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Mass problem for tensor mesons*

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A model Lagrangian combining Yang-Mills fields with tensor fields is suggested for the dynamical generation of masses of all particles involved, without giving rise to the spin-zero ghost of Deser and Boulware.

It is well known that there exists no generally covariant Lagrangian model for a massive spin-2 particle. Less well known is the fact that, even if the demand of general covariance is given up, there exists no model Lagrangian for a massive spin-2 particle which does not contain a spin-zero ghost. The classical treatment given by Pauli and Fierz, while perfectly satisfactory for free fields, meets with difficulties when interactions are turned on. This was proved by Boulware and Deser,[1] who showed that the addition of any mass term—including in particular the Pauli-Fierz expression—to the Lagrangian of general relativity leads to inconsistencies even at the classical level: An extra ghostlike degree of freedom appears and causes the energy to be unbounded below.

When quantum effects are taken into account this effect persists. We can summarize the position by remarking that the Pauli-Fierz Lagrangian works for free fields owing to a peculiarly delicate balance in the mass term, a balance which in general would be disturbed by interactions. The point is easily illustrated by evaluating the propagator for the Lagrangian for a symmetric tensor $\phi_{\mu\nu}$ interacting with a source $J_{\mu\nu}$,

$$\mathcal{L}=\tfrac{1}{4}(\phi_{\mu\nu,\lambda}\phi_{\mu\nu,\lambda}-2\phi_{\mu\nu,\nu}\phi_{\mu\lambda,\lambda}+2\phi_{\mu\nu,\nu}\phi_{\lambda\lambda,\mu}-\phi_{\mu\mu,\lambda}\phi_{\nu\nu,\lambda})-\tfrac{1}{4}M^2(\phi_{\mu\nu}\phi_{\mu\nu}-b\phi_{\mu\mu}\phi_{\nu\nu})+\tfrac{1}{2}J_{\mu\nu}\phi_{\mu\nu}\ ,$$

which reduces to the Pauli-Fierz form when the parameter b is set equal to unity. The equations of motion are

$$\partial^2\phi_{\mu\nu}-\partial_\nu\partial_\lambda\phi_{\mu\lambda}-\partial_\mu\partial_\lambda\phi_{\nu\lambda}+\partial_\mu\partial_\nu\phi_{\lambda\lambda}-\eta_{\mu\nu}\partial^2\phi_{\lambda\lambda}+\eta_{\mu\nu}\partial_\lambda\partial_\sigma\phi_{\lambda\sigma}+M^2(\phi_{\mu\nu}-b\eta_{\mu\nu}\phi_{\lambda\lambda})=J_{\mu\nu}$$

and they are most easily solved by taking a Fourier transform and for, say, timelike 4-momentum referring to the rest frame:

$$-k^2(\tilde\phi_{\mu\nu}-\eta_{0\nu}\tilde\phi_{\mu 0}-\eta_{0\mu}\tilde\phi_{\nu 0}+\eta_{0\mu}\eta_{0\nu}\tilde\phi_{\lambda\lambda}-\eta_{\mu\nu}\tilde\phi_{\lambda\lambda}+\eta_{\mu\nu}\tilde\phi_{00})+M^2(\tilde\phi_{\mu\nu}-b\eta_{\mu\nu}\tilde\phi_{\lambda\lambda})=\tilde J_{\mu\nu}\ .$$

One finds in a general frame

$$\tilde\phi_{\mu\nu}=\frac{1}{M^2-k^2}\left[\left(\eta_{\mu\lambda}-\frac{k_\mu k_\lambda}{M^2}\right)\left(\eta_{\nu\sigma}-\frac{k_\nu k_\sigma}{M^2}\right)-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{M^2}\right)\left(\eta_{\lambda\sigma}-\frac{k_\lambda k_\sigma}{M^2}\right)\right]J_{\lambda\sigma}$$
$$+\frac{1-b}{2(1-b)k^2+(1-4b)M^2}\,\frac{1}{3}\left(\eta_{\mu\nu}+\frac{2k_\mu k_\nu}{M^2}\right)\left(\eta_{\lambda\sigma}+\frac{2k_\lambda k_\sigma}{M^2}\right)J_{\lambda\sigma}\ ,$$

which reveals the presence of a massive particle of spin 2 and a ghost of spin 0. For simplicity, suppose that the current $J_{\mu\nu}$ is conserved, i.e. $\tilde J_{0\nu}=0$ in the rest frame. Then the effect of one exchange is proportional to

$$-\tilde J_{ij}\tilde\phi_{ij}=\frac{1}{k_0^{\,2}-M^2}(\tilde J_{ij}\tilde J_{ij}-\tfrac{1}{3}\tilde J_{ii}\tilde J_{jj})$$
$$-\frac{b-1}{2(b-1)k_0^{\,2}+(4b-1)M^2}\,\frac{1}{3}\tilde J_{ii}\tilde J_{jj}\ .$$

If the overall sign is adjusted so that the spin-2 particle is normal then the spin-0 particle must be a ghost. The only escape is to choose $b=1$.

It is precisely this Pauli-Fierz requirement, $b=1$, that seems to be unstable against interactions. Some sort of symmetry principle is needed to enforce its stability and here we propose a tentative solution. Our main contention is that the tensor mass should arise from a dynamical process, i.e. that it should result from a nonperturbative self-consistent calculation.[2] This is of course a somewhat diffuse remark, and we shall attempt to make it more precise although in view of the unrenormalizability of the theory we shall not be able to make numerical estimates.

A second reason for preferring dynamical mass generation is that it should lead to a softening of the short-distance behavior—hopefully to the same extent as for zero-mass Einstein theory, though presumably not enough to make the theory conventionally renormalizable.

The generally noncovariant model we propose is given by the following Lagrangian:

$$\begin{aligned}\mathcal{L} = \tfrac{1}{4}(&\phi_{\mu\nu,\lambda}\phi_{\mu\nu,\lambda} - 2\phi_{\mu\nu,\nu}\phi_{\mu\lambda,\lambda} \\ &+2\phi_{\mu\nu,\nu}\phi_{\lambda\lambda,\mu} - \phi_{\mu\mu,\lambda}\phi_{\nu\nu,\lambda}) \\ &+\frac{1}{4\mu^2}\phi_{\mu\kappa}\phi_{\nu\lambda}\vec{F}_{\mu\nu}\cdot\vec{F}_{\kappa\lambda}\,, \end{aligned} \tag{1}$$

where $\vec{F}_{\mu\nu}$ denotes a Yang-Mills field [for SU(2), say]

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu\,.$$

The equations of motion are

$$\begin{aligned}\partial^2\phi_{\mu\nu} - \partial_\nu\partial_\lambda\phi_{\mu\lambda} - \partial_\mu\partial_\lambda\phi_{\nu\lambda} + \partial_\mu\partial_\nu\phi_{\lambda\lambda} - \eta_{\mu\nu}\partial^2\phi_{\lambda\lambda}& \\ +\eta_{\mu\nu}\partial_\kappa\partial_\lambda\phi_{\kappa\lambda} + \frac{1}{\mu^2}\vec{F}_{\mu\kappa}\cdot\vec{F}_{\nu\lambda}\phi_{\kappa\lambda} = 0\,,& \end{aligned} \tag{2}$$

$$(\partial_\mu + g\vec{A}_\mu\times)\left(\frac{1}{\mu^2}\phi_{\mu\kappa}\phi_{\nu\lambda}\vec{F}_{\kappa\lambda}\right) = 0\,. \tag{3}$$

At the classical level neither of these equations is very satisfactory. Neither equation has a mass term and the vector has no kinetic term in the usual linear sense. However, we shall suppose that, as a result of quantum effects, the operator products here develop c-number parts[3]

$$A^i_\mu A^j_\nu \to -\alpha\eta_{\mu\nu}\delta^{ij} + (A^i_\mu A^j_\nu)_{\text{quantum}}\,, \tag{4}$$

$$\begin{aligned}\phi_{\kappa\lambda}\phi_{\mu\nu} \to \beta(&\eta_{\kappa\mu}\eta_{\lambda\nu} + \eta_{\kappa\nu}\eta_{\lambda\mu} - \tfrac{2}{3}\eta_{\kappa\lambda}\eta_{\mu\nu}) \\ &+ (\phi_{\kappa\lambda}\phi_{\mu\nu})_{\text{quantum}}\,, \end{aligned} \tag{5}$$

where α and β are positive real numbers with the dimensions of (mass)^2. They represent tadpole graphs and are, of course, cutoff dependent.

The c-number part of $\vec{F}_{\mu\kappa}\cdot\vec{F}_{\nu\lambda}$ reduces[4] to

$$\begin{aligned}(\vec{F}_{\mu\kappa}\cdot\vec{F}_{\nu\lambda})_c &= g^2(\vec{A}_\mu\times\vec{A}_\kappa\cdot\vec{A}_\nu\times\vec{A}_\lambda)_c \\ &= 6g^2\alpha^2(\eta_{\mu\nu}\eta_{\kappa\lambda} - \eta_{\mu\lambda}\eta_{\kappa\nu})\,. \end{aligned} \tag{6}$$

With this approximation the tensor equation reduces to the linear Pauli-Fierz form (i.e., $b=1$) with mass given by

$$M^2 = \frac{6g^2\alpha^2}{\mu^2}\,. \tag{7}$$

Notice that the fact that $(F\cdot F)_c$ is nonzero is a consequence of the non-Abelian gauge nature of the Yang-Mills Lagrangian, while the special value of the parameter $b=1$ is a direct consequence of the antisymmetry $\vec{F}_{\mu\nu} = -\vec{F}_{\nu\mu}$ and should therefore be stable.

Our discussion of the tensor equation is concluded. Now consider what happens to the vector equation (3) when it is linearized according to prescriptions (4) and (5);

$$\begin{aligned}(\partial_\mu + g\vec{A}_\mu\times)&\left(\frac{1}{\mu^2}\phi_{\mu\kappa}\phi_{\nu\lambda}\vec{F}_{\kappa\lambda}\right) \\ &\approx -\frac{5}{3}\frac{\beta}{\mu^2}(\partial_\mu\vec{F}_{\mu\nu} + g\vec{A}_\mu\times\vec{F}_{\mu\nu}) \\ &\approx -\frac{5}{3}\frac{\beta}{\mu^2}[\partial_\mu(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu) + g^2\vec{A}_\mu\times(\vec{A}_\mu\times\vec{A}_\nu)] \\ &\approx -\frac{5}{3}\frac{\beta}{\mu^2}[\partial_\mu(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu) + 6g^2\alpha\vec{A}_\nu]\,. \end{aligned} \tag{8}$$

Thus, the Yang-Mills field acquires both a kinetic energy and a mass,

$$M_{\text{YM}}{}^2 = 6g^2\alpha\,. \tag{9}$$

From the positivity of norms of physical states, one expects α and β to be positive. If the theory were renormalizable, α and β could be computed by the self-consistent Hartree-Fock method. A program for such computations in a renormalizable model has been presented by Cornwall, Jackiw, and Tomboulis.[5] This program is perfectly feasible for a nonrenormalizable theory as well, except that α and β, like all other parameters in the theory, will depend on a cutoff.[6]

These ideas could be applied to strong-gravity theory with

$$\frac{1}{\mu}\phi^{\kappa\lambda} = f^{\kappa\lambda} - g^{\kappa\lambda}\,,$$

where $g^{\kappa\lambda}$ is the metric tensor of space-time and $f^{\kappa\lambda}$ is a similar tensor associated with a strong-gravity short-range force. A possible Lagrangian[6] might be

$$\mathcal{L}=\sqrt{-g}\left(\frac{1}{\kappa_g{}^2}R(g)\right)+\sqrt{-f}\left(\frac{1}{\kappa_f{}^2}R(f)+\tfrac{1}{4}\phi^{\mu\kappa}\phi^{\nu\lambda}\vec{\mathrm{F}}_{\mu\nu}\cdot\vec{\mathrm{F}}_{\kappa\lambda}\right), \qquad (10)$$

where R is the Einstein scalar. Through the mechanism described above, this system will describe one massless graviton, one massive 2^+ particle plus a triplet of massive Yang-Mills particles.

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[1]D. G. Boulware and S. Deser, Phys. Rev. D 6, 3368 (1972).

[2]M. J. Duff, Phys. Rev. D 12, 3969 (1975).

[3]P. G. O. Freund, Ann. Phys. (N.Y.) 84, 440 (1974).

[4]We have ignored the derivative terms $(\partial_\mu\vec{A}_\kappa\cdot\partial_\nu\vec{A}_\lambda)_c$ in evaluating (6). If we had not done this, the parameter α appearing in (6) would be independent of the one introduced in (4). However, the antisymmetric tensorial form in (6) must persist in any case.

[5]J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D 10, 2428 (1974).

[6]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 3, 867 (1971). These authors have shown [Phys. Lett. 46B, 407 (1973)] that for Lagrangians of the type (10) the cutoff is intrinsically determined to be $(\kappa_f)^{-1}$.