Comments on the production and decay of charged heavy leptons*

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Several issues concerning charged heavy leptons ($L^{\pm}$) are considered for the production and decay sequence $e^+ + e^- \rightarrow L^+ + L^- \rightarrow$decay products. We note that certain angular distributions of decay products contain information on the L^+L^- production process independent of any details of the decays. For leptonic decays $L \rightarrow \nu_L + l + \bar{\nu}_l$ ($l = e$ or μ) other aspects of the joint energy and angular distributions provide further information concerning the helicity structure of the weak leptonic interactions. Several other issues concerning semileptonic decays are also discussed. In particular some special cases are described where parity-violating effects would uniquely signal the presence of second-class currents.

I. PRODUCTION AND LEPTONIC DECAYS

Close to 100 events of the type $e^+ + e^- \rightarrow \mu^{\pm} + e^{\mp}$, unaccompanied by other charged particles, have been seen to date at SPEAR.[1,2] Only a very few of the events, at most, can have accompanying π^0's or K^0's (none have actually been seen). On these and other grounds a plausible but as yet by no means certified explanation of the μe phenomenon invokes the production and decay of heavy leptons $L^{\pm}$: $e^+ + e^- \rightarrow L^+ + L^-$, $L \rightarrow l + \bar{\nu}_l + \nu_L$ ($l = e$ or μ). The present experimental bounds on the mass m of $L^{\pm}$ are $1.6 \lesssim m \lesssim 2.0$ GeV. Analysis of the evidence from neutrino reactions suggests that ν_L is most probably not to be identified with the μ-type neutrino ν_μ.[3]

Already some years ago, speculation that heavy leptons might be needed in gauge theories of the weak interactions spurred theoretical inquiry into the production and decay properties of such particles.[4] The more recent developments just described have added new impetus to these issues. In this note we make a few additional comments on the subject.

An immediate question is whether $L^{\pm}$ are indeed leptons, i.e., spin-$\frac{1}{2}$ particles that do not participate directly in strong interactions. An interesting alternative that has recently been proposed[5] involves production of heavy fermions $(F, \bar{F})$, where F and $\bar{F}$ can decay leptonically but where strong forces act between F and $\bar{F}$. One obvious distinction between the two schemes has to do with the production cross section in e^+, e^- collisions. For the case of pointlike heavy leptons one has the familiar expression

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_0}{4\pi(3+\lambda)}(1 + \lambda\cos^2\vartheta), \tag{1.1}$$

where

$$\sigma_0(s) = \frac{4\pi}{3}\frac{\alpha^2}{s}\left(1 + \frac{2m^2}{s}\right)\left(1 - \frac{4m^2}{s}\right)^{1/2}, \tag{1.2}$$

$$\lambda(s) = \frac{s - 4m^2}{s + 4m^2} = \frac{\beta^2}{2 - \beta^2} \tag{1.3}$$

(β is the laboratory velocity of the heavy leptons). On the scheme involving strongly interacting fermion pairs, the angular distribution would have the same *form*, but in general both $\lambda(s)$ and $\sigma_0(s)$ would be expected to display a different dependence on the energy variable s. Of course one does not see the heavy leptons (or $F, \bar{F}$ fermions) directly. However, the shape of the curve of decay events versus s obviously preserves the shape of $\sigma_0(s)$. So too $\lambda(s)$ can be extracted from a study of the angular distribution of a decay product, i.e., of the particle x in L (or F)$\rightarrow x + \cdots$. The decay product x might be an electron or a muon, although any other light particle, e.g., a pion, would do as well. Provided only that x can be treated as effectively massless, then independent of any details of the decay reaction one has (θ is the angle between the beam direction and the direction of particle x)

$$\frac{d\sigma_x}{d\Omega} = B_x\frac{\sigma_0}{4\pi}\left\{1 + \frac{3\lambda}{3+\lambda}\left[\frac{3-2\beta^2}{\beta^2} - \frac{3(1-\beta^2)}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right] \times (\cos^2\theta - \tfrac{1}{3})\right\}, \tag{1.4}$$

where B_x is the branching ratio for (L or F)$\rightarrow x + \cdots$ and $\beta = (1 - 4m^2/s)^{1/2}$. An integration over all energies of x is implied here. Notice that, as $\beta \rightarrow 1$, the distribution approaches the form $1 + \cos^2\theta$ if $\lambda(s)$ is as given in Eq. (1.3). Our purpose in recording this result is to show how the fermion-*production* cross-section quantities $\sigma_0(s)$ and $\lambda(s)$ can be extracted from a study of decay products and therefore used to test whether the production is really pointlike.

On the interpretation that the μe phenomenon is associated with production and decay of heavy leptons, we turn next to the question of the nature of the weak couplings responsible for $L \to l + \bar{\nu}_l + \nu_L$. There are two general issues here: the mass of ν_L and the space-time character (V,A,S,P,T) of the presumably local interactions. The diagnostics have been discussed by a number of authors, in greatest generality by Pi and Sanda.[6,7] What has been particularly considered is the laboratory energy spectrum of e or μ, and, when both L^+ and L^- decay leptonically, the distribution in laboratory angle between e and μ. The spectra do depend on the "Michel parameters" that characterize the weak decay couplings but not, as it turns out, in a strongly sensitive way. Even if one entertains the healthy prejudice that the coupling is of current-current form, with the $(\nu_e e)$ and $(\nu_\mu \mu)$ currents of the usual $V-A$ sort, there still remain two important open issues: whether the $(\nu_L L)$ current is $V-A$ or $V+A$, the natural options associated with a massless ν_L, and, furthermore, whether or not ν_L is *indeed* massless. (If it is not, then more general V,A mixtures must also be contemplated.) In the following we restrict ourselves to a massless ν_L and to the pure helicity cases, $V-A$ or $V+A$.

As already said, as part of a more comprehensive analysis of possible coupling types various authors have focused on the e or μ energy distributions and collinearity-angle distributions for purposes of diagnosing the coupling types.[6,7] Here we wish to add some remarks about the joint energy and angular distribution (relative to the e^+, e^- beam axis) of e or μ coming from one of the heavy leptons. For definiteness let us speak of a muon decay product. For the sequence $e^+ + e^- \to L^+ + L^-$, $L \to \mu + \bar{\nu}_\mu + \nu_L$ one has

$$\frac{d\sigma}{dx d\Omega} = B_\mu \frac{\sigma_0}{4\pi}\left[A(x) + \frac{3\lambda}{3+\lambda} B(x)(\cos^2\theta - \tfrac{1}{3})\right], \tag{1.5}$$

where $\sigma_0(s), \lambda(s)$ are given in Eqs. (1.2) and (1.3) and θ is the angle between the muon and the e^+, e^- axis. The variable x is defined by

$$x = \frac{E_\mu}{E_\mu^{\max}}, \quad E_\mu^{\max} = \frac{m}{2}\left(\frac{1+\beta}{1-\beta}\right)^{1/2}, \tag{1.6}$$

where E_μ is the muon laboratory energy and $E_\mu^{\max}$ is its maximum possible energy (we are ignoring the muon mass). The normalization

$$\int_0^1 A(x)dx = 1$$

is implied, and then, from Eq. (1.4), we have also

$$\int_0^1 B(x)dx = \frac{3-2\beta^2}{\beta^2} - \frac{3(1-\beta^2)}{2\beta^3}\ln\frac{1+\beta}{1-\beta}.$$

We report here the distribution function $B(x)$, for the two cases $V-A$ and $V+A$ (the mass of ν_L assumed zero). For convenience we shall also give the distribution function $A(x)$ which has been studied before by several authors.[6,7] Let

$$x_0 \equiv \frac{1-\beta}{1+\beta}. \tag{1.7}$$

Then

(i) $V-A$:

$$A(x) = \begin{cases} \dfrac{2x^2(1+\beta)}{3(1-\beta)^3}[9(1-\beta) - 2x(3+\beta^2)], & 0 \le x \le x_0 \quad (1.8) \\[2ex] \dfrac{1+\beta}{6\beta}(1-x)(5+5x-4x^2), & 1 \ge x \ge x_0 \quad (1.9) \end{cases}$$

$$B(x) = \begin{cases} \dfrac{-8}{15}\dfrac{\beta^2(1+\beta)x^3}{(1-\beta)^3}, & 0 \le x \le x_0 \quad (1.10) \\[2ex] \dfrac{(1+\beta)(1-x)}{120\beta^3 x^2}[3(1-x)^2(21+3x-4x^2) - 18\beta(1-x)(7+4x+x^2-2x^3) + \beta^2(63+63x+13x^2+13x^3-32x^4)], & 1 \ge x \ge x_0 \quad (1.11) \end{cases}$$

(ii) $V+A$:

$$A(x) = \begin{cases} \dfrac{4x^2(1+\beta)}{(1-\beta)^3}[3(1-\beta) - x(3+\beta^2)], & 0 \le x \le x_0 \quad (1.12) \\[2ex] \dfrac{1+\beta}{\beta}(1-x)^2(1+2x), & 1 \ge x \ge x_0 \quad (1.13) \end{cases}$$

$$B(x)=\begin{cases}\dfrac{-8}{5}\,\dfrac{\beta^2(1+\beta)x^3}{(1-\beta)^3}\,, & 0\le x\le x_0 \qquad (1.14)\\[2ex] \dfrac{(1+\beta)(1-x)^2}{20\beta^3x^2}[3(1-x)^2(3+2x)-6\beta(1-x)(3+4x+3x^2)+\beta^2(9+18x+17x^2+16x^3)], & 1\ge x\ge x_0. \qquad (1.15)\end{cases}$$

All the A and B distributions vanish at $x=0$ and $x=1$. In addition $B(x)$ vanishes at an internal point of the interval and so passes through both a maximum and a minimum.

We have examined the distribution function $B(x)$ for the $V-A$ and $V+A$ cases at various beam energies s (i.e., various values of β). The distinctions between the two helicity cases are neither much more nor much less pronounced for $B(x)$ than for $A(x)$; i.e., the two distributions may be of comparable and complementary value as diagnostics.

II. SEMILEPTONIC DECAYS

The remaining questions that we wish to comment on have to do with semileptonic decays of heavy leptons. It may be, in principle, that an e^+,e^- annihilation event leading to production of a heavy-lepton pair would be signaled reliably by detection of a single prompt μ or e from leptonic decay of one of the heavy leptons. In such an event *all* accompanying hadrons would then be associated with a semileptonic decay of the other heavy lepton: $L\to\nu_L+\text{hadrons}$ (up to electromagnetic corrections). Of course the existence of other "background" sources of single detected leptons would make this kind of association hazardous.

Even if such background problems are surmounted there remain further difficulties in the theoretical analysis of semileptonic decays. Indeed we have been struck time and again by the loss of information consequent upon integration over the variables referring to the neutrinos produced in the decay processes. These difficulties are compounded by the possible creation of neutral mesons in semileptonic decays. This is particularly unfortunate since some of the most interesting issues for semileptonic decays are best tackled by a study of select exclusive channels. One may of course also look upon semileptonic decays in an inclusive fashion. But then, in our experience, one often "integrates away" the answers to interesting questions. However, we are given to understand that the next generation of detectors will be able to cope much better with the selection of exclusive channels than is presently possible. We therefore permit ourselves to consider certain exclusive channel issues. Let us then turn next to some of the theoretical issues.

Even if it is supposed that the new leptons $(\nu_L L)$ couple in current form to hadronic matter, there is the question whether the *hadronic* currents to which they couple are the same ones encountered in the usual $(e\nu_e),(\mu\nu_\mu)$ semileptonic interactions. The study of exclusive channels would shed light on this matter. For example, the very detection of the exclusive decay $L^\pm\to\nu_L+\pi^\pm$ would already be informative in revealing the existence of an axial-vector piece in the hadronic current. Moreover, if the new leptons couple with the usual strength to the usual hadronic currents, *and* if ν_L is massless, the branching ratio $(L\to\nu_L+\pi)/(L\to\nu_L+\mu+\overline{\nu}_\mu)$ can be predicted from the known rates for $\mu\to e+\overline{\nu}_e+\nu_\mu$ and $\pi\to\mu+\nu$ decays.[4,7] Similarly the detection of the exclusive process $L^\pm\to\nu_L+\pi^\pm+\pi^0$ would reveal the existence of a vector piece in the hadronic current.[8] We may also note that any asymmetry between the charged and neutral pions in $L^\pm\to\nu_L+\pi^\pm+\pi^0$ decay could arise only as a result of isotopic-spin interference—between the expected $I=1$ and a possible $I=2$ current. This leads us to the following more general remarks.

(i) *Isospin considerations*. If the $(\nu_L L)$ current couples to the usual charge-changing hadronic current, then the semileptonic decays will be strongly dominated by $|\Delta I|=1$, $\Delta S=0$ transitions. To within small $\Delta S\neq 0$ corrections this implies a unmber of inequalities. Let

$$\begin{aligned} L^+&\to\overline{\nu}_L+\pi^++\cdots\,, \quad (R^+),\\ &\to\overline{\nu}_L+\pi^0+\cdots\,, \quad (R^0),\\ &\to\overline{\nu}_L+\pi^-+\cdots\,, \quad (R^-),\end{aligned}$$

denote inclusive pion production, where the R's are rate symbols. Then a pure $I=1$ structure of the currents implies[9]

$$R^-\le 2R^0\le 2R^++\tfrac{2}{3}R^-. \qquad (2.1)$$

For the $\Delta S=0$ modes

$$\begin{aligned} L^+&\to\overline{\nu}_L+K^++\cdots\,, \quad (r^+),\\ &\to\overline{\nu}_L+K^0+\cdots\,, \quad (r^0),\end{aligned}$$

one has

$$r^0\le 3r^+, \qquad (2.2)$$

and likewise for $K^+\to\overline{K}^0$, $K^0\to K^-$. Analogous relations can easily be derived for $\Delta I=1/2$, $|\Delta S|=1$

channels. More detailed relations obtain for pure pionic channels: $L^+ \to \bar{\nu}_L N\pi$. Let R_m^N denote the rate for production of N pions, of which m are neutral. Then[10]

$$\tfrac{1}{4}R_0^3 \le R_2^3 \le R_0^3, \tag{2.3}$$

$$R_3^4 \le \tfrac{2}{3}R_1^4, \tag{2.4}$$

etc.

(ii) *Tests for CP invariance*. Semileptonic decays of $L^\pm$ have a unique advantage for tests of CP invariance, under the condition being considered here where L^+L^- pairs are created in e^+, e^- annihilation. Namely, since L^+ and L^- are both produced in the same reaction, with the same laboratory energies, the CP comparisons (in principle at least) ought to be unusually direct.

The tests as such are obvious. One compares conjugate decay reactions, whether exclusive or inclusive, e.g.

$$L^+ \to \bar{\nu}_L + \pi^+ + \cdots$$

vs

$$L^- \to \nu_L + \pi^- + \cdots$$

or $L^+ \to \bar{\nu}_L + \pi^- + \cdots$ vs $L^- \to \nu_L + \pi^+ + \cdots$ etc. The spectra for the conjugate processes must be identical if CP invariance obtains (with the obvious interchange particle $\leftrightarrow$ antiparticle, and with all momenta reversed).

(iii) *Inclusive decay* $L \to \nu_L + K^+ + \bar{K}^0 + pions$. As we have already noted, the detection of certain exclusive decay processes, e.g., $L \to \nu_L + \pi$, $L \to \nu_L + \pi + \pi$, would serve to reveal the presence of axial and vector hadronic currents, respectively. Such a simple question as this is much harder to get at with inclusive reactions. We note here, however, a special case where it is possible to test for interference between V and A hadronic currents. Actually what is tested is interference between even- and odd-G-parity currents; if the currents are exclusively first-class, this means V,A interference. We will return to the question of second-class currents later on. Now consider the inclusive reactions $L \to \nu_L + K^+ + \bar{K}^0 +$ any number of pions (the same reasoning will hold for $L \to \nu_L + K^- + K^0 +$ any number of pions). Let k_1 and k_2 be the momenta, respectively, of K^+ and $\bar{K}^0$. Suppose that the current is purely vector (even G parity). Then channel by channel, for any definite number N of accompanying pions, the $K^+\bar{K}^0$ system must be in a state of definite G parity (even if N is even, odd if N is odd). But definite G parity for the $K^+\bar{K}^0$ system implies *symmetry* under $k_1 \leftrightarrow k_2$ in the decay spectrum. Clearly the same conclusion follows if the current is pure axial vector (odd G parity). Thus, the detection of any asymmetry between the K^+ and $\bar{K}^0$ spectra would signal the existence of V,A interference (or more generally, interference between currents of opposite G parity). It will be obvious that this diagnostic works for $K^+\bar{K}^0$ or K^-K^0 pairs, but *not* for other $K\bar{K}$ combinations, e.g. K^+K^-.

(iv) *Tests for second-class currents*. The preceding discussion already draws attention to the fact that G-parity considerations play an important role in the selection rules for the semileptonic decay of heavy leptons. This led us to consider whether there is any way to test uniquely for the presence of second-class currents (i.e., vector currents of odd G parity, axial-vector currents of even G parity). The question is of special interest since recent experiments on nuclear β decay have raised the spector of such currents.[11] We are aware that second-class currents are particularly unpalatable from the point of view of gauge models of the weak interactions. However that may be, heavy leptons offer a new opportunity to study the question experimentally.

In connection with the inclusive decays $L \to \nu_L + K^+ + \bar{K}^0 +$ pions we have seen that any asymmetry in the spectra of K^+ and $\bar{K}^0$ would signal the interference of currents of opposite G parity—on a most conservative interpretation this could be associated with purely first-class V,A interference, although the effect would also arise from first- (I) and second- (II) class interferences $V_{\rm I}V_{\rm II}, A_{\rm I}A_{\rm II}$. More decisive tests arise for channels involving particles of definite G parity. For example, in an N-pion channel, in the absence of second-class currents only the vector or the axial hadronic current can contribute (depending on whether N is even or odd), so that quite generally the observation of hadronic V,A interference here would signal the presence of second-class currents. This might suggest for such channels a search for parity-violating effects. However, such effects can arise not only from hadronic V,A interference but also from V,A interference at the leptonic vertex, so that, in the general case, parity violation does not provide a unique signal. However, for certain simple channels parity violation *does* provide a unique test for second-class contributions.

Consider, for example, the exclusive channel

$$L^\pm \to \nu_L + \pi^\pm + \phi, \quad \phi \to K + \bar{K}$$

(notice that only $I=1$ currents come into play here). In the absence of second-class interactions only the hadronic vector current can contribute to this process and in this situation there can be no effects arising from V,A interference at the lepton vertex, i.e., no parity-violating effects whatever. To see this it is enough to consider the structure of the matrix element of the current between the vacuum and $\phi\pi$ states. For a vector current V_μ the matrix element has the unique form $\epsilon_{\mu\nu\rho\sigma}\,\xi_\nu p_\rho q_\sigma$, where ξ and p are the polarization and momentum vectors of the ϕ meson, and q is the momentum of the pion. Namely, the $\phi\pi$ system is in a definite (p-wave) orbital state and therefore there cannot be any parity-violating effects arising from V,A interference at the lepton vertex. Parity violation can arise only if there is a (second-class) axial-vector contribution to the amplitude, hence the detection of parity-violating effects would signal not only the existence of a second-class current but but more specifically the existence of a second-class axial-vector current. Parity violation, in turn, would reveal itself via a term in the decay spectrum of the form $(\vec{k}\times\vec{\bar{k}})\cdot\vec{q}$, where $\vec{k},\vec{\bar{k}},\vec{q}$ are the momenta of $K,\bar{K},\pi$. Insofar as CP invariance holds true, such a correlation term also relies on the existence of final-state interactions for the $\phi\pi$ system. It is obvious that similar considerations hold for other special channels, e.g. $L\to\nu_L+\omega+\pi^-$, $\omega\to 3\pi$.

In still other situations potential signatures for second-class currents are again available but the tests become more subtle. We conclude with an example:

$$L^\pm\to\nu_L+\pi^\pm+\pi^\pm+\pi^\mp\,.$$

Let $k^{(1)},k^{(2)},k^{(3)}$ be the momenta of the three pions, let $q=k^{(1)}+k^{(2)}+k^{(3)}$, and let K be the total four-momentum of the colliding e^+ and e^- particles. Even in the absence of second-class currents a parity-violating term of the form

$$f(K\cdot q,k^{(i)})\epsilon_{\alpha\beta\gamma\delta}K_\alpha k_\beta^{(1)}k_\gamma^{(2)}k_\delta^{(3)}$$

can now arise, from leptonic V,A interference. However, a parity-violating term of the sort

$$g(K\cdot q,k^{(i)})K(k^{(i)}-k^{(j)})\epsilon_{\alpha\beta\gamma\delta}K_\alpha k_\beta^{(1)}k_\gamma^{(2)}k_\delta^{(3)},$$

with $i\neq j$, can arise only from *hadronic* V,A interference, hence only from interference between first- and second-class currents.

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