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Application of a single-particle-theory calculation of $g-2$ to spin one

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It is illustrated how, in principle, one may apply the Heisenberg-equation-of-motion technique in a single-particle theory, recently discussed in connection with the calculation of the electron's anomalous magnetic moment, to a charged spin-one Yang-Mills particle.

There is considerable current interest in attempting to understand the anomalous magnetic moment of an electron as a resonance precession effect,[1] calculating the anomalous moment from the radiative frequency shift of the resonant spin-flip transition. In spite of its difficulties[2] in practice, the basic idea is very attractive, and since virtual transitions to negative-energy spin states are important, one begins with the Dirac Hamiltonian, later taking the low-energy limit. In order to simplify the equations of motion so obtained, a Foldy-Wouthuysen transformation[3] of the complete Dirac Hamiltonian is first carried out to eliminate *Zitterbewegung* effects. Such a transformation diagonalizes the unperturbed Hamiltonian and transforms the interaction Hamiltonian in closed form.

The purpose of this note is to show how the same idea can, in principle, be applied to a spin-one charged particle governed by the Sakata-Taketani equation.[4] The unperturbed Hamiltonian is

$$H_0=\left(m+\frac{\pi_0^{\,2}-qB_0S_3}{2m}\right)\rho_3+\left[\frac{\pi_0^{\,2}-qB_0S_3}{2m}-\frac{(\vec{S}\cdot\vec{\pi}_0)^2}{m}\right]i\rho_2-\kappa\,\frac{qB_0S_3}{2m}\,(\rho_3+i\rho_2)\,. \qquad (1)$$

The matrices $\rho_{1,2,3}$ have the algebra of the Pauli spin matrices and $\vec{S}$ are a representation of spin-one matrices. The particle has spin one, mass m, charge q, and "anomalous" g factor κ. A constant magnetic field $\vec{B}_0$ is applied along the z axis so $\vec{\pi}_0=\vec{P}-q\vec{A}_0$ with $\vec{P}=i\vec{\nabla}$, $\vec{A}_0=(-\frac{1}{2}B_0y,\frac{1}{2}B_0x,0)$, and $\vec{B}_0=\vec{\nabla}\times\vec{A}_0$. It is much more convenient to introduce the operator $\alpha\equiv\pi_0^{\,2}-2qB_0S_3$, and to rewrite the Hamiltonian as

$$H_0=\left(m+\frac{\alpha}{2m}\right)\rho_3+\left[\frac{\alpha}{2m}-\frac{(\vec{S}\cdot\vec{\pi}_0)^2}{m}\right]i\rho_2+\frac{qB_0S_3}{2m}\,(1-\kappa)\,(\rho_3+i\rho_2)\,. \qquad (2)$$

α is important because it commutes with $\vec{S}\cdot\vec{\pi}_0$, $\vec{S}\cdot\vec{B}_0$, and $\pi_0^{\,2}$ and so can be chosen to be diagonal when one looks for the eigenvalues of H_0. Equation (2) also shows the simplification that occurs if $\kappa=1$. This value corresponds to a g factor of 2, and it is the "normal" value appropriate to a charged Yang-Mills-type vector particle[5] interacting with the magnetic field. The result is

$$H_0=\left(m+\frac{\alpha}{2m}\right)\rho_3+\left[\frac{\alpha}{2m}-\frac{(\vec{S}\cdot\vec{\pi}_0)^2}{m}\right]i\rho_2\,, \qquad (3)$$

which describes a spin-one particle with a normal g factor of 2 (and expected not to have strong interactions) interacting with a constant, homogeneous magnetic field.

With the similarity transformation operator

$$U=e^{\rho_2\rho_3\phi/2}=\cos\phi+\rho_2\rho_3\sin\phi\,, \qquad (4)$$

one obtains

$$UH_0U^{-1}=\left\{\cos\phi\left(m+\frac{\alpha}{2m}\right)-i\sin\phi\left[\frac{\alpha}{2m}-\frac{(\vec{S}\cdot\vec{\pi}_0)^2}{m}\right]\right\}\rho_3+\left\{i\cos\phi\left[\frac{\alpha}{2m}-\frac{(\vec{S}\cdot\vec{\pi}_0)^2}{m}\right]+\sin\phi\left(m+\frac{\alpha}{2m}\right)\right\}\rho_2\,. \qquad (5)$$

With the choice

$$\tan\phi=\frac{-i[\alpha/2m-(\vec{S}\cdot\vec{\pi}_0)^2/m]}{m+\alpha/2m}\,, \qquad (6)$$

the coefficient of the ρ_2 term is zero, and the resulting Hamiltonian is

$$H_0'=\left(m^2+\alpha-\frac{qB_0P_3}{m^2}\vec{S}\cdot\vec{\pi}_0\right)^{1/2}\rho_3\,. \qquad (7)$$

If the momentum is small (nonrelativistic approximation), the $\vec{S}\cdot\vec{\pi}_0$ term is unimportant and the resulting Hamiltonian is

$$H_0'\cong\left(m+\frac{\alpha}{2m}\right)\rho_3\,. \qquad (8)$$

Thus the nonrelativistic approximation to this spin-one theory has exactly the same operator form as the $g=2$ Dirac-theory nonrelativistic approximation.

Of course when the quantized radiation field $\vec{A}_R$ is introduced as in Ref. 1, the transformed interaction Hamiltonian is rather involved, having the form $UH_{\text{int}}U^{-1}$, where

$$H_{\text{int}} = \frac{1}{2m}[q^2\vec{A}_R\cdot\vec{A}_R - 2q\vec{A}_R\cdot\vec{\pi}_0 - 2q\vec{S}\cdot(\vec{\nabla}\times\vec{A}_R)](\rho_3 + i\rho_2) - \frac{1}{m}[(\vec{S}\cdot\vec{A}_R)^2 - 2q\vec{S}\cdot\vec{A}_R\vec{S}\cdot\vec{\pi}_0 - iq\vec{S}\cdot(\vec{A}_R\times\vec{\pi}_0) + iq(\vec{S}\cdot\vec{\nabla}\vec{S}\cdot\vec{A}_R)]i\rho_2\,. \tag{9}$$

Nevertheless, one may, in principle, apply the same methods to a calculation of the spin-one Yang-Mills particle's anomalous g factor, as may be used for the spin-$\frac{1}{2}$ calculation.

It is worth noting that one may readily obtain the exact energy eigenvalues of Eq. (7) in the following way. One first squares H_0' so that the eigenvalue equation is

$$(E^2 - m^2 - \alpha)\psi = -\frac{qB_0P_3}{m^2}\vec{S}\cdot\vec{\pi}_0\psi\,, \tag{10}$$

with E the eigenvalues of H_0'. Operating on ψ twice more with the right-hand side of Eq. (10) yields the cubic equation

$$\left[(E^2 - m^2 - \alpha)^3 - \left(\frac{qB_0P_3}{m^2}\right)^2\alpha(E^2 - m^2 - \alpha) + \left(\frac{qB_0P_3}{m^2}\right)^3 qB_0P_3\right]\psi = 0\,. \tag{11}$$

Equation (11) may be solved by taking ψ to be a simultaneous eigenfunction of α and P_3, and applying the usual methods for obtaining solutions of a cubic equation.[6] Reality of all the resulting eigenvalues depends on the expression $-\frac{1}{27}\alpha^3 + (qB_0P_3/2)^2$ being negative or zero. The eigenvalues of α are $qB_0(2n+1-2m_S) + P_3^2$ with $n = 0, 1, 2, \ldots$ and $m_S = 0, \pm 1$. It turns out that for $n > 0$ and any P_3 and m_S, all the solutions for E are real, but for $n = 0$ and $m_S = \pm 1$, E^2 can be negative for some values of P_3 and B_0.

[1]S. B. Lai, P. L. Knight, and J. H. Eberly, Phys. Rev. Lett. 32, 494 (1974), and references contained therein.

[2]H. Grotch and E. Kazes, Phys. Rev. Lett. 35, 124 (1975); S. B. Lai, P. L. Knight, and J. H. Eberly, *ibid.* 35, 126 (1975); K. O. Mikaelian, Phys. Rev. D 11, 3626 (1975).

[3]K. M. Case, Phys. Rev. 95, 1323 (1954).

[4]M. Taketani and S. Sakata, Proc. Phys. Math. Soc. Jpn. 22, 757 (1939).

[5]C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954).

[6]See, for example, G. A. Korn and T. M. Korn, *Mathematical Handbook for Sceintists and Engineers* (McGraw-Hill, New York, 1961), p. 23.