Electromagnetic interference in J/ψ decays

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We point out the importance of properly separating the electromagnetic and "direct" contributions to ψ-particle exclusive decay modes before a meaningful comparison with theoretical predictions can be made. This is illustrated using the decays $J/\psi \rightarrow \rho\pi$ and $J/\psi \rightarrow K K^*(892)$, with particular attention to tests of the SU(3) properties of J/ψ. An experiment is suggested to observe the effects of electromagnetic interference in $e^+e^- \rightarrow \rho\pi$ just below the J/ψ mass, and its implications are discussed.

It has been widely recognized, ever since the discovery of the ψ particles, that the branching ratio for the process $\psi \rightarrow \gamma \rightarrow$ hadrons is much larger than could be naively expected, simply because the direct hadronic decays of the ψ particles are suppressed.[1] The theoretical estimate for $J/\psi(3095)$ (which we hereafter abbreviate as ψ),

$$\Gamma(\psi \rightarrow \gamma \rightarrow \text{hadrons} \simeq R(\sqrt{s} = 3\text{ GeV}) \times \Gamma(\psi \rightarrow \mu^+\mu^-)$$
$$\simeq 12\text{ keV},$$

together with the observed total width $\Gamma_{\text{tot}}(\psi) \simeq 69$ keV, leads to a branching ratio

$$\frac{\Gamma(\psi \rightarrow \gamma \rightarrow \text{hadrons}) + \Gamma(\psi \rightarrow \text{lepton pair})}{\Gamma_{\text{tot}}(\psi)} \simeq 32\%,$$

i.e., only 68% of the observed ψ width is due to *direct* decay into hadrons. Similarly large electromagnetic effects are expected for individual exclusive decay channels; these effects should be extracted from the raw experimental numbers for partial widths before a meaningful comparison can be made with theoretical models for ψ decays. We demonstrate this by first considering the decay $\psi \rightarrow \rho\pi$. For this and other simple two-body decays, we expect an interesting complication: The direct and electromagnetic decay amplitudes should maintain their definite phase relation and yield a sizable interference effect.

The decay of $\psi(3095)$ to $\rho\pi$ through an intermediate photon is described by the amplitude

$$G_{\text{em}}(\psi \rightarrow \rho\pi) = g_{\gamma\psi} m_\psi^{-2} G_{\gamma\rho\pi}(m_\psi^2). \tag{1}$$

We estimate $G_{\gamma\rho\pi}$ using vector-meson dominance; the largest contribution comes from an intermediate ω,

$$G_{\gamma\rho\pi}(s) = g_{\gamma\omega}(s - m_\omega^2)^{-1} g_{\omega\rho\pi}. \tag{2}$$

The photon–vector-meson couplings are extracted directly from lepton-pair decay widths:

$$g_{\gamma V}^2 = 3\alpha^{-1} m_V^3 \Gamma(V \rightarrow e^+e^-). \tag{3}$$

We take $g_{\omega\rho\pi} = 16\text{ GeV}^{-1}$ as determined from ω decays; $\alpha = e^2/4\pi \simeq \frac{1}{137}$ is the fine-structure constant. [As a check of the validity of Eq. (2) for $s \gg m_\omega^2$, we have calculated the cross section for the process $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ for $s = 4\text{ GeV}^2$. We find a value of approximately 1 nb, comparable to the experimental result, $\simeq$2 nb, given by the Adone $\gamma\gamma$ group.[2]] Equation (1) then gives[3]

$$|G_{\text{em}}(\psi \rightarrow \rho\pi)| \simeq 0.6 \times 10^{-3}\text{ GeV}^{-1}. \tag{4}$$

The $\psi \rightarrow \rho\pi$ width is calculated from the effective interaction

$$\mathcal{L}_{\psi\rho\pi} = G_{\text{eff}} \epsilon^{\mu\nu\lambda\sigma} \delta^{ab} \partial_\mu \psi_\nu \partial_\lambda \rho_\sigma^a \pi^b,$$

$$G_{\text{eff}} \equiv G_{\text{em}}(\psi \rightarrow \rho\pi) + G_{\text{dir}}(\psi \rightarrow \rho\pi),$$

which gives

$$\Gamma(\psi \rightarrow \rho\pi) = G_{\text{eff}}^2 \lambda^{3/2}(m_\psi^2, m_\rho^2, m_\pi^2)/32\pi m_\psi^3,$$

with the usual definition of the kinematical function $\lambda(x, y, z) = (x - y - z)^2 - 4yz$. Experimentally,[4] $\Gamma(\psi \rightarrow \rho\pi) \simeq 0.9 \pm 0.4$ keV, so that

$$|G_{\text{em}} + G_{\text{dir}}| \simeq (1.9 \pm 0.4) \times 10^{-3}\text{ GeV}^{-1}, \tag{5}$$

while $|G_{\text{em}}| \simeq 0.6 \times 10^{-3}\text{ GeV}^{-1}$. We must now distinguish two cases for real coupling constants,[5] namely

(a) $G_{\text{em}}/G_{\text{dir}} > 0$,

(b) $G_{\text{em}}/G_{\text{dir}} < 0$.

Case (a) yields $|G_{\text{dir}}| \simeq (1.3 \pm 0.4) \times 10^{-3}\text{ GeV}^{-1}$. In the absence of electromagnetic effects, this would correspond to a width for direct $\psi \rightarrow \rho\pi$ decay of 0.4 keV, less than half the observed partial width. In case (b), we find $|G_{\text{dir}}| \simeq (2.5 \pm 0.4) \times 10^{-3}$ GeV^{-1}.

Here we have, then, a clear instance of large electromagnetic interference effects in an exclusive decay mode of ψ. The importance of properly isolating the nonelectromagnetic part of decay widths before comparing them with the predictions of theoretical models for direct decays is thus apparent. This point seems to have been consistently overlooked until now.

Such interference phenomena may be more directly relevant to the interpretation of data. Consider, for instance, the evidence bearing on the SU(3) character of the ψ. None of the two-body or quasi-two-body decay modes forbidden to an SU(3)-singlet state have been observed, while the allowed modes $\rho\pi$, $KK^*(892)$, and $K^*(892)K^*(1420)$ have been identified.[6] These results would seem to indicate that the ψ is an SU(3) singlet. However, it has been claimed that this assignment is rendered doubtful by the fact that the further prediction for an SU(3) singlet ψ,

$$\frac{\Gamma(\psi\to\rho\pi)}{\Gamma(\psi\to KK^*(892))}=\frac{3}{4},$$

was badly violated.[6] We wish to point out that electromagnetic interference is very likely present in these decays and should be separately considered.

If the $\psi(3095)$ is an SU(3) singlet, the following relations hold between direct-decay coupling constants:

$$G_{\rm dir}\equiv G_{\rm dir}(\psi\to\rho\pi)=G_{\rm dir}(\psi\to K^{*+}K^-)=G_{\rm dir}(\psi\to K^{*0}K^0)\,. \tag{6}$$

Further, using the well-known results of Okubo,[7] we obtain relations between electromagnetic amplitudes, assuming similar off-shell behaviors, as follows:

$$\begin{aligned}G_{\rm em}\equiv G_{\rm em}(\psi\to\rho\pi)&=G_{\rm em}(\psi\to K^{*+}K^-)\\&=-\tfrac{1}{2}G_{\rm em}(\psi\to K^{*0}K^0)\,.\end{aligned}\tag{7}$$

Effective coupling constants for $\psi\to K^*K$ can be extracted using

$$\begin{aligned}G_{\rm eff}^{\,2}(\psi\to K^{*+}K^-)&=G_{\rm eff}^{\,2}(\psi\to K^{*-}K^+)\\&=48\pi m_\psi^{\,3}\lambda^{-3/2}(m_\psi^{\,2},m_{K^*}^{\,2},m_K^{\,2})\\&\quad\times\Gamma(\psi\to K^{*\pm}K^\mp)\,,\end{aligned}\tag{8}$$

and the same relation for $G_{\rm eff}^{\,2}(\psi\to K^{*0}K^0)$. Using data from Ref. 4, we readily find, from Eq. (8),

$$|G_{\rm eff}(\psi\to K^{*+}K^-)|\simeq(1.3\pm.3)\times10^{-3}\ {\rm GeV}^{-1}\,, \tag{9}$$

$$|G_{\rm eff}(\psi\to K^{*0}K^0)|\simeq(1.1\pm.2)\times10^{-3}\ {\rm GeV}^{-1}\,. \tag{10}$$

(We consider effective coupling constants rather than partial widths in order to make the obvious correction for mass differences in kinematic and phase-space factors.)

We first notice that $|G_{\rm eff}(\psi\to\rho\pi)|>|G_{\rm eff}(\psi\to K^{*0}K^0)|$, which, in view of Eqs. (6) and (7) and the definition $G_{\rm eff}(\psi\to\rho\pi)=G_{\rm dir}+G_{\rm em}$, yields immediately $G_{\rm dir}G_{\rm em}>0$. This corresponds to case (a) above. Further, one obtains the relations

$$G_{\rm dir}=\tfrac{1}{3}[2G_{\rm eff}(\psi\to\rho\pi)+G_{\rm eff}(\psi\to K^{*0}K^0)]\,, \tag{11}$$

$$G_{\rm em}=\tfrac{1}{3}[G_{\rm eff}(\psi\to\rho\pi)-G_{\rm eff}(\psi\to K^{*0}K^0)]\,, \tag{12}$$

and the prediction

$$G_{\rm eff}(\psi\to\rho\pi)=G_{\rm eff}(\psi\to K^{*+}K^-)\,, \tag{13}$$

which does not require the detailed separation of direct and electromagnetic amplitudes in order to be tested. One must expect corrections to the above relations to be at most of order 15%, the typical level of SU(3) symmetry violation in strong interactions.

Equations (11) and (12) allow us to extract values for $G_{\rm dir}$ and $G_{\rm em}$ from the observed effective coupling constants. Because of ambiguities in the signs of $G_{\rm eff}(\psi\to\rho\pi)$ and $G_{\rm eff}(\psi\to K^{*0}K^0)$, we are led to the following possibilities:

$$G_{\rm em}=\pm(0.3\pm0.2)\times10^{-3}\ {\rm GeV}^{-1}\quad\text{or}\ \pm(1.0\pm0.2)\times10^{-3}\ {\rm GeV}^{-1}$$

and, correspondingly,

$$G_{\rm dir}=\pm(1.6\pm0.3)\times10^{-3}\ {\rm GeV}^{-1}\quad\text{or}\ \pm(0.9\pm0.3)\times10^{-3}\ {\rm GeV}^{-1}\,.$$

These numbers are consistent with our previous estimate for these quantities, based only on $\psi\to\rho\pi$ and a vector-meson-dominance pole model for $G_{\rm em}$ [case (a)]. Equation (13), on the other hand, does not seem especially well satisfied by presently available data. However, one should be careful not to draw premature conclusions from this, in view of the large experimental uncertainties associated with these quantities: The measurement of $\psi\to K^*K$ is made particularly difficult by the problem of identifying high-momentum charged kaons in the SPEAR detector.[4,6] It is clear that much more accurate data will be necessary in order to make a meaningful test of Eq. (13).

We conclude, therefore, that sizable electromagnetic interference effects are very likely present in these decays. Our primary concern is to emphasize that such effects may produce large apparent violations of SU(3) symmetry. Experimenters should recognize and subtract such contributions; we have given above a prescription for doing this. However, one should ask whether there is a more direct way of isolating the electromagnetic amplitudes.

It turns out that the interference effect in $\psi\to\rho\pi$ is large enough to be observed experimentally in the reaction $e^+e^-\to\rho\pi$ just below the ψ mass. We must consider the interference of three contributions, viz., $e^+e^-\to\gamma\to\rho\pi$, $e^+e^-\to\gamma\to\psi\to\rho\pi$, and

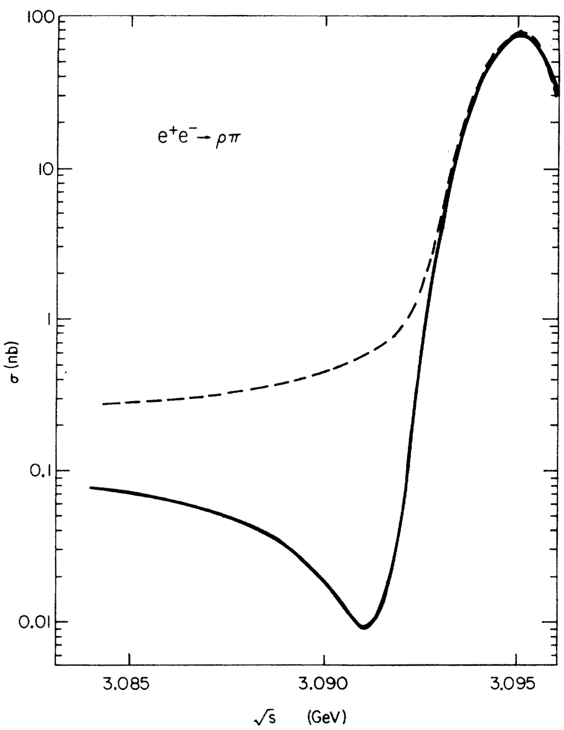

FIG. 1. The cross section for $e^+e^- \to \rho\pi$ versus total c.m. energy just below the ψ mass. The solid curve corresponds to the favored case, $G_{em}/G_{dir} > 0$ [case (a): see text following Eq. (10)]. The dashed curve corresponds to the case $G_{em}/G_{dir} < 0$, included here for purposes of comparison. We have used the value $|G_{em}(\psi \to \rho\pi)| \simeq 0.6 \times 10^{-3}$ GeV^{-1} in the calculations.

$e^+e^- \to \gamma \to \psi \to \gamma \to \rho\pi$, including "renormalization" effects due to the $\gamma\psi$ coupling. Completely standard manipulations then lead to the amplitude

$$M(e^+e^- \to \rho\pi) = e\bar{v}(p_+)\gamma_\mu u(p_-)\tfrac{1}{s}\epsilon^{\mu\nu\lambda\sigma}\epsilon_\nu(k)K_\lambda k_\sigma F(s),$$

where u, p_- and v, p_+ are the electron and positron spinors and four-momenta, respectively, $K = p_- + p_+$, $s = K^2$ is the total center-of-mass energy squared, and $\epsilon_\nu(k)$ and k are, respectively, the polarization vector and the four-momentum of the outgoing ρ meson. $F(s)$ is given by the following [with $G_{\gamma\rho\pi}(s)$ given in Eq. (2)]:

$$F(s) = G_{\gamma\rho\pi}(s) + D_\psi(s)(g_{\gamma\psi}G_{dir} + g_{\gamma\psi}{}^2 G_{\gamma\rho\pi}(s)/m_\psi{}^2), \tag{14}$$

with $D_\psi(s) = (s - m_\psi{}^2 + im_\psi\Gamma_\psi)^{-1}$. The cross section for $e^+e^- \to \rho\pi$ near the ψ mass is then given by

$$\sigma_{e^+e^- \to \rho\pi}(s) = \alpha|F(s)|^2\lambda^{3/2}(s, m_\rho{}^2, m_\pi{}^2)/8s^3. \tag{15}$$

We have calculated $\sigma_{e^+e^- \to \rho\pi}$ for both constructive [case (a)] and destructive [case (b)] interference, folding in a Gaussian beam-energy resolution function of the form

$$\mathcal{G}(W, W') = (2\pi\Delta^2)^{-1/2}\exp[-(W - W')^2/2\Delta^2]$$

($W \equiv s^{1/2}$), using $\Delta \simeq 0.8$ MeV, typical of a machine of SLAC-SPEAR dimensions. We have not included the effects of radiative corrections, as these only modify the ψ peak height and the magnitude of the tail above the ψ mass. The results are shown in Fig. 1; the interference effect is seen to be quite dramatic. With a machine luminosity of 10^{32} cm^{-2} sec^{-1}, one can hope for 4–40 events/hour in the region of the dip predicted for case (a). The results of such an experiment (which seems quite feasible, for example, at the new Orsay colliding-beam facility DCI) would be of great interest.

In closing, we would like to point out that, quite apart from the determination of G_{dir}, the sign of the interference term might be of use in constraining theoretical models of the ψ. At the moment, although a plethora of such models are available,[8] there are no indications from experiment as to the nature of the new quark(s) from which the ψ particles are built. The interference effect discussed here allows determination of the sign of the product $g_{\gamma\omega}g_{\gamma\psi}g_{\omega\rho\pi}G_{dir}(\psi \to \rho\pi)$. Now, the sign of $g_{\omega\rho\pi}$ can be determined in a variety of ways, for example, through a careful analysis of the reaction $e^+e^- \to \pi^0\gamma$ using vector-meson dominance.[9] Should an independent determination of the sign of G_{dir} (or of the relative sign of $g_{\omega\rho\pi}$ and G_{dir}) be possible, one would know the relative sign of $g_{\gamma\psi}$ and $g_{\gamma\omega}$, which is related in a well-known way to the sign of the charge of the constituent quarks of ψ. One might then fix the sign of the charge of the new quark without needing to locate and characterize a large number of elusive particles with new quantum numbers. Of course, the experimental determination of the sign of G_{dir} (or of the relative sign of $g_{\omega\rho\pi}$ and G_{dir}) may prove to be just as difficult, but we feel that the possibility of such tests of theoretical models of the ψ is certainly worthy of attention.

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[1]This occurred to many people: See, for example, J. Bjorken and S. Brodsky, in SLAC Report No. SLAC-PUB-1515, 1974 (unpublished); and Michael S. Chanowitz, Phys. Rev. D 12, 918 (1975).

[2]C. Bernardini, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Cornell University, Ithaca, N. Y., 1971).

[3]The vector-meson pole model used to obtain this number entails an error of typically 10–20%, which we do not explicitly quote everywhere but which should be borne in mind through the remainder of the paper. All errors given arise from experimental uncertainties.

[4]A. Boyarski *et al.*, SLAC report SLAC-PUB-1599, paper submitted to the Palermo Conference, 1975, by the SLAC-LBL Magnetic Detector Collaboration (unpublished).

[5]A similar pole model, with *real* coupling constants, was found to provide an excellent fit to the data obtained in a recent experiment on ω-ϕ interference near the ϕ mass; see G. Parrour *et al.*, report (unpublished).

[6]See, for example, Harvey L. Lynch, lectures delivered at the Cargèse Summer Institute, Cargèse, Corsica, France, 1975, SLAC Report No. SLAC-PUB-1643, 1975 (unpublished).

[7]S. Okubo, Phys. Lett. 4, 14 (1963); *ibid.* 5, 165 (1963). Note that while some of the relations derived by Okubo from unitary symmetry do not seem to fare too well experimentally at present [for example, one predicts $\Gamma(\rho\to\pi\gamma)=\frac{1}{9}\Gamma(\omega\to\pi\gamma)\simeq 100$ keV, while the latest number from experiment is 35 ± 10 keV: B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974)], the ones that are of interest to us, relating $\gamma\rho\pi$ and γK^*K, are found to hold quite well [see W. C. Carithers *et al.*, Phys. Rev. Lett. 35, 349 (1975)].

[8]Haim Harari, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies*, Stanford, edited by W. T. Kirk (SLAC, Stanford University, Stanford, California, 1976), p. 317; J. Ellis, and F. E. Close, lectures given at the XV Cracow School of Physics, Zakopane, Poland, 1975 [CERN Reports Nos. CERN TH. 1996 and CERN TH. 2041, respectively (unpublished)].

[9]F. M. Renard, Nucl. Phys. B82, 1 (1974).