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New approach to collective phenomena in superconductivity models*

Tohru Eguchi†
The Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637
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A new approach to collective phenomena in superconductivity models is presented. In this approach nonlinear spinor models are converted into equivalent theories involving fermions and collective bosonic states. For a wide class of nonlinear spinor theories, interactions among fermions and collective states are of the renormalizable kind, and hence these models themselves turn out to be renormalizable. The equivalence of various four-fermion theories to known renormalizable models is pointed out. The original Nambu–Jona-Lasinio model, for instance, is shown to be equivalent to the linear σ model. Detailed discussions are given on the origin of massless fields and the local gauge invariance. Implications of Pauli-Gürsey-type symmetry in spinor theories are also expounded.

I. INTRODUCTION

The superconductivity model of elementary particles, first put forth by Nambu and Jona-Lasinio,[1] has been a useful theoretical laboratory in helping us to understand collective phenomena in quantum field theories. Within a certain approximation we can explicitly solve Bethe-Salpeter-type equations and demonstrate the existence of bound states in various channels. In the original model the existence of scalar, pseudoscalar, vector, and axial-vector bound states was noted. However, the nature and properties of these collective excitations have remained relatively unknown except for the massless pseudoscalar state, which received extensive studies in connection with partial conservation of axial-vector current (PCAC). It has not been well understood, for instance, in which channels what kinds of collective modes are generally expected. In particular, practically nothing has been known about the nature of interactions among these bosonic bound states.

Recently, however, a simple method was discovered[2] to analyze the dynamics of collective states and to derive an effective Lagrangian which governs their mutual interactions. In Ref. 2 a nonlinear spinor theory with scalar, pseudoscalar, vector, and axial-vector interactions was considered, and it was pointed out that the axial-vector, scalar, and pseudoscalar bound states interact with each other like a gauge field and the real and imaginary parts of a Higgs-type complex field, respectively. Hence these collective modes can be described by a Higgs-type renormalizable Lagrangian. A Hartree-Fock self-consistency method was used in this analysis.

The above derivation of an effective Lagrangian has more recently been improved by Kikkawa,[3] who has introduced collective coordinates as integration variables in the path-integral formulation of the theory. He thereby avoids certain difficulties associated with Poincaré invariance in the previous Hartree-Fock treatment.[3a]

The purpose of this paper is to further study the dynamics of the collective states. In particular, we shall show that in a wide class of nonlinear spinor theories interactions among collective states as well as those between collective states and fermions are of the renormalizable kind and hence the nonlinear spinor theories themselves are renormalizable. In our treatment of the theory the original perturbation series in four-fermion coupling constants is converted into a renormalizable series in the induced boson-fermion and boson-boson coupling constants. In physical terms the original four-fermion interaction can be made sufficiently mild and renormalizable if a part of its strength is exhausted in forming bound states.

One example of the above mechanism has been known for some time. Soon after the proposal of the superconductivity approach Bjorken and others[4,5] studied a theory with a vector-type interaction and concluded that if a collective state is excited in the vector channel it behaves exactly like a gauge field and the resulting theory becomes equivalent to spinor electrodynamics. In this paper this kind of equivalence will be extended to other theories. We shall show that the original Nambu-Jona-Lasinio theory[1] is equivalent to the linear σ model,[6] the theory of Ref. 2, and its non-Abelian generalization[7] to Abelian and non-Abelian Higgs-type theories,[8] respectively, and so on. In these examples S matrices of the corresponding theories become identical, although their Green's functions are not necessarily the same because of certain differences in the definition of renormalization parameters.

The contents of this paper are as follows. In Sec. II we explain our general procedure making use of the Nambu-Jona-Lasinio model as an example. In Sec. III the method of Sec. II is applied to other theories. Detailed discussions will be given on the origin of massless particles and local gauge invariance. Sec. IV is devoted to an analysis of implications of the Pauli-Gürsey-type symmetry in spinor theories. Sec. V is used for discussions and comments. Throughout this paper we shall employ the method of Ref. 3 for the introduction of collective coordinates.

II. σ MODEL

Let us start with the following Lagrangian consisting of an isodoublet of spinor fields:

$$\mathcal{L}=i\bar{\psi}\gamma\cdot\partial\psi+\frac{G_0}{2}[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\vec{\tau}\psi)^2], \tag{2.1}$$

where G_0 is a bare four-fermion coupling constant and is assumed to be positive. The theory has a symmetry of chiral U(2)×SU(2). Its generating functional is given as usual,

$$W[\eta,\bar{\eta}]=\frac{1}{N}\iint \exp\{i[\mathcal{L}(\psi,\bar{\psi})+\bar{\eta}\psi+\bar{\psi}\eta]\}d\psi\, d\bar{\psi}. \tag{2.2}$$

N is a normalization factor which will be suppressed hereafter. Following Kikkawa[3] our first task is to introduce integration variables σ and $\vec{\pi}$ and a new Lagrangian $\mathcal{L}'$ in such a way that the above generating functional is expressed as

$$W[\eta,\bar{\eta}]=\int \exp\{i[\mathcal{L}'(\psi,\bar{\psi},\sigma,\vec{\pi})+\bar{\eta}\psi+\bar{\psi}\eta]\}\times d\psi\, d\bar{\psi}\, d\sigma\, d\vec{\pi}. \tag{2.3}$$

$\mathcal{L}'$ is easily found and is given by

$$\mathcal{L}'=\bar{\psi}[i\gamma\cdot\partial-g_0(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})]\psi -\tfrac{1}{2}\delta\mu_0^2(\sigma^2+\vec{\pi}^2), \tag{2.4}$$

where g_0 and $\delta\mu_0$ are related to G_0 by

$$G_0=\frac{g_0^2}{\delta\mu_0^2}. \tag{2.5}$$

The purpose of this section is to demonstrate that our theory is equivalent to the linear σ model. Therefore, let us write down the Lagrangian of the σ model $\mathcal{L}_\sigma$ and compare it with Eq. (2.4),

$$\begin{aligned}\mathcal{L}_\sigma=&\bar{\psi}[i\gamma\cdot\partial-g_0'(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})]\psi\\ &+\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2-\mu'^2(\sigma^2+\vec{\pi}^2)]\\ &-\tfrac{1}{4}\lambda_0'(\sigma^2+\vec{\pi}^2)^2-\tfrac{1}{2}\delta\mu_0'^2(\sigma^2+\vec{\pi}^2), &(2.6)\\ \equiv&\mathcal{L}_f+\mathcal{L}_{bf}+\mathcal{L}_b. &(2.7)\end{aligned}$$

Here μ' is the symmetric renormalized mass of σ and pion and we have decomposed $\mathcal{L}_\sigma$ into three pieces: the fermion kinetic term $\mathcal{L}_f$, the fermion-boson interaction part $\mathcal{L}_{bf}$, and the purely bosonic sector $\mathcal{L}_b$. So in our theory Eq. (2.4) we have both $\mathcal{L}_f$ and $\mathcal{L}_{bf}$, but the whole of $\mathcal{L}_b$ is missing except for the mass counterterms. However, we shall see in the following, it is generally possible to create $\mathcal{L}_b$ out of $\mathcal{L}_f$ and $\mathcal{L}_{bf}$ if we make use of the quantum fluctuations in quantum field theories.[9] The observation of this phenomenon constitutes the principal ingredient of this paper.

In Eq. (2.4) we expect that the field σ has in general a nonvanishing vacuum expectation value v_0. Hence we introduce a field s,

$$\sigma=s+v_0, \tag{2.8}$$

and treat g_0v_0 as a bare fermion mass. Next we perform integrations over fields ψ and $\bar{\psi}$ in Eq. (2.3).[3] Using the standard formula we obtain

$$W[\eta,\bar{\eta}]=\iint d\sigma\, d\vec{\pi}\exp\left(i\left\{-\tfrac{1}{2}\delta\mu_0^2(\sigma^2+\vec{\pi}^2)-i\,\mathrm{Tr}\ln\left[1-\frac{1}{i\gamma\cdot\partial-g_0v_0}g_0(s+i\gamma_5\vec{\pi}\cdot\vec{\tau})\right]+\bar{\eta}\frac{1}{i\gamma\cdot\partial-g_0(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})}\eta\right\}\right). \tag{2.9}$$

The second term is expanded into a power series in g_0,

$$U\equiv -i\,\mathrm{Tr}\ln\left[1-\frac{1}{i\gamma\cdot\partial-g_0v_0}g_0(s+i\gamma_5\vec{\pi}\cdot\vec{\tau})\right]=\sum_{n=1}^{\infty}U^{(n)}, \tag{2.10}$$

where

$$U^{(n)}\equiv\frac{i}{n}\,\mathrm{Tr}\left[\frac{1}{i\gamma\cdot\partial-g_0v_0}g_0(s+i\gamma_5\vec{\pi}\cdot\vec{\tau})\right]^n. \tag{2.11}$$

A diagrammatic expression of the above series is shown in Fig. 1. We note that these diagrams happen to be the same as those of the lowest-order radiative corrections to the effective action in the conventional formulation of the functional method.[10,11] However, in our treatment the external lines in Fig. 1 are quantized fields s and $\vec{\pi}$ and not their expectation values. In the present case $U^{(i)}$ $(i=1,2,3,4)$ give ultraviolet divergences while $U^{(n)}$ $(n\geq 5)$ are all convergent. After an explicit evaluation of the divergent part of the diagrams,[2] the exponent of Eq. (2.9) is given by

$$\bar{\mathcal{L}} = -\tfrac{1}{2}(\delta\mu_0{}^2 - 2I_2 g_0{}^2)(\sigma^2+\vec{\pi}^2) + I_0 g_0{}^2\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2] + I_0 g_0{}^2(g_0 v_0)^2(\sigma^2+\vec{\pi}^2)$$
$$-\tfrac{1}{2}I_0 g_0{}^4(\sigma^2+\vec{\pi}^2)^2 + \mathcal{L}_c(g_0, v_0) + \bar{\eta}\frac{1}{i\gamma\cdot\partial - g_0(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})}\eta, \qquad (2.12)$$

where

$$\mathcal{L}_c = \sum_{i=2}^{4} U_c^{(i)} + \sum_{i=5}^{\infty} U^{(i)}, \qquad (2.13)$$

and $U_c^{(i)}$ is the convergent part of $U^{(i)}$. The separation of $U^{(i)}$ ($i=2,3,4$) into convergent and divergent parts is done in a convenient way. I_2 and I_0 are quadratically and logarithmically divergent integrals, respectively.

$$I_2 = 4i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-(g_0v_0)^2}, \qquad (2.14)$$

$$I_0 = -4i\int \frac{d^4p}{(2\pi)^4}\frac{1}{[p^2-(g_0v_0)^2]^2}. \qquad (2.15)$$

In deriving Eq. (2.12) we have first calculated diagrams using the field s and in the end we have eliminated s in favor of σ. Although Eq. (2.12) contains divergent coefficients, we have in fact created kinetic and interaction terms of bosons out of radiative corrections. In order to find out an appropriate renormalization prescription for Eq. (2.12) let us next perform a similar calculation in the case of the σ model. After the integration

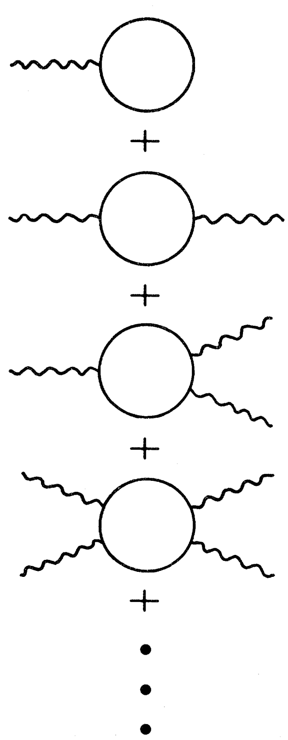

FIG. 1. Feynman diagrams of the series Eq. (2.10). External wavy lines represent the s or the $\vec{\pi}$ field.

over ψ and $\bar{\psi}$ fields in

$$W_\sigma[\eta,\bar{\eta}] = \int \exp\{i[\mathcal{L}_\sigma(\psi,\bar{\psi},\sigma,\vec{\pi}) + \bar{\eta}\psi + \bar{\psi}\eta]\}$$
$$\times\, d\psi\, d\bar{\psi}\, d\sigma\, d\vec{\pi}, \qquad (2.16)$$

we find that the exponent is given by

$$\bar{\mathcal{L}}_\sigma = -\tfrac{1}{2}(\delta\mu_0'^2 + \mu'^2 + \lambda_0' v_0'^2 - 2I_2' g_0'^2)(\sigma^2+\vec{\pi}^2)$$
$$+ (1 + I_0' g_0'^2)\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2]$$
$$+ \tfrac{1}{2}[\lambda_0' v_0'^2 + 2I_0' g_0'^2(g_0'v_0')^2](\sigma^2+\vec{\pi}^2)$$
$$-\tfrac{1}{4}(\lambda_0' + 2I_0' g_0'^4)(\sigma^2+\vec{\pi}^2)^2$$
$$+ \mathcal{L}_c(g_0', v_0') + \bar{\eta}\frac{1}{i\gamma\cdot\partial - g_0'(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})}\eta. \qquad (2.17)$$

Here $\mathcal{L}_c(g_0', v_0')$ is obtained from $\mathcal{L}_c(g_0, v_0)$ by substituting g_0' and v_0' for g_0 and v_0. Similarly I_2' and I_0' are given by Eqs. (2.14) and (2.15) with g_0 and v_0 replaced by g_0' and v_0'. Note that $\bar{\mathcal{L}}_\sigma$ has exactly the same operatorial structure as $\bar{\mathcal{L}}$.

Renormalization prescriptions for the linear σ model are well known.[12,13] By the introduction of wave-function and vertex renormalization factors Z_M and Z_λ, infinites in Eq. (2.17) are absorbed into renormalization parameters,

$$1 + I_0' g_0'^2 = \frac{1}{Z_M}, \qquad (2.18)$$

$$\lambda_0' + 2I_0' g_0'^4 = \frac{\lambda_0}{Z_\lambda}, \qquad (2.19)$$

$$-\delta\mu_0'^2 - \mu'^2 - \lambda_0' v_0'^2 + 2I_2' g_0'^2 = 0. \qquad (2.20)$$

Defining renromalized fields and vertices as

$$\sigma_R = Z_M{}^{-1/2}\sigma, \quad \vec{\pi}_R = Z_M{}^{-1/2}\vec{\pi}, \qquad (2.21)$$

$$v' = Z_M{}^{-1/2} v_0', \qquad (2.22)$$

$$\lambda' = Z_\lambda{}^{-1} Z_M{}^2 \lambda_0', \qquad (2.23)$$

we obtain the following expression of $\bar{\mathcal{L}}_\sigma$:

$$\bar{\mathcal{L}}_\sigma = \tfrac{1}{2}[(\partial_\mu\sigma_R)^2 + (\partial_\mu\vec{\pi}_R)^2 + \lambda' v'^2(\sigma_R{}^2 + \vec{\pi}_R{}^2)]$$
$$-\tfrac{1}{4}\lambda'(\sigma_R{}^2+\vec{\pi}_R{}^2)^2 + \mathcal{L}_c(g_0', v_0')$$
$$+ \bar{\eta}\frac{1}{i\gamma\cdot\partial - g_0'(\sigma+i\gamma_5\vec{\pi}\cdot\vec{\tau})}\eta. \qquad (2.24)$$

Referring to the above procedure, we find that our theory should be renormalized as follows:

$$I_0 g_0{}^2 = \frac{1}{Z_M}, \qquad (2.25)$$

$$2I_0 g_0^{\,4} = \frac{\lambda_0}{Z_\lambda}\,, \tag{2.26}$$

$$-\delta\mu_0^{\,2} + 2I_2 g_0^{\,2} = 0\,, \tag{2.27}$$

$$\sigma_R = Z_M^{-1/2}\sigma, \quad \vec{\pi}_R = Z_M^{-1/2}\vec{\pi}\,, \tag{2.28}$$

$$v = Z_M^{-1/2} v_0\,, \tag{2.29}$$

$$\lambda = Z_\lambda^{-1} Z_M^{\,2}\lambda_0\,. \tag{2.30}$$

By Eq. (2.26) we have introduced a new parameter λ_0 into our theory. Then the exponent $\overline{\mathcal{L}}$ is given by

$$\begin{aligned}\overline{\mathcal{L}} &= \tfrac{1}{2}[(\partial_\mu \sigma_R)^2 + (\partial_\mu \vec{\pi}_R)^2 + \lambda v^2(\sigma_R^{\,2} + \vec{\pi}_R^{\,2})] \\ &\quad -\tfrac{1}{4}\lambda(\sigma_R^{\,2} + \vec{\pi}_R^{\,2})^2 + \mathcal{L}_c(g_0, v_0) \\ &\quad + \overline{\eta}\,\frac{1}{i\gamma\cdot\partial - g_0(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})}\,\eta\,.\end{aligned} \tag{2.31}$$

Hence generating functionals of two theories have exactly the same structure,

$$W = \mathcal{W}(g_0, \lambda_0, v_0)\,, \tag{2.32}$$

$$W_\sigma = \mathcal{W}(g_0', \lambda_0', v_0')\,, \tag{2.33}$$

where $\mathcal{W}$ is a certain functional. Differences between Eqs. (2.18)–(2.20) and Eqs. (2.25)–(2.27) are easy to understand. Since originally we had no kinetic terms or interaction terms for bosons in our theory, this explains the absence of terms such as 1 or λ_0 in the left-hand sides of Eqs. (2.25)–(2.27). However, these missing terms have now been created out of radiative corrections and the difference between our model and the σ model has been absorbed into the relations between renormalization parameters and bare quantities. Since no renormalization parameters or bare quantities appear in S-matrix elements, both theories predict the same S matrix to all orders in perturbation theory.

A convenient feature of the above method lies in the fact that only those calculations corresponding to lowest-order radiative corrections are needed in order to demonstrate the equality of the S matrix to all orders in perturbation theory. However there is a more formal argument by means of which we can infer the equivalence even without doing any explicit calculations. In fact, as we see from the foregoing discussions our essential observation is to exploit the ambiguities in decomposing renormalized quantities into bare terms and counterterms. We can rewrite, for instance, the renormalized Lagrangian of the σ model in terms of bare quantities in two different ways:

$$\mathcal{L}_{\sigma R} = \overline{\psi}_R i\gamma\cdot\partial\psi_R - g\overline{\psi}_R(\sigma_R + i\gamma_5\vec{\pi}_R\cdot\vec{\tau})\psi_R + \tfrac{1}{2}[(\partial_\mu\sigma_R)^2 + (\partial_\mu\vec{\pi}_R)^2 - \mu^2(\sigma_R^{\,2} + \vec{\pi}_R^{\,2})] - \tfrac{1}{4}\lambda(\sigma_R^{\,2} + \vec{\pi}_R^{\,2})^2 \tag{2.34}$$

$$\begin{aligned}&= \{\overline{\psi} i\gamma\cdot\partial\psi - g_0\overline{\psi}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi + \tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2 - \mu^2(\sigma^2 + \vec{\pi}^2)] - \tfrac{1}{2}\delta\mu_0^{\,2}(\sigma^2 + \vec{\pi}^2) - \tfrac{1}{4}\lambda_0(\sigma^2 + \vec{\pi}^2)^2\} \\ &\quad + \Big[\Big(\frac{1}{Z_F} - 1\Big)\overline{\psi} i\gamma\cdot\partial\psi - \Big(\frac{1}{Z_g} - 1\Big) g_0\overline{\psi}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi \\ &\quad + \Big(\frac{1}{Z_M} - 1\Big)\tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2 - \mu^2(\sigma^2 + \vec{\pi}^2)] + \tfrac{1}{2}\delta\mu_0^{\,2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda_0}{4}\Big(\frac{1}{Z_\lambda} - 1\Big)(\sigma^2 + \vec{\pi}^2)^2\Big]\end{aligned} \tag{2.35}$$

$$\begin{aligned}&= [\overline{\psi} i\gamma\cdot\partial\psi - g_0\overline{\psi}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi - \tfrac{1}{2}\delta\mu_0^{\,2}(\sigma^2 + \vec{\pi}^2)] \\ &\quad + \Big\{\Big(\frac{1}{Z_F} - 1\Big)\overline{\psi} i\gamma\cdot\partial\psi - \Big(\frac{1}{Z_g} - 1\Big) g_0\overline{\psi}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi + \frac{1}{Z_M}\tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2 - \mu^2(\sigma^2 + \vec{\pi}^2)] \\ &\quad + \tfrac{1}{2}\delta\mu_0^{\,2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda_0}{4}\frac{1}{Z_\lambda}(\sigma^2 + \vec{\pi}^2)^2\Big\}\,.\end{aligned} \tag{2.36}$$

Equation (2.35) is the usual prescription, where terms in the curly brackets are regarded as the bare Lagrangian of the theory and those in the large square brackets its radiative corrections. On the other hand, Eq. (2.36) corresponds to our treatment, where the bare Lagrangian consists only of the three terms of the first square brackets. Of course such kind of ambiguities in quantum field theories is well known and forms the basis of the renormalization-group approach. However, its relevance in nonlinear spinor theories has not been well appreciated. If we take a variation of the boson fields in the bare Lagrangian of Eq. (2.35) we obtain the usual equation of motion,

$$\Box\sigma + (\mu^2 + \delta\mu_0^{\,2})\sigma + g_0\overline{\psi}\psi + \lambda_0\sigma(\sigma^2 + \vec{\pi}^2) = 0\,, \tag{2.37}$$

and a similar one for $\vec{\pi}$. On the other hand, the bare Lagrangian of Eq. (2.36) give us the Euler equations,

$$\delta\mu_0^{\,2}\sigma + g_0\overline{\psi}\psi = 0\,, \tag{2.38}$$

$$\delta\mu_0^{\,2}\vec{\pi} + g_0\overline{\psi} i\gamma_5\vec{\tau} = 0\,. \tag{2.39}$$

In Eq. (2.37) the σ field has its own degrees of freedom while in Eqs. (2.38) and (2.39) the boson fields appear to be entirely dependent on the spinor field. However, in quantum field theories, it is not meaningful to argue about the distinction between the two so far as both theories predict the same S

matrix. Thus boson fields can be at the same time elementary and composite.

Apparently the above arguments can be applied to theories other than the σ model and most renormalizable models involving fermions and bosons are realized as various kinds of nonlinear spinor theories.

However, it is important to not that the above phenomenon occurs only if we have sufficiently strong ultraviolet divergences in the theory so that all the renormalization parameters, Z's, are equal to zero. Fro instance, in the case of 2 or 3 (space + time) dimensions the Z's turn out to be finite and we do not have the equivalence of four-fermion theories with models involving bosons and fermions.

III. SPONTANEOUSLY BROKEN GAUGE THEORIES

In this section we consider the consequences of possible symmetries of nonlinear spinor theories. If the primary interaction of four-fermion theory possesses a certain symmetry, its equivalent renormalizable theory will also exhibit a corresponding symmetry. Here the interesting phenomenom is that sometimes the latter induced symmetry turns out to be higher than the original one. In particular, whenever a collective mode is excited in a channel of a conserved current we necessarily arrive at a local gauge symmetry starting from a globally invariant one. Let us next see how this happens in the model of Bjorken.[4,5]

The model is given by the Lagrangian

$$\mathcal{L}=\bar{\psi}(i\gamma\cdot\partial-m_0)\psi-\frac{G_0'}{2}(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \tag{3.1}$$

having a global U(1) symmetry. Applying the same technique as in Sec. II we obtain

$$\mathcal{L}'=\bar{\psi}(i\gamma\cdot\partial-m_0)\psi-e_0\bar{\psi}\gamma_\mu\psi A^\mu+\tfrac{1}{2}\delta\mu_0^{\,2}A_\mu A^\mu\,, \tag{3.2}$$

where

$$G_0'=\frac{e_0^{\,2}}{\delta\mu_0^{\,2}}\,. \tag{3.3}$$

The equivalence of this theory to QED is well known. In fact, if we perform a similar calculation as in the σ model, we find that

$$\begin{aligned}\bar{\mathcal{L}}&=\tfrac{1}{2}[\delta\mu_0^{\,2}-\tfrac{1}{2}e_0^{\,2}(I_2+m_0^{\,2}I_0)]\\&\quad+(\tfrac{1}{3}e_0^{\,2}I_0)(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu})+\mathcal{L}_c\\&\quad+(\text{fermion source term}).\end{aligned} \tag{3.4}$$

Here $F_{\mu\nu}$ is the usual curl of A_μ and we have explicitly exhibited the photon mass term coming from the lowest-order vacuum polarization diagram. This is to be canceled against the photon mass counterterm $\frac{1}{2}\delta\mu_0^{\,2}A_\mu A^\mu$. Then via the renormalization presciption,

$$\delta\mu_0^{\,2}-\tfrac{1}{2}e_0^{\,2}(I_2+m_0^{\,2}I_0)=0\,, \tag{3.5}$$

$$\tfrac{1}{3}e_0^{\,2}I_0=\frac{1}{Z_3}\,, \tag{3.6}$$

ultraviolet infinites are eliminated and we obtain

$$\bar{\mathcal{L}}=-\tfrac{1}{4}F_{\mu\nu R}F^{\mu\nu R}+\mathcal{L}_c+(\text{fermion source term}), \tag{3.7}$$

which has exactly the same structure as the corresponding expression in QED.

It is possible to interpret Eq. (3.5) as a gap equation,[4]

$$1=\tfrac{1}{2}G_0'(I_2+m_0^{\,2}I_0)\,. \tag{3.8}$$

In the Hartree-Fock method it is derived by assuming that

$$\frac{e_0}{\delta\mu_0^{\,2}}\langle\bar{\psi}\gamma_\mu\psi\rangle=\langle A_\mu\rangle=\eta_\mu\neq0\,, \tag{3.9}$$

where η_μ is a certain constant vector. In this context the above equation is sometimes regarded as implying a spontaneous breakdown of Lorentz invariance. From this point of view of photon is a Nambu-Goldstone boson associated with the breakdown of Lorentz symmetry.[14] However, we have to keep in mind that even in this interpretation the actual Lorentz symmetry is not broken at all since Eq. (3.9) can always be understood as a particular gauge condition on A_μ.[15] Moreover, if in the above we had made use of a gauge-invariant regularization scheme instead, the photon mass term would have been automatically eliminated in the evaluation of the vacuum polarization diagram. In this case we have to set $\delta\mu_0=0$ in Eq. (3.2). This choice seems a bit awkward but it is certainly legitimate. Then Eq. (3.5) reduces to a trivial one, $0=0$. Thus the form of Eq. (3.5) has a certain dependence on prescriptions on how to handle divergent quantities. Hence in this paper we do not adopt the above interpretation of the photon as a Nambu-Goldstone boson but regard Eq. (3.5) simply as a renormalization prescription. This method of renormalization which makes use of gauge noninvariant counterterms and regulators consistently with the Ward identity is know to work in both Abelian and non-Abelian gauge theories.[16]

From the above example it is already apparent how we can create a local gauge symmetry starting from a global invariance. In Eq. (3.2) we have a collective excitation A_μ which is coupled to a conserved current $\bar{\psi}\gamma_\mu\psi$. Hence $\mathcal{L}'$ is invariant under

$$A_\mu\to A_\mu+\partial_\mu\Lambda\,, \tag{3.10}$$

with an arbitrary Λ. Although the mass counterterm $\frac{1}{2}\delta\mu_0{}^2 A_\mu A^\mu$ seems to spoil this invariance, it in fact eliminates the gauge-noninvariant photon mass term coming from radiative corrections and preserves the gauge invariance.

The non-Abelian analog of the above mechanism is quite similar. If we take as our starting Lagrangian

$$\mathcal{L} = \bar\psi(i\gamma\cdot\partial - m_0)\psi - \frac{G_0'}{2}(\bar\psi\gamma_\mu\lambda_\alpha\psi)(\bar\psi\gamma^\mu\lambda_\alpha\psi), \quad (3.11)$$

where λ_α are SU(n) matrices, and introduce an excitation A_μ^α in the channel $\bar\psi\gamma_\mu\lambda_\alpha\psi$, then it behaves exactly like an SU(n) Yang-Mills field and we arrive at the so-called quantum chromodynamics.

With the above preliminaries it is now possible to tell which kind of nonlinear spinor theories lead to which kind of renormalizable models even without going into detailed calculations. For example, let us take the model considered in Ref. 2. It is given by

$$\mathcal{L} = \bar\psi i\gamma\cdot\partial\psi + \tfrac{1}{2}G_0[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\psi)^2] - \tfrac{1}{2}G_0'[(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma^\mu\psi) + (\bar\psi\gamma_5\gamma_\mu\psi)(\bar\psi\gamma_5\gamma^\mu\psi)]. \quad (3.12)$$

The theory possesses a symmetry U(1) × U(1) and hence a conserved vector and axial-vector current. Therefore, if we introduce collective fields according to

$$\mathcal{L}' = \bar\psi i\gamma\cdot\partial\psi - g_0\bar\psi(\phi_S + i\gamma_5\phi_P)\psi - e_0\bar\psi(\gamma_\mu V^\mu + \gamma_5\gamma_\mu A^\mu)\psi + (\text{mass counterterms}), \quad (3.13)$$

then V_μ and A_μ behave like a vector and an axial-vector gauge field. Since mesons ϕ_S, ϕ_P, V_μ, and A_μ do not carry a baryonic charge, V_μ will couple only to the spinor field. On the other hand, under a chiral rotation fields transform as

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad (3.14)$$

$$\phi \equiv \frac{\phi_S + i\phi_P}{\sqrt{2}} \to e^{-2i\alpha}\phi, \quad (3.15)$$

$$V_\mu, A_\mu \to V_\mu, A_\mu. \quad (3.16)$$

Hence both spinor and ϕ fields are axially charged and interact with A_μ. Since ϕ develops a vacuum expectation value as in the σ model A_μ acquires a mass due to the Higgs mechanism. Therefore, our model will be equivalent to a broken gauge theory of chiral U(1) × U(1) symmetry where only the axial U(1) gauge is spontaneously broken. In fact, after a detailed calculation we find that the equivalent theory is described by the Lagrangian

$$\mathcal{L}_{\text{Higgs}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + i\bar\psi\gamma\cdot\partial\psi + |\partial_\mu\phi + 2ie_0A_\mu|^2 - g_0\bar\psi(\phi_S + i\gamma_5\phi_P)\psi - e_0\bar\psi(\gamma_\mu V^\mu + \gamma_5\gamma_\mu A^\mu)\psi - \mu_0{}^2|\phi|^2 - \lambda_0|\phi|^4, \quad (3.17)$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad (3.18)$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (3.19)$$

It is also easy to discuss the non-Abelian analog of the above example.[7] The Lagrangian is given by

$$\mathcal{L} = \bar\psi i\gamma\cdot\partial\psi + \tfrac{1}{2}G_0[(\bar\psi\psi)^2 + (\bar\psi\lambda_\alpha\psi)^2 + (\bar\psi i\gamma_5\psi)^2 + (\bar\psi i\gamma_5\lambda_\alpha\psi)^2] - \tfrac{1}{2}G_0'[(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma^\mu\psi) + (\bar\psi\gamma_\mu\lambda_\alpha\psi)(\bar\psi\gamma^\mu\lambda_\alpha\psi) + (\bar\psi\gamma_5\gamma_\mu\psi)(\bar\psi\gamma_5\gamma^\mu\psi) + (\bar\psi\gamma_5\gamma_\mu\lambda_\alpha\psi)(\bar\psi\gamma_5\gamma^\mu\lambda_\alpha\psi)]. \quad (3.20)$$

The theory has a symmetry U(n) × U(n). After a similar treatment as in the previous case, the model is shown to be equivalent to a spontaneously broken gauge theory of chiral U(n) × U(n) symmetry.

Physical spectra of the theory which we obtain via the Higgs mechanism depend on the choice of vacuum expectation values. In the case of $n=2$, for instance, if we take $\langle\phi_S\rangle \neq 0$ and $\langle\vec\phi_S\rangle = 0$ then we obtain a theory of massive ϕ_S, $\vec\phi_S$, A_μ, and $\vec{A}_\mu$ fields interacting with massless V_μ and $\vec{V}_\mu$ mesons. In this choice the axial U(2) symmetry is completely broken while the vector U(2) gauge is left invariant. Note that the number and representations of the Higgs fields are fixed by our construction and are not at all arbitrary as in the usual treatment of the Higgs mechanism.

IV. PAULI-GÜRSEY TRANSFORMATION

It is well known that the fermion kinetic term $i\bar\psi\gamma\cdot\partial\psi$ is invariant under a certain transformation which mixes particles and antiparticles, the Pauli-Gürsey transformation.[17] If we make use of this invariance, it is possible to introduce collective excitations in the fermion-fermion (on antifermion-antifermion) channels as well as in the fermion-antifermion channels.

In the Abelian case the transformation is defined as

$$\psi \to a\psi + b\gamma_5\psi^c, \quad (4.1)$$

$$\psi^c \to a^*\psi^c + b^*\gamma_5\psi, \quad (4.2)$$

where $|a|^2 + |b|^2 = 1$. This is very similar to the Bogoliubov transformation which gives rise to a coherent mixture of particles and holes in the theory of superconductors. Let us consider the model

$$\mathcal{L} = \bar{\psi} i\gamma \cdot \partial\psi + \tfrac{1}{2}G_0[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2] - \tfrac{1}{2}G_0'[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) + (\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma^\mu\psi)] \tag{3.11}$$

which we have already encountered in Sec. III. Here, however, instead of directly introducing collective modes to Eq. (2.11) we first perform a change of integration variables,

$$\psi \to \chi = \frac{1}{\sqrt{2}}(\psi + \gamma_5\psi^c), \tag{4.3}$$

$$\psi^c \to \chi^c = \frac{1}{\sqrt{2}}(\psi^c + \gamma_5\psi), \tag{4.4}$$

and then form bound states in various $\chi\bar{\chi}$ channels as

$$W[\eta, \eta'] = \int \exp\{i[\mathcal{L}'(\chi, \bar{\chi}, \phi, V_\mu, A_\mu) + \bar{\eta}\chi + \bar{\chi}\eta]\} \times d\chi\, d\bar{\chi}\, d\phi\, dV_\mu dA_\mu, \tag{4.5}$$

$$\mathcal{L}'(\chi, \bar{\chi}, \phi, V_\mu, A_\mu) = \bar{\chi} i\gamma\cdot\partial\chi - g_0\bar{\chi}(\phi_S + i\gamma_5\phi_P)\chi - e_0\bar{\chi}(\gamma_\mu V^\mu + \gamma_5\gamma_\mu A^\mu)\chi + \text{(mass counterterms)}. \tag{4.6}$$

Here the field χ is an equal mixture of the Dirac spinor ψ and its charge conjugate and hence is very much like a Majorana field. Using its definition we find that

$$\bar{\chi}\chi = \tfrac{1}{2}(\psi C^{-1}\gamma_5\psi + \bar{\psi}\gamma_5 C\bar{\psi}), \tag{4.7}$$

$$\bar{\chi}i\gamma_5\chi = \tfrac{1}{2}(\psi i C^{-1}\psi + \bar{\psi} i C\bar{\psi}), \tag{4.8}$$

$$\bar{\chi}\gamma_\mu\chi = \tfrac{1}{2}(\psi C^{-1}\gamma_5\gamma_\mu\psi + \bar{\psi}\gamma_\mu\gamma_5 C\bar{\psi}), \tag{4.9}$$

$$\bar{\chi}\gamma_5\gamma_\mu\chi = \bar{\psi}\gamma_5\gamma_\mu\psi. \tag{4.10}$$

Hence only the field A_μ couples to both χ and ψ while the others do not have a Yukawa type coupling to ψ. These fields ϕ_S, ϕ_P, and V_μ are interpreted as diquark (qq) [or dilepton $(l\bar{l})$]-type bound states. Therefore, our theory describes a situation where we have collective states both in $q\bar{q}$ and qq (or $\bar{q}\bar{q}$) channels. Still in this case the Higgs mechanism works as we clearly see in the χ representation, Eq. (4.6). The axial-vector field A_μ absorbs the massless pseudoscalar diquark ϕ_P and converts into a massive axial-vector meson. This phenomenon was first discovered in the study of superconductors.[18]

Non-Abelian analogs can be worked out in a similar way. In the case of an isodoublet of quarks the transformation is given by

$$\psi \to a\psi + b\psi^G, \tag{4.11}$$

$$\psi^G \to a^*\psi^G + b^*\psi, \tag{4.12}$$

$$|a|^2 + |b|^2 = 1, \tag{4.13}$$

where ψ^G is a G conjugate of ψ,

$$\psi^G = C\tau_2\bar{\psi}. \tag{4.14}$$

In this case bilinear forms $\bar{\psi}\psi$, $\bar{\psi}i\gamma_5\psi$, $\bar{\psi}\gamma_\mu\vec{\tau}\psi$, and $\bar{\psi}\gamma_5\gamma_\mu\psi$ are invariant under the transformation while, on the other hand, if we define χ by

$$\chi = \frac{1}{\sqrt{2}}(\psi + \psi^G), \tag{4.15}$$

$$\chi^G = \frac{1}{\sqrt{2}}(\psi^G + \psi), \tag{4.16}$$

the other bilinears transform as

$$\bar{\chi}\vec{\tau}\chi = \tfrac{1}{2}(\bar{\psi}^G\vec{\tau}\psi + \bar{\psi}\vec{\tau}\psi^G), \tag{4.17}$$

$$\bar{\chi}i\gamma_5\vec{\tau}\chi = \tfrac{1}{2}(\bar{\psi}^G i\gamma_5\vec{\tau}\psi + \bar{\psi}i\gamma_5\vec{\tau}\psi^G), \tag{4.18}$$

$$\bar{\chi}\gamma_\mu\chi = \tfrac{1}{2}(\bar{\psi}^G\gamma_\mu\psi + \bar{\psi}\gamma_\mu\psi^G), \tag{4.19}$$

$$\bar{\chi}\gamma_5\gamma_\mu\vec{\tau}\chi = \tfrac{1}{2}(\bar{\psi}^G\gamma_5\gamma_\mu\vec{\tau}\psi + \bar{\psi}\gamma_5\gamma_\mu\vec{\tau}\psi^G). \tag{4.20}$$

Hence $\vec{\phi}_S$, $\vec{\phi}_P$, V_μ, and $\vec{A}_\mu$ are diquarks. If we take $\langle\phi_S\rangle \neq 0$, we obtain a broken gauge theory of massive mesons ϕ_S, $\vec{\phi}_S$, A_μ, and $\vec{A}_\mu$ interacting with massless vector fields V_μ and $\vec{V}_\mu$.

A curious feature in the above construction of diquarks is that we always obtain only one combination of qq and $\bar{q}\bar{q}$ while *a priori* it seems possible to have both $qq \pm \bar{q}\bar{q}$. Charge-conjugation properties of diquarks are determined according to the choice of the mixed field χ. With our choice Eqs. (4.3) and (4.15) diquarks have opposite charge conjugations to their $q\bar{q}$ counterparts.

Apparently the above models cannot be realistic ones; however, in view of the recent speculations on the unified gauge theories of weak interactions,[19] it will be still worthwhile to look at the possibility of constructing a broken gauge theory of (quark or lepton) number conservation along the lines we have described.

V. DISCUSSIONS

In the preceding sections we have discussed spinor theories with scalar, pseudoscalar, vector, and axial-vector interactions. Then how about tensor interactions? In fact, recently there has been some speculation on the relevance of antisymmetric tensor fields in various physical contexts.[20,21] We have looked at this possibility but unfortunately could not find interesting theories involving tensor fields. Here the basic difficulty comes from the noninvariance of the antisymmetric field under chiral rotations. In fact, $\bar{\psi}\sigma_{\mu\nu}\psi$ transforms as

$$\bar{\psi}\sigma_{\mu\nu}\psi \to \cos 2\alpha\,\bar{\psi}\sigma_{\mu\nu}\psi + \sin 2\alpha\,\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi, \tag{5.1}$$

under an axial rotation. The invariant form is then given by

$$(\bar{\psi}\sigma_{\mu\nu}\psi)^2 + (\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi)^2 \,; \tag{5.2}$$

however, the above combination vanishes identically. Thus it is impossible to introduce an antisymmetric tensor in a chiral-invariant manner. Once the chiral symmetry is broken the axial-vector field A_μ no longer interacts in a gauge-invariant way and this causes certain difficulties.

Readers may have already wondered whether it is meaningful to discuss various four-Fermi interactions separately, since they transform into each other by Fierz rearrangements. For instance, in the Bjorken model if we first apply a Fierz transformation and then introduce collective coordinates, we will obtain a theory with scalar, pseudoscalar, vector, and axial-vector fields which is apparently different from the electrodynamics.

The solution to this puzzle is given as follows. As we have seen in preceding sections the condition for the elimination of self-energies for each collective field is given in the form of a gap equation,

$$1 = G_i I_i \quad (i = S, P, \cdots) \,. \tag{5.3}$$

Here G_i is the coupling constant for the type-i four-Fermi interaction and I's are certain divergent integrals which in general differ from each other. Hence if G's are given as a result of a Fierz transformation applied to a particular four-Fermi interaction, it is generally impossible to satisfy the above equations. Thus the requirement of the elimination of self-energies fixes the choice of collective modes and the type of the corresponding renormalizable theory.

As we see in the above not all of the spinor theories are made renormalizable even in our treatment. For instance, a theory of scalar and pseudoscalar interactions with different coupling constants cannot be renormalized. A similar situation obtains if a theory involves vector and axial-vector couplings with different coefficients. On the other hand, most renormalizable theories of fermions and bosons seem to be realized as nonlinear spinor theories by our method. At least in the case of renormalizable models in four dimensions with no dimensional coupling constants no counterexample has been found so far as the author could check.

In this paper we have developed a new approach to collective phenomena in superconductivity models and have revealed unknown relations between apparently nonrenormalizable theories and renormalizable models and also theories of one field and those of many fields. We hope that our approach may be useful in our further studies on collective phenomena in quantum field theories.

Added note: After submitting this paper for publication I came to know that Dr. N. Snyderman has obtained similar conclusions as mine in the Nambu–Jona-Lasinio model making use of a somewhat different method.

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