

Monopole-track characteristics in plastic detectors

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Total and restricted energy loss rates are calculated for magnetic monopoles of charge $g = 137e$ in Lexan polycarbonate. Range-energy curves are also presented. The restricted-energy-loss model is used to estimate the appearance of a monopole track in plastic detectors. The results are applied to the event observed by Price *et al.* and identified by them as a monopole. It is found that the observed etch rate is consistent with what one would expect for a slow magnetic monopole ($\beta \sim 0.5$, $g = 137e$). These results should also be of use to other investigators for both the design and analysis of monopole experiments.

It has been shown that one can describe the electromagnetic interaction of magnetic monopoles of charge g and velocity βc with matter by the following procedure: Solve the analogous problem of electric charges of charge Ze and velocity βc and then make the substitution $g\beta$ for Ze .^{1,2} This leads to an approximately constant energy-loss rate $[(Ze/\beta)^2 - g^2]$. I will advance further arguments for this prescription in this paper. I will also demonstrate that this result does not imply that a magnetic monopole of charge $g = 137e$ will appear at all velocities like a minimum-ionizing electric charge with $Z = 137$ in dielectric track recorders. This fact seems to have escaped the attention of monopole investigators in the past.

I. ELEMENTARY TREATMENT OF ENERGY LOSS OF ELECTRICALLY AND MAGNETICALLY CHARGED PARTICLES

Let us first consider the electron production spectra for electric and magnetic charges. We will consider projectiles much more massive than an electron so that the classical kinematic limit of energy transfer is given by $w_m = 2m_e c^2 \beta^2 \gamma^2$. Letting $dn/dw dx$ denote the electron production spectrum per unit length per unit energy, we note that

$$\frac{dn}{dw dx} = \frac{4\pi N}{w_m} \frac{d\sigma}{d\Omega'} \quad (1)$$

where N is the electron number density and $d\sigma/d\Omega'$ is the differential cross section for the scattering of a free electron by the heavy particle in the center-of-momentum frame. $d\sigma/d\Omega'$ can be expressed as a function of θ' , the center-of-momentum scattering angle, by the relation $w = w_m \sin^2(\frac{1}{2}\theta')$.

The specific energy loss can be calculated immediately:

$$\frac{dE}{dx} = \int_{w_{\min}}^{w_{\max}} w \frac{dn}{dw dx} dw.$$

Atomic and quantum effects are taken into ac-

count by carefully considering the limits of integration.³ Clearly $w_{\max} = w(b_{\min})$ and $w_{\min} = w(b_{\max})$, where b is the impact parameter, because close collisions result in the largest energy transfers. Classically, one expects that $w_{\max} = w_m = 2m_e c^2 \beta^2 \gamma^2$. However, if b is small enough so that the wave packet of the electron overlaps with the (assumed massive and hence fixed) force center, then we expect quantum-mechanical limitations of the energy transfer, corresponding to $b_{\min} = \hbar/m_e v \gamma$, the de Broglie wavelength of the electron in the center-of-momentum frame. So w_{\max} is the smaller of the two quantities w_m and $w(b_{\min} = \hbar/m_e v \gamma)$.

The maximum impact parameter is determined by the adiabatic limit. The fields of a moving electric charge vary like $(b^2 + \gamma^2 v^2 t^2)^{-3/2}$, where $t=0$ when the charge is a distance b from the electron.³ The symmetry of Maxwell's equations implies the same dependence for magnetic charges.

Hence, for both magnetic and electric charges, the fields at the electron are appreciable for a time $t \sim b/\gamma v$. When $t \gtrsim 1/\omega$ ($1/\omega$ being a typical orbital time for an atomic electron), the force turns on and off so slowly that the electron is adiabatically perturbed with no net change of state. So energy transfer requires that

$$b < \frac{\gamma v}{\omega} = b_{\max}$$

and $w_{\min} = w(b_{\max})$.

Using the classical Rutherford cross section for an electric charge in Eq. (1) yields the familiar result

$$\left. \frac{dn}{dw dx} \right|_e = \frac{2\pi N Z^2 e^4}{m_e c^2 \beta^2 w^2}$$

(using the more correct Mott cross section⁴ would modify this result by a factor $1 - \beta^2 w/w_m$). Hence for electric charges, the results can be summarized as

$$\left. \frac{dE}{dx} \right|_e = \frac{\omega_p^2 Z^2 e^2}{2c^2 \beta^2} \ln \frac{w_{\max}}{w_{\min}}$$

where

$$\omega_p^2 = \frac{4\pi N e^2}{m_e},$$

$$w(b) = \frac{2Z^2 e^4}{m v^2 b^2} \quad (\text{for Rutherford scattering}),$$

$$w_{\max} = \begin{cases} 2m_e c^2 \beta^2 \gamma^2 & \text{if } Z/\beta > 137, \\ \frac{2Z^2 e^4 \gamma^2 m_e}{\hbar^2} & \text{if } Z/\beta < 137, \end{cases}$$

$$w_{\min} = \frac{2Z^2 e^4 \omega^2}{m_e v^4 \gamma^2}.$$

For $Z/\beta < 137$, the case we will be considering,

$$\left. \frac{dE}{dx} \right|_e = \frac{\omega_p^2 Z^2 e^2}{c^2 \beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{\hbar \omega}.$$

This result approximates the more accurate Bethe-Bloch relation which includes the effects of atomic or molecular binding and relativistic corrections,

$$\frac{dE}{dx} = \frac{\omega_p^2 Z^2 e^2}{c^2 \beta^2} \left[\ln \left(\frac{2m_e c^2 \beta^2 \gamma^2}{\hbar \langle \omega \rangle} \right) - \beta^2 \right].$$

The scattering of an electron by a magnetic monopole can be treated in a straightforward manner using relativistic classical mechanics⁵: There are no bound states and the orbit of the electron is a spiral on the surface of a cone. The center-of-momentum differential cross section is⁵

$$\frac{d\sigma}{d\Omega'} = \frac{4}{\pi^2} b^2 \frac{\delta^4}{(\sin^2 \delta - \delta \cos \delta \sin \delta) [(2\delta/\pi)^2 - 1]},$$

where

$$\cos(\tfrac{1}{2}\theta') = \pi \sin \delta / 2\delta$$

and

$$\delta = \frac{\pi}{2} \left(1 + 137^2 \frac{r_0^2}{\beta^2 \gamma^2 b^2} \right)^{1/2},$$

where r_0 is the classical electron radius, b is the impact parameter, and the charge of the monopole is taken to be $g = 137e$. One can find $d\sigma/d\Omega'$ as a function of w by direct computation:

$$\frac{dn}{dw dx} = \frac{2\pi N e^4 (137)^2}{m_e c^2 w^2} (1 + \xi),$$

$$\left(\frac{dE}{dx} \right)_b = \begin{cases} \frac{\omega_p^2 Z^2 e^2}{c^2 \beta^2} \left[\ln \left(\frac{1.123 \beta c \gamma (\epsilon - 1)^{1/2}}{\omega_p b \sqrt{\epsilon}} \right) - \frac{\beta^2}{2} \right], & \text{if } \beta < \epsilon^{-1/2} \\ \frac{\omega_p^2 Z^2 e^2}{c^2 \beta^2} \left[\ln \left(\frac{1.123 \beta c}{\omega_p b} \right) - \frac{(1 - \beta^2)}{2(\epsilon - 1)} \right], & \text{if } \beta > \epsilon^{-1/2}. \end{cases}$$

By utilizing the symmetry of Maxwell's equations, and extending Fermi's treatment to include permeability effects (this is essential since the

with $\xi \approx 0.08$ near w_{\max} (always determined by the quantum-mechanical, rather than kinematic, limit) falling rapidly to zero as w goes to zero. This result is independent of velocity (from $\beta = 0.05$ to $\beta = 0.95$). Thus, we can approximate

$$\left. \frac{dn}{dw dx} \right|_m = \frac{2\pi N e^4 (137)^2}{m_e c^2 w^2},$$

with an error no greater than 0.5% in dE/dx . We should note that for small scattering angles in the center-of-momentum frame, the cross section for a magnetic charge approaches the Rutherford cross section for an electric charge with the replacement $g\beta$ for Ze . Hence

$$w_{\min} = \frac{2g^2 e^2 \omega^2}{m_e v^2 c^2 \gamma^2}.$$

For $0.05 < \beta < 0.95$ it is found that $w_{\max} \approx 0.69 w_m$, so

$$\left. \frac{dE}{dx} \right|_m = \frac{\omega_p^2 e^2 (137)^2}{c^2} \ln \frac{0.83 m_e c^2 \beta^2 \gamma^2}{\hbar \omega},$$

where we have used $ge = \hbar c$. This is virtually identical to the electrical case except for the replacement of Ze by $g\beta$.

Since our ultimate goal will be to try to estimate what relativistic electric charge will mimic a monopole (this is the appropriate question one must ask if one is analyzing an event in which a particle apparently does not slow) we should attempt to formulate a consistent way of looking at the relativistic ionization of electric and magnetic charges. The method given above does not take into account the microscopic polarizability of the medium (i.e., the density effect) since it is based only on the microscopic collision cross section.

Since these effects are important at relativistic energies, we now consider the more accurate but less intuitive approach due to Fermi which includes these effects. Fermi's⁶ treatment of the energy loss of a moving electric charge in a medium which can be characterized by its low-frequency dielectric constant (ϵ) incorporates these macroscopic effects, based as it is on Maxwell's equations for the fields of the moving charge. Fermi obtained for the energy loss due to collisions with impact parameter greater than b

transcription of electric results to magnetic results requires an interchange of ϵ with μ , Tompkins⁷ obtained the analogous result for monopoles

$$\left(\frac{dE}{dx}\right)_b = \begin{cases} \frac{\omega_p^2 g^2}{c^2} \left[\ln \left(\frac{1.123 \beta c \gamma (\epsilon - 1)^{1/2}}{b \omega_p} \right) - \frac{1}{2} \right], & \text{if } \beta < \epsilon^{-1/2} \\ \frac{\omega_p^2 g^2}{c^2} \left[\ln \left(\frac{1.123 c}{b \omega_p} \right) + \frac{1 - 1/\beta^2}{2(\epsilon - 1)} \right], & \text{if } \beta > \epsilon^{-1/2}. \end{cases}$$

This applies to nonabsorbing (ϵ is real), non-permeable ($\mu = 1$) media. For total energy loss we let b be given by $\hbar/m_e v \gamma$, which was previously shown to take precedence over the kinematic limit for monopoles. To obtain ϵ we note that for $\beta < \epsilon^{-1/2}$, with $b = \hbar/m_e v \gamma$, Fermi's formula reads

$$\frac{dE}{dx} = \frac{\omega_p^2 Z^2 e^2}{c^2 \beta^2} \left[\ln \left(\frac{1.123 m_e c^2 \beta^2 \gamma^2}{\hbar \omega_p (\epsilon / \epsilon - 1)^{1/2}} \right) - \frac{\beta^2}{2} \right],$$

which is identical to the Bethe-Bloch formula with $(1/1.123) \hbar \omega_p [\epsilon / (\epsilon - 1)]^{1/2} = \frac{1}{2} \hbar \langle \omega \rangle$ (aside from the $\beta^2/2$ term instead of β^2). Here we will only be interested in Lexan polycarbonate, which has $\hbar \langle \omega \rangle = 69.5$ eV (this is the value of the logarithmic

mean excitation energy which one obtains for the composition of Lexan, $C_{16}H_{14}O_3$, by neglecting the effects of chemical binding¹⁷). For Lexan, $\hbar \omega_p = 22.8$ eV (which is obtained by using $N = 3.77 \times 10^{23}/\text{cm}^3$, which is appropriate for Lexan), so $\epsilon = 1.52$. (This is quite different from the true low-frequency dielectric constant for Lexan, $\epsilon = 3.17$,⁸ showing how poorly a single-oscillator model describes this complex polymeric material. However, we can safely apply this "fitted" ϵ to the description of the energy loss of a monopole, because ϵ relates to the properties of the medium, not the particle.) Using $g = 137e$, we obtain for the total energy-loss rate of a monopole in Lexan

$$\frac{dE}{dx} = \begin{cases} 3.00 \text{ GeV}/(\text{g}/\text{cm}^2) \times [9.31 + 2 \ln \beta \gamma], & \beta < 0.81 \\ 3.00 \text{ GeV}/(\text{g}/\text{cm}^2) \times [11.09 + \ln \beta \gamma - 0.96/\beta^2], & \beta > 0.81 \end{cases}$$

II. FORMATION OF ETCHABLE TRACKS IN LEXAN

The reader is referred to Ref. 9 for a complete description of the principles of particle detection and identification by means of observations of their tracks in dielectric solids. Unless stated otherwise, the following information on tracks is extracted from that reference.

When heavily ionizing particles pass through certain dielectric solids they leave semipermanent records of their passage by the formation of tracks: localized regions of intense radiation damage. These latent tracks, being smaller than the wavelengths of visible light, can be made observable by chemical etching which causes the damaged region to be removed at a rate V_T which is greater than the general rate of removal V_G . This results in the formation of a cone of half angle θ given by $\sin \theta = V_G/V_T$. The cone can be observed through an optical microscope. The length of the cone is used to determine V_T which is then used to identify the particle.

Many models have been proposed for the description of track formation, but none has enjoyed great quantitative success. Table 1-5 in Ref. 9 lists the more attractive models. At present no particular model fits observed data over all atomic numbers and velocities. For Lexan polycarbonate detectors, power laws of the form $V_T = A(Z^*/\beta)^6$ are used to fit the data. (Z^* is the effective charge

of the nucleus; $Z^* \rightarrow Z$ for large β .) Since this is not derived from some physical model it is not surprising that δ varies from batch to batch of Lexan, from exposure to exposure, and even as a function of V_T . Even though no firm track-formation model exists, several qualitative features are known. It seems that the primary mechanism for track formation is electronic excitation and ionization rather than atomic displacement, although the latter may be relevant at energies of the order 1 keV/amu. It is also known that the size of the latent track is very small. Transmission electron microscopy has been used in the thickness-contrast mode to measure a latent track diameter of ~ 60 Å for fission fragments in mica. Electrical conductance measurements of freshly etched cylinders at the time of breakthrough give a pore radius of 33 Å for fission-fragment tracks in mica and ~ 50 Å for fission-fragment tracks in Lexan.

Using these facts we want to make the best guess as to the nature of the track formed by a monopole with $g = 137e$ and velocity βc . It is not sufficient to replace Z^*/β with 137 in the power-law fit mentioned above. This would give an etch rate totally independent of velocity, a result which cannot be correct since $\beta \rightarrow c/137$ ionization, and hence radiation damage, must cease [of course, if $\delta = \delta(\beta)$, then $V_T = V_T(\beta)$]. Rather than do this, we want to find the property which characterizes a

track, to calculate the nature of this property for monopoles, and to determine the corresponding electric charges which would produce the same etch rate.

The restricted-energy-loss model (see Table 1–5 of Ref. 9) is convenient in this regard. This model holds that the character of the track is determined by the energy loss which contributes to δ rays with energy less than some fixed amount. This model is attractive because it is consistent with the observed narrow track widths. For Lexan the value of the fixed amount is typically taken to be $w_0 = 350$ eV. (This number comes from empirical fits of accelerator data.)¹⁰ The restricted-energy-loss model neglects the effects of high-energy δ rays. Strictly, this is inappropriate. However, a 500-keV electron deposits only ~ 2 eV in a typical Lexan track core, whereas a 350-eV electron will excite a random walk and deposit all of its energy in the core. These notions are consistent with those involving a critical dose for track recording. It is well known in radiation chemistry that organic solids suffer degradation (chain scission for polymers, formation of free radicals, etc.) when subjected to intense doses of radiation.¹¹ Typically, if the dose is less than some critical value, little change in material properties is observed, whereas doses exceeding the critical value lead to extremely rapid deterioration (this is even true for human beings; there

is a fine line between apparently mild radiation sickness and almost certain death). If one assumes that latent track formation is determined by the size of the cylinder in which a critical dose is attained, then it seems reasonable that high-energy δ rays can be neglected in evaluating track formation. There are relatively low numbers of these high-energy electrons and their energy is dissipated over much larger volumes than is the energy of low-energy electrons.

We let $(dE/dx)|_{350}$ denote the restricted energy loss for a monopole in Lexan. The impact parameter corresponding to 350 eV is $b_{\min} \sim \hbar/(2m_e w_0)^{1/2}$, where $w_0 = 350$ eV. This is so because if the impact parameter were smaller than this, the uncertainty principle would imply that we could not know that the energy transfer was less than w_0 . In order that we treat electric and magnetic charges consistently with respect to each other we will take $b_{\min} = 1.123\hbar/(2m_e w_0)^{1/2}$. The factor 1.123 is introduced to get agreement with a quantum-mechanical treatment of restricted energy loss for electric charges (see Ref. 3, p. 442). Since the minimum impact parameter is determined from quantum-mechanical kinematic constraints, it is the same for both electric and magnetic charges.

Inserting this value for b_{\min} into Tompkin's formula we have for a monopole in Lexan

$$b_{\min} = 0.117 \text{ \AA},$$

$$\left(\frac{dE}{dx}\right)_{350} = \begin{cases} 3.00 \text{ GeV}/(\text{g}/\text{cm}^2) \times [5.90 + \ln\beta\gamma], & \beta < 0.81 \\ 3.00 \text{ GeV}/(\text{g}/\text{cm}^2) \times [7.68 - 0.96/\beta^2], & \beta > 0.81. \end{cases}$$

For relativistic electric charges we obtain from Fermi's formula

$$\left(\frac{dE}{dx}\right)_{350}^{(e)} = 1.08Z_e^2 \text{ MeV}/(\text{g}/\text{cm}^2).$$

This result is in agreement with Eq. (13.80) of Ref. 3.

We now define the effective electric charge Z_e by $1.08Z_e^2 \text{ MeV}/(\text{g}/\text{cm}^2) = (dE/dx)_{350}(\beta)$. Z_e is the atomic number of a relativistic nucleus which would produce a cone identical to that of a magnetic monopole of speed β .

Figure 1 gives the functional dependence of Z_e on β . In Fig. 1 we also plot the restricted and total energy-loss rates for monopoles with $g = 137e$ in Lexan. It is seen that at $\beta \approx 0.01$ ionization reaches a very low level. The precise way in which dE/dx approaches zero is difficult to calculate. Nevertheless, at these low velocities energy

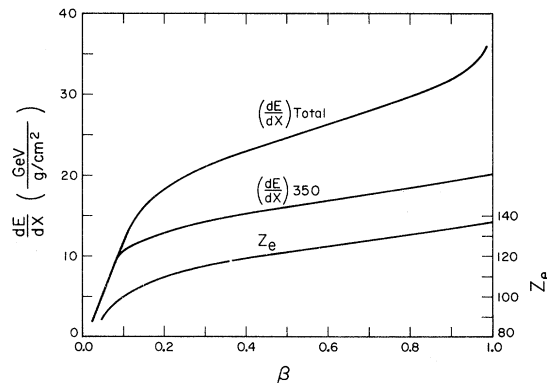


FIG. 1. Total and restricted energy loss rate for monopole with charge $g = 137e$ in Lexan. $Z_e(\beta)$ is the electric charge with $\beta = 1$ which would produce a track identical to that of a monopole with velocity βc .

losses due to collisions with nuclear Coulomb potentials and with the diamagnetic repulsive potentials of the inner core electrons of atoms dominate. It is these elastic collisions which rapidly thermalize the monopole. The curve for total energy loss may also be in error at large β : at $\beta = 0.95$, $b = \hbar/m_e v \gamma = 0.33 \hbar/m_e c < \hbar/m_e c$, so our treatment of close collisions is suspect if relativistic quantum effects are important.

By integrating $(dE/dx)_{\text{total}}$ we can find range-energy curves for monopoles of various masses,

$$R(\beta) = Mc^2 \int_{\beta}^{0.01} \frac{\beta d\beta}{(1-\beta^2)^{3/2} (dE/dx)(\beta)}.$$

We choose $\beta = 0.01$ as the lower limit on velocity since this is the effective limit for ionization. For any ionization-sensitive instrument, this is the effective end-of-range, since further motion will not register.

In Fig. 2 we plot $R(\beta)/Mc^2$ as a function of $\gamma - 1$. Note the slight upturn in this function at low velocities due to the decreased energy-loss rate.

If one assumes the restricted-energy-loss model to be accurate, then the principle sources of error in our calculations are uncertainties in w_0 and ϵ as well as the usual problems with calculating dE/dx (are close and distant collisions treated properly?). Jackson¹² has estimated that these model-independent errors cause an uncertainty in Z_e of ± 5 charge units at every velocity. Of course, we cannot place a numerical degree of uncertainty on the model-dependent errors, since we do not know what the true behavior is. Hence, knowledge of the true track-formation mechanism is needed to evaluate the overall accuracy of our results.

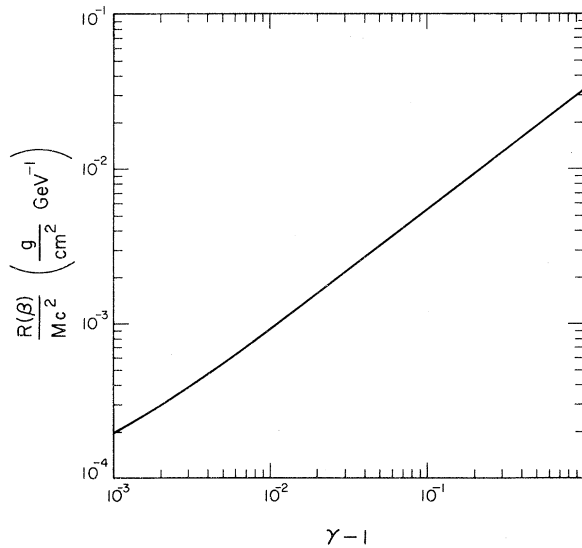


FIG. 2. Range-energy curve for a magnetic monopole with $g = 137e$ in Lexan.

III. APPLICATIONS TO THE "MONOPOLE" EVENT

Price *et al.*¹³ have recently reported a cosmic-ray event which they interpret as a moving magnetic monopole ($\beta = 0.5_{-0.05}^{+0.1}$) traversing their detectors. Figure 2 of Ref. 13 indicates the apparent atomic number Z_e to be 137. Subsequently, experimental calibrations with iron cosmic-ray nuclei have reduced the measured effective charge to ~ 114 .¹⁴ The results of my calculations (Fig. 2), indicate that for a monopole with $\beta = 0.45$, $Z_e = 120$ and for a monopole with $\beta = 0.6$, $Z_e = 125$. If one uses Jackson's error estimate of ± 5 for Z_e , then it is seen that a measured effective charge of 114 is consistent with that value predicted from a restricted-energy-loss model of track formation.

In Fig. 3 we plot the data of Price *et al.* with a slight modification. The scale for the depth has been changed to agree with corrections subsequent to Ref. 13 involving the construction of the detectors. Also, we make no distinction between the triangles and solid black dots (20-h and 30-h etch times) since the respective iron calibrations are reported to show no systematic differences between the two sets of data.¹⁴

If one assumes that the curve which fits the data best is smooth (i.e., that the event is not a fragmenting nucleus, which case has been treated elsewhere),¹⁵ then the best-fit curve is the straight line:

$$V_T = \left(2.91 - 0.0406 \frac{X}{\text{g/cm}^2} \right) \frac{\mu}{\text{h}} \quad (X = \text{depth}).$$

The error at 1 standard deviation on the slope is $\pm 0.0658 (\mu/\text{h})/(\text{g/cm}^2)$.

If the event truly was a monopole, one can place limits on the mass (which are considerably more stringent than that of $200m_p$ in Ref. 13) by requiring that the slope be within an appropriate confidence interval. We have

$$\frac{dV_T}{dx} = \frac{dV_T}{d\beta} \frac{d\beta}{dE} \frac{dE}{dx}.$$

If we assume $V_T \sim (dE/dx)_{350}^{2.53}$, consistent with power-law fits of the form $V_T \sim (Z/\beta)^{5.07}$ (the value

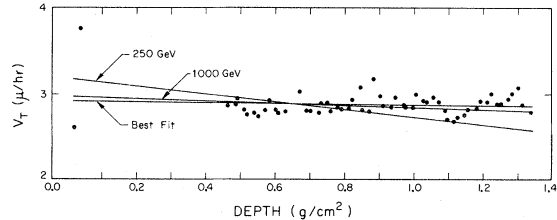


FIG. 3. Data of the event observed by Price *et al.* Superimposed are curves of etch rate vs depth which one would expect for a monopole with mass 250 GeV, 1000 GeV.

5.07 is taken from Ref. 14), then $V_T = K(5.9 + \ln\beta\gamma)^{2.53}$, where K is a constant determined by the data,

$$\frac{dV_T}{d\beta} = 2.53(5.9 + \ln\beta\gamma)^{1.53} \frac{\gamma^2}{\beta} \times K$$

and

$$\frac{d\beta}{dE} = \frac{1}{Mc^2\beta\gamma^3}$$

with

$$\frac{dE}{dx} = -3 \frac{\text{GeV}}{\text{g/cm}^2} \times (9.31 + 2 \ln\beta\gamma),$$

assuming $\beta < 0.81$, which is greater than the limit demanded by the published interpretation of the Čerenkov film evidence.¹³

I will assume that β changes slowly enough so that $V_T(X)$ can be regarded as a straight line. Then

$$K = 2.91 \frac{\mu}{h} \times [5.9 + \ln\beta\gamma]^{-2.53}$$

and

$$-\frac{dV_T}{dx} = 22.1 \frac{\text{GeV}}{Mc^2} \frac{1}{\beta^2\gamma} \frac{(9.31 + 2 \ln\beta\gamma)}{(5.9 + \ln\beta\gamma)} \frac{\mu/h}{\text{g/cm}^2}.$$

If the event is a monopole, its slope must be less than 0. At the 84% coincidence level, its slope cannot be less than $-0.106 (\mu/h)/(\text{g/cm}^2)$. This means that

$$\frac{Mc^2}{\text{GeV}} > 208 \frac{1}{\beta^2\gamma} \frac{(9.31 + 2 \ln\beta\gamma)}{(5.9 + \ln\beta\gamma)}$$

at the 84% confidence level. Some examples are

$$Mc^2 > \begin{cases} 1800 \text{ GeV} & \text{if } \beta = 0.4, \\ 1110 \text{ GeV} & \text{if } \beta = 0.5, \\ 719 \text{ GeV} & \text{if } \beta = 0.6, \\ 478 \text{ GeV} & \text{if } \beta = 0.7. \end{cases}$$

These large masses are consistent with the suggestion by 't Hooft¹⁶ that certain gauge theories imply the existence of monopoles with rest mass on the order of $137 M_w$, where M_w is the mass of the weak intermediate vector boson.

In Fig. 3 we sketch approximate curves of V_T vs X for various masses at $\beta = 0.5$.

IV. REVELANCE OF RESULTS TO MONOPOLE SEARCH EXPERIMENTS

The results of this paper have direct bearing on two classes of monopole experiments: those which look for moving magnetic monopoles with ionization-sensitive instruments and those which look for trapped monopoles by extraction from minerals with magnetic fields (which invariably use ionization-sensitive instruments for the detection of the monopoles after extraction). For both types of experiments, care should be taken that gains and thresholds of the ionization-sensitive detectors are adequate for the conditions of the experiment (for example, Lexan polycarbonate would not even detect monopoles if $\beta \approx 0.05$, because of the reduction of specific energy loss at low velocities). One should also consider the effects of the possibly high mass of the monopole with regard to an extraction-type experiment. If the mass is large, there could be two consequences: (1) The monopole would not follow field lines if it was rigid enough; (2) extracted velocities might be insufficient to trigger ionization detectors.

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