1}{2}$) must exclude $\sin^2\theta_W$ from about 0.45 to 0.55 and above 0.70 to avoid conflict with neutrino-electron scattering data, but most other values of $\sin^2\theta_W$ are allowed if κ is chosen appropriately. The HYMC model[3,4,9] has the right-handed coupling (ν, E^-) and (ν, M^-) where E and M are heavy leptons, but has no coupling to e^- (or μ^-) so $\tau_e = 0$. Therefore, g_A and g_V for the HYMC model are the same as for the W-S-GIM model; if $\kappa = 1$ and $\sin^2\theta_W = 0.4$, then the HYMC model is on the edge of the allowed region. However, if κ is slightly less than one then $\sin^2\theta_W = 0.2$ or 0.3 are allowed. $\sin^2\theta_W$ greater than 0.5 are not allowed by this neutrino-electron scattering data.

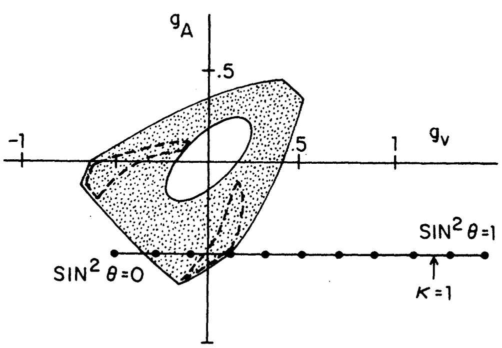

FIG. 5. The allowed domains (90% confidence level in shaded area) of g_V and g_A from experiment (see Ref. 6). The line with dots at every one-tenth value of $\sin^2\theta_W$ shows the theoretical values of g_V and g_A for the W-S-GIM model (the HYMC model is the same) where $\kappa = 1$. Recently reported data (not discussed in the text) may be used to further limit the allowed regions to those inside the dashed curves (see Ref. 32).

Note added. While preparing this manuscript, we received a paper by Barger and Nanopoulos[33] reporting similar calculations. We have also learned that Albright, Quigg, Schrock, and Smith have completed similar work.[34]

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APPENDIX A

One can obtain predictions for the relative factors between axial-vector and vector terms, and between isoscalar and isovector terms by considering the following SU(6) quark representations of protons and neutrons. The order of quarks in a term indicates the color, and the upper (lower) case letters indicate spin up (down).

$$\text{proton} = \frac{1}{\sqrt{18}}(2UdU + 2UUd + 2dUU - UuD - UDu - uUD - uDU - DUu - DuU)\,, \tag{A1}$$

$$\text{neutron} = \frac{1}{\sqrt{18}}(-2DuD - 2DDu - 2uDD + DdU + DUd + dDU + dUD + UDd + UdD)\,. \tag{A2}$$

Taking the nonrelativistic limit, γ_μ reduces to the unit operator and $\gamma_\mu\gamma_5$ reduces to the spin operator σ_3. Then using representations (A1) and (A2), and summing over quarks, one finds for the isovector case (charged or neutral)

$$\frac{\langle N' | J_\mu^A | N\rangle}{\langle N' | J_\mu^{(1)} | N\rangle} = \frac{5}{3}\,, \tag{A3}$$

where $J_\mu^{(1)}$ refers to the γ_μ (or F_1) part of the vector current. It is presumably possible to make relativistic corrections to bring this closer to the experimental value which is 1.23.

For the neutral currents, one also needs to consider isoscalar currents. Using 1 and τ_3 for the isoscalar and isovector currents (no approximation needed) with Eq. (A1) (summing over quarks)

$$\frac{\langle p | J_\mu^V(I=0) | p\rangle}{\langle p | J_\mu^V(I=1) | p\rangle} = 3\,, \tag{A4}$$

$$\frac{\langle p | J_\mu^A(I=0) | p\rangle}{\langle p | J_\mu^A(I=1) | p\rangle} = \frac{3}{5}\,. \tag{A5}$$

For neutrons the isovector terms change sign.

There is in addition a factor of $\frac{1}{2}$ in the neutral currents relative to the charged currents. This is found from the ratio of Clebsch-Gordan coefficients for isovector neutral to charged currents:

$$\frac{\mathrm{Tr}(\frac{1}{2}\tau_3)^2}{\mathrm{Tr}[\frac{1}{2}(\tau_1 + i\tau_2)\frac{1}{2}(\tau_1 - i\tau_2)]} = \frac{1}{2}\,. \tag{A6}$$

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