

Comment on the associated multiplicity in semi-inclusive reactions*

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The average charged multiplicity \bar{n}_c for the semi-inclusive reaction $p_1 p_2 \rightarrow p_3 p_4 X$ is studied in the context of a previously proposed two-component model. Recent data for \bar{n}_c at 28.5 GeV/c for $p_{3T} \leq 1$ GeV/c are shown to be satisfactorily described by the soft component, without requiring any contribution from hard scattering. Reasonable assumptions for the hard component are made in order to estimate \bar{n}_c as a function of p_{3T} .

In a previous paper¹ we proposed a two-component model for the average charged multiplicity \bar{n}_c and single-particle distributions in inclusive reactions. Our work was motivated by the need to understand the behavior of \bar{n}_c for the reactions $pp \rightarrow pX$ and $pp \rightarrow \pi^+ X$ observed² at $p_{1ab} = 28.5$ GeV/c. The steplike dependence on the transverse momentum p_T of the trigger for fixed missing mass in these data does not seem to be of kinematic origin,³ but rather suggests a transition in the dynamics of particle production at $p_T \approx 1$ GeV/c. In our model, the steplike behavior of \bar{n}_c is attributed to the transition from the "soft" coherent-scattering regime at $p_T \leq 1$ GeV/c to the region where "hard", incoherent scattering from the hadronic constituents dominates at $p_T \geq 1$ GeV/c.

The advent of recent data⁴ on the associated multiplicity in the semi-inclusive reaction

$$p(p_1) + p(p_2) - p(p_3) + p(p_4) + X \quad (1)$$

poses a further challenge for our model. Consequently, in the present note we analyze the data of Ref. 4 in our picture.

For completeness, we first recall the relevant results of paper I. Particle production in an inclusive reaction is assumed to proceed through either of two mechanisms:

- (i) incoherent, hard scattering from the hadronic constituents,
- (ii) coherent, soft scattering from the constituents.

Process (ii) dominates at small p_T and corresponds to a Mueller-Regge description of particle production, whereas process (i) takes over at large p_T because of its power-law dependence on p_T . The invariant single-particle distribution for $p(p_1) + p(p_2) - p(p_3) + X$ is given by

$$f(p_3) \equiv E_3 \frac{d\sigma}{d^3p_3} = f_s(p_3) + f_h(p_3), \quad (2)$$

where $f_s(p_3)$ and $f_h(p_3)$ are the soft and hard components, respectively. The associated mean total charged multiplicity is obtained by averaging the contributions from soft and hard components with appropriate weights,

$$\bar{n}_c(p_3) f(p_3) = \bar{n}_s(p_3) f_s(p_3) + \bar{n}_h(p_3) f_h(p_3). \quad (3)$$

The multiplicity associated with the soft component is assumed to have a simple logarithmic dependence on the missing mass M ,

$$\bar{n}_s(p_3) = a + b \ln M^2, \quad (4)$$

where $a = b = 1$ was found in paper I to give a good fit to data⁵ at 205 GeV/c (we do not include the trigger proton in this expression). We note that (4) can be obtained in multiperipheral models⁶ and corresponds to the idea that the asymptotic multiplicity in a system of particles is determined principally by the invariant mass of the system.⁷ We mean this statement to apply *asymptotically* to soft processes only. For instance, at a low invariant mass the multiplicity for a B (baryon number) = 2 system is necessarily lower than that for a $B = 0$ system.

For the hard component, the multiplicity is given by

$$\bar{n}_h(p_3) = a + \ln(\sqrt{s} - \sqrt{s'})^2 + a_1 + b_1 |\vec{p}_3|, \quad (5)$$

where a is the same as in (4), and a_1 and b_1 are given by¹ $a_1 = 0.7$, and $b_1 = 0.5$ (GeV/c)⁻¹. The variable s' is the invariant mass squared of the irreducible subprocess. For a full discussion of (5), including the determination of the parameter values, we refer the reader to paper I.⁸

We now turn to the application of these ideas to the semi-inclusive reaction (1). Analogously to (2) and (3), we have for the invariant distribution and charged multiplicity

$$g(p_3, p_4) \equiv E_4 \frac{d\sigma}{d^3p_3 d^3p_4} = g_s(p_3, p_4) + g_h(p_3, p_4) \quad (2')$$

and

$$\bar{n}_c(p_3, p_4) g(p_3, p_4) = \bar{n}_s(p_3, p_4) g_s(p_3, p_4) + \bar{n}_h(p_3, p_4) g_h(p_3, p_4), \quad (3')$$

respectively. For $\bar{n}_s(p_3, p_4)$ we shall make the reasonable assumption, discussed above, that the multiplicity depends on the missing mass in the usual way,

$$\bar{n}_s(p_3, p_4) = a' + \ln M_{34}^2, \quad (6)$$

where

$$M_{34}^2 = (p_1 + p_2 - p_3 - p_4)^2. \quad (7)$$

The parameter a' will be determined below. To calculate \bar{n}_c in (3') we need to know $\bar{n}_h(p_3, p_4)$ and the ratio $g_h(p_3, p_4)/g_s(p_3, p_4)$. In the absence of theoretical information on this ratio we make the following assumption in the phase-space region of Ref. 4:

$$\frac{g_h(p_3, p_4)}{g_s(p_3, p_4)} \ll 1. \quad (8)$$

This is based on the following arguments:

(i) Disregarding \vec{p}_3 , p_{4T} is too small in the kinematic range of Ref. 4 for dominance of the hard component (see Fig. 3 of Ref. 4).

(ii) In Fig. 1 we plot, using the results of paper I, the ratio $f_h(p_3)/f(p_3)$ for $M_3^2 \equiv (p_1 + p_2 - p_3)^2 = 3.5^2 \text{ GeV}^2$, and $p_{3T} \leq 1 \text{ GeV}/c$. These are the values of M_3 and p_{3T} used in Ref. 4. We see from Fig. 1 that for these values of p_{3T} the hard com-

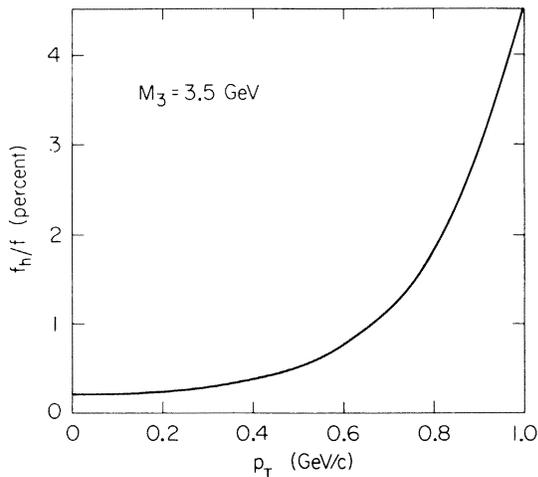


FIG. 1. The ratio of hard to total inclusive cross section at 28.5 GeV/c plotted versus p_T for $M_3 = 3.5 \text{ GeV}$ in the reaction $pp \rightarrow pX$.

ponent gives a minor contribution when \vec{p}_4 is averaged over.

(iii) In Fig. 2, along with the data of Ref. 4, we plot the empirical mean charged multiplicities for soft and hard components (dashed-dotted lines) for the process $pp \rightarrow pX$ at $M_3 = 3.57 \text{ GeV}$. From this figure it is clear that the data of Ref. 4 do not correspond to typical hard or soft events because of their generally very different multiplicities.

Argument (ii) shows that the hard component is almost zero for $p_{3T} < 1 \text{ GeV}/c$, and argument (iii) suggests that the constraint on \vec{p}_4 made in Ref. 4 is insufficient to select the rare hard events because of their ambiguous characterization as hard or soft. Consequently, we assume (8) in what follows. To make (8) more precise, we would require knowledge of the semi-inclusive distributions $g_s(p_3, p_4)$ and $g_h(p_3, p_4)$ (see Ref. 9); however, we show below that (8) gives a satisfactory description of the data of Ref. 4. Therefore, we have

$$\bar{n}_c(p_3, p_4) = \bar{n}_s(p_3, p_4), \quad (9)$$

with $\bar{n}_s(p_3, p_4)$ given by (6).

To apply this picture to the 28.5 GeV/c data of Ref. 4 we take $p_{3T} = 0.25 \text{ GeV}/c$, and find that $a' = 1.5$ gives the solid line in Fig. 2.¹⁰ Choosing different values of p_{3T} in the range $0 \leq p_{3T} \leq 1 \text{ GeV}/c$ (the region covered by the experiment) results only in small changes in the normalization constant a' ,¹¹ while the essential θ_4 behavior is always the same. In Fig. 2 we also show as the

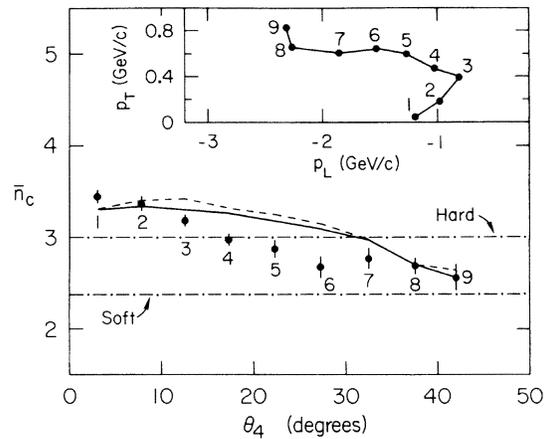


FIG. 2. Data from Ref. 4 for \bar{n}_c (trigger protons not included) plotted versus θ_4 , the average laboratory scattering angle of p_4 in the reaction $p_1 p_2 \rightarrow p_3 p_4 X$ at $\bar{M}_3 = 3.5 \text{ GeV}$. The inset shows the corresponding c.m. momentum of p_4 , and the solid line is our fit as explained in the text. The dashed line is the parametrization $\bar{n}_c = -0.2 + \ln M_4^2$. We also show the empirical charged multiplicities for soft and hard components when $M_3 = 3.57 \text{ GeV}$ in the reaction $pp \rightarrow pX$ (dashed-dotted lines).

dashed line the parametrization $\bar{n}_c = -0.2 + \ln M_4^2$, where⁴ $M_4^2 \equiv (p_1 + p_2 - p_4)^2$. The two parametrizations are similar, although (6) is in slightly better agreement with the data. The fact that (6) is the natural prediction of our model, together with its reasonable success in fitting the data in Fig. 2 [taking into account the fact that M_{34}^2 is always too small ($\leq 7 \text{ GeV}^2$) for us to expect the asymptotic expression (6) to provide detailed agreement with the data] encourages us to believe that M_{34} is the important variable here.

As further support for the identification of \bar{n}_c with a general soft-component multiplicity we show in Fig. 3 the data of Ref. 4 (open circles) along with data on $e^+e^- \rightarrow \text{hadrons}$ (closed circles, Ref. 12), $\pi^-p \rightarrow pX$ (closed squares, Ref. 6), and $\bar{p}p \rightarrow \text{hadrons}$ (closed triangles, Ref. 13). For comparison, we also include data for $pp \rightarrow pX$ (open squares) from Ref. 5. The abscissa in Fig. 3 is the invariant mass squared of the unobserved system in each case. The data for $pp \rightarrow ppX$ are seen to follow the general trend of the e^+e^- , $\bar{p}p$, and π^-p data, although the $pp \rightarrow pX$ data (with $B=1$) lie consistently lower. All of these data correspond to a general soft component because either they are averaged over all final states ($e^+e^- \rightarrow X$, $\bar{p}p \rightarrow X$) and the soft contribution is therefore dominant, or the p_T value of the observed particle is very small and the same conclusion holds ($\pi^-p \rightarrow pX$, $pp \rightarrow pX$). We also show the parametrization $\bar{n}_c = 1.5 + \ln M_X^2$ as the solid line in Fig. 3.

The experimentalists⁴ have also measured \bar{n}_c for reaction (1) at fixed M_3^2 for $\theta_4 < 15^\circ$ ($|\vec{p}_4| \leq 1 \text{ GeV}/c$) and $0 \leq p_{3T} \leq 2 \text{ GeV}/c$. They find that \bar{n}_c is independent of p_{3T} , in contrast to the result when p_4 is not constrained.¹⁴ In our picture, for p_{3T} smaller than the value at which the break in \bar{n}_c occurs in the inclusive data and for a \vec{p}_4 value

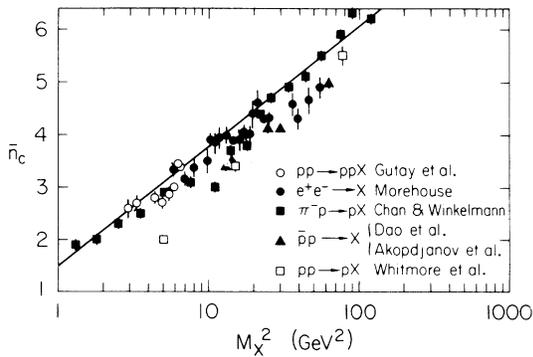


FIG. 3. Data for \bar{n}_c plotted versus the invariant mass squared of the unobserved system M_X^2 ; $pp \rightarrow ppX$ (open circles, Ref. 4), $e^+e^- \rightarrow X$ (closed circles, Ref. 12), $\pi^-p \rightarrow pX$ (closed squares, Ref. 6), $\bar{p}p \rightarrow X$ (closed triangles, Ref. 13), and $pp \rightarrow pX$ (open squares, Ref. 5). The solid line is the parametrization $\bar{n}_c = 1.5 + \ln M_X^2$.

in the phase-space region of Ref. 4, we have that $\bar{n}_c(p_3, p_4) = \bar{n}_s(p_3, p_4)$. In the region where both contributions are comparable, even if we are able to give an expression for $\bar{n}_h(p_3, p_4)$,¹⁵ we do not know $g_s(p_3, p_4)$ and $g_h(p_3, p_4)$ and therefore it is impossible to predict \bar{n}_c . When the hard contribution is dominant $\bar{n}_c(p_3, p_4) = \bar{n}_h(p_3, p_4)$. In this case we shall assume that for $(p_{4L}, p_{4T}) \approx (-1.0, 0.2) \text{ GeV}/c$, corresponding to $\theta_4 \approx 8^\circ$, $\bar{n}_h(p_3, p_4) \approx \bar{n}_h(p_3)$, because this value of \vec{p}_4 is probably not very different from that of the “average” hard event.¹⁶

Using this method we show our estimate for $M_3 = 3.57, 4.56, \text{ and } 5.47 \text{ GeV}$ as the solid lines in Fig. 4. The dashed lines represent smooth interpolations joining the estimated multiplicities in the transition region. We see that the p_{3T} dependence is slight for the largest values of M_3 , the numerical value of $\bar{n}_c(p_3, p_4)$ coinciding with $\bar{n}_c(p_3)$ for the reaction $pp \rightarrow pX$ after the rise in agreement with the behavior reported in Ref. 4. The absence of a step in the semi-inclusive data results, in our model, from the selection of soft events with $|\vec{p}_4|$ small and, consequently, M_{34}^2 large. It is perhaps reasonable that events with low $|\vec{p}_4|$ should have a higher multiplicity than the “average” soft event, because in the former case more energy is available for pion production.

In conclusion, we have applied our two-component model for inclusive reactions to a limited phase-space region of the semi-inclusive case by means of reasonable, though perhaps simplistic, assumptions. In this picture, our essential results for the data⁴ at $28.5 \text{ GeV}/c$ are the following:

(i) The low- p_{3T} data are reasonably well described by pure soft scattering with a multiplicity depending only on the overall missing mass. The

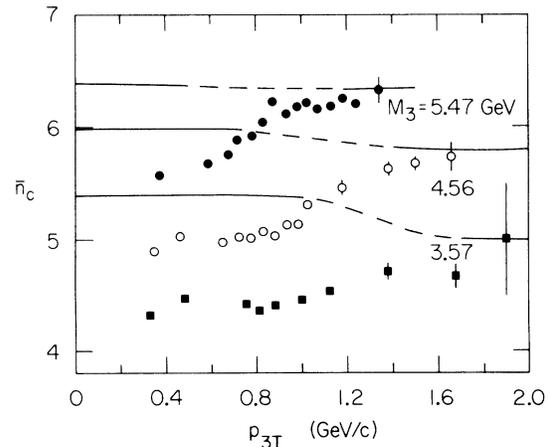


FIG. 4. Estimates for \bar{n}_c at $28.5 \text{ GeV}/c$ plotted versus p_{3T} for various fixed values of M_3 and for $(p_{4L}, p_{4T}) = (-1.0, 0.2) \text{ GeV}/c$. We show, for comparison, data for \bar{n}_c in the reaction $pp \rightarrow pX$ from Ref. 2.

rise in multiplicity at small \vec{p}_4 corresponds to the general idea that \bar{n}_c increases when going from diffractive to nondiffractive production.

(ii) Our picture is consistent with the almost complete absence of a step in \bar{n}_c as a function of p_{3T} at fixed M_3 and \vec{p}_4 (for the largest values of M_3), and its value is roughly equal to \bar{n}_c in the reaction $pp - pX$ above the rise for the same value of M_3 . However, as M_3 becomes smaller varia-

tions in \bar{n}_c as a function of p_{3T} are expected.

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⁶C.-F. Chan and F. C. Winkelmann, Phys. Rev. D **10**, 3645 (1974).

⁷See, for example, E. L. Berger, in Proceedings of Ecole d'Ete de Physique des Particules, Gif-Sur-Yvette, France, 1973 (unpublished).

⁸In paper I we assumed, for simplicity, that the c.m. system for the irreducible collision coincides with the c.m. system for the pp collision. This assumption and the reasonable supposition that the multiplicity associated with the irreducible collision depends asymptotically only on s' (in our picture, on $|\vec{p}_3|$ in the c.m. system for the irreducible collision) are essentially the reasons why (5) depends only on M^2 . For fixed \vec{p}_3 , the c.m. system for the irreducible process which minimizes s' (and which therefore favors the cross section) is that system for which $s' \approx (2p_{3T})^2$, and is not, in general, the overall c.m. system. Thus, independent of the dynamics of the irreducible process, expression (5) oversimplifies the true expression for $\bar{n}_h(p_3)$. More experimental and theoretical work is needed to clarify this point, and a possible p_{3T} dependence beyond that coming from the transition from one mechanism to the other should be kept in mind. In this connection, see R. O. Raitio and G. A. Ringland, Phys. Rev. D (to be published). Further details are discussed by D. Sivers, S. J. Brodsky, and

R. Blankenbecler, Phys. Rep. **23C**, 1 (1976).

⁹For an example of the difficulties encountered in obtaining a simple parametrization of the semi-inclusive distribution, see the study of the correlation coefficients in $pp \rightarrow \pi^0 \pi^0 X$ performed by D. Schiff, A. P. Contogouris, and J. L. Alonso, Phys. Lett. **55B**, 87 (1975).

¹⁰Here we have assumed that the protons p_3 and p_4 are observed on opposite sides of the beam (i.e., $\Delta\phi = 180^\circ$). This corresponds to the experimental setup [see, for example, J. R. Ficenc *et al.*, in *Experimental Meson Spectroscopy*, edited by C. Baltay and A. Rosenfeld (Columbia University Press, New York, 1970), p. 581].

¹¹The variations in a' are of the order of 0.05 when p_{3T} changes within its range.

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¹³F. T. Dao *et al.*, Phys. Lett. **51B**, 505 (1974); G. A. Akopdjanov *et al.*, Nucl. Phys. **B75**, 401 (1974).

¹⁴In Ref. 4 this result is reported but not shown. In fact, we know that at $M_3 = 4.6$ MeV, \bar{n}_c is almost independent of p_{3T} (L. J. Gutay, private communication); however, we do not know the experimental situation for M_3 different from this value.

¹⁵This is, perhaps, not too difficult. This case is different from that studied in paper I in that now, in $\bar{n}_R(p_3)$, \vec{p}_4 is not averaged over, and therefore one must "separate" this \vec{p}_4 from the remaining energy in a similar way to the separation made for the soft component.

¹⁶Here, we take $|\vec{p}_4|$ small because we have assumed in our model that only one proton is involved in the irreducible subprocess, and that the remaining energy is too small to favor $|\vec{p}_4|$ large. In particular, we take $\theta_4 \approx 8^\circ$ because this corresponds to the central value of the experimental range ($\theta_4 < 15^\circ$). Our results are not significantly altered by using other values of $\theta_4 < 15^\circ$.