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Model for relativistic bound-state perturbation theory

Y. S. Kim
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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A model for relativistic bound-state perturbation theory is presented. For the bound-state wave function needed in the perturbation formula, the covariant-harmonic-oscillator wave function is used. As for the perturbing potential, the Feynman propagator corresponding to one-meson exchange is used. This model produces a relativistic derivation of the Breit-Fermi formula, which was recently used by De Rújula, Georgi, and Glashow in their study of hadronic masses.

I. INTRODUCTION

Most of the calculations in nonrelativistic quantum mechanics require one or another kind of perturbation theory. For scattering processes, we often use a Born-type series starting from plane-wave solutions. For bound-state problems, we calculate energy shifts using normalizable bound-state wave functions.

Let us review the attempts to make a Lorentz generalization of these procedures. The plane-wave expansion eventually becomes the covariant perturbation theory in which the S matrix is written as a sum of Feynman amplitudes. This is possible because the Lorentz generalization of the plane wave is simple and accepts Feynman's interpretation of propagation. As for the bound state, we do not as yet have any working perturbation theory. This is because there are no normalizable covariant bound-state wave functions. One possible way to avoid this difficulty once appeared to be a bound-state perturbation theory in terms of the functions associated with the S matrix which are covariant quantities.[1] However, in this approach, there are very serious difficulties in maintaining proper asymptotic behavior of the bound-state wave function.[2]

These facts lead us to look into the possibility of formulating a Lorentz-invariant eigenvalue problem and of finding normalizable wave functions carrying a covariant probability interpretation. For this purpose, Kim and Noz presented a model based on covariant harmonic oscillators.[3-5] The purpose of this note is to present a covariant bound-state perturbation theory based on the harmonic-oscillator wave functions.

In formulating a covariant perturbation theory, we also need a covariant perturbing potential. The concept of a covariant weak potential is not strange in modern physics. This potential arises from the exchange of a meson by the constituent particles. Therefore, the Feynman propagator for this intermediary meson is the ideal candidate for the covariant perturbing potential.[6]

While the primary purpose of this paper is to formulate a covariant perturbation theory, it is unavoidable to examine further physical implications of the covariant harmonic oscillator. Like all theories, the oscillator formalism we use in this paper started from phenomenology. However, unlike other relativistic models, the wave functions in the oscillator formalism can be given a covariant probability interpretation. This feature forces us to look into possibilities of reexamining the present form of the quantum superposition principle. In collaboration with Vasavada[7] and Noz[3,4] this author has been dealing with this problem in previous publications.[8] The present paper is also a part of the continuing discussion on this important problem.[9]

In this paper, we first formulate the abovementioned perturbation theory, and then apply this formalism to the mass difference calculations of current interest. In Sec. II, we present the formalism of covariant harmonic oscillators and discuss the physical implications which are needed in formulating a bound-state perturbation theory. In Sec. III, we discuss the first-order mass shift due to the exchange of a meson. In Sec. IV, we derive a relativistic Breit-Fermi formula, which is needed for studying hadronic mass spectra.

II. CONSTRUCTION OF COVARIANT HARMONIC OSCILLATORS

We consider a system of two quarks bound together by a harmonic-oscillator force. We use the organizational framework of Feynman *et al.*[10] and start with the equation

$$\{2[\Box_1 + \Box_2] - \tfrac{1}{16}\omega^2(x_1 - x_2)^2 + m_0{}^2\}\,\phi(x_1, x_2) = 0\,, \tag{1}$$

where ω is the spring constant. In this bound system, the quarks can never be separated, and their masses do not carry any meaning. Their masses are intrinsically contained in $\Box_1$ and $\Box_2$.

They do not have to be equal. Following Feynman *et al.*,[10] we make the following coordinate transformation:

$$X = \tfrac{1}{2}(x_1 + x_2), \quad x = \frac{1}{2\sqrt{2}}(x_1 - x_2). \tag{2}$$

In terms of these new variables, we can write Eq. (1) as

$$\left[\frac{\partial^2}{\partial X_\mu{}^2} + m_0{}^2 + \frac{1}{2}\left(\frac{\partial^2}{\partial x_\mu{}^2} - \omega^2 x_\mu{}^2\right)\right]\phi(X, x) = 0. \tag{3}$$

We can now separate the variables X and x, and write

$$\phi(X, x) = f(X)\psi(x). \tag{4}$$

Then $f(X)$ and $\psi(x)$ satisfy the following equations respectively:

$$\left(\frac{\partial^2}{\partial X_\mu{}^2} + m_0{}^2 + \lambda\right) f(X) = 0, \tag{5}$$

$$\frac{1}{2}\left(\frac{\partial^2}{\partial x_\mu{}^2} - \omega^2 x_\mu{}^2\right)\psi(x) = \lambda\psi(x). \tag{6}$$

The differential equation for $f(X)$ is a Klein-Gordon equation in the X variables, which can be regarded as the hadronic coordinate. The physics of the Klein-Gordon equation is well known. Equation (6) is a relativistic harmonic-oscillator equation for the constituent quarks.

There are many different solutions to Eq. (6) depending on boundary conditions. The set of wave functions proposed recently by Kim and Noz[3,4] satisfy all the requirements of nonrelativistic quantum mechanics in the hadronic rest frame. The basic advantage of using these wave functions is the fact that they carry a covariant probability interpretation.[11]

The construction of Kim and Noz goes like this: The hyperbolic differential equation of Eq. (6) is separable in the x, y, z, t variables. It is also separable in their Lorentz-transformed variables

$$\begin{aligned} x' &= x, \quad y' = y, \\ z' &= (1-\beta^2)^{-1/2}(z - \beta t), \\ t' &= (1-\beta^2)^{-1/2}(t - \beta z), \end{aligned} \tag{7}$$

where β is the velocity parameter of the hadron. The normalizable solution then becomes

$$\psi(x, P) = H_n(x', y', z')\exp[-\tfrac{1}{2}\omega(x'^2 + y'^2 + z'^2 + t'^2)], \tag{8}$$

where $H_n(x', y', z')$ is a product of Hermite polynomials corresponding to excitations along the x', y', and z' directions. P is the four-momentum of the hadron. We can suppress the timelike oscillation along the t' direction by imposing the familiar subsidiary condition

$$P^\mu\left(\omega x_\mu + \frac{\partial}{\partial x^\mu}\right)\psi(x, P) = 0. \tag{9}$$

We can now define the inner product of the two wave functions as

$$(\psi(x,P), \psi'(x,P')) = \int d^4x\, \psi^*(x,P)\psi'(x,P'). \tag{10}$$

If the four-vectors P and P' represent the same velocity, then both wave functions are in the same Lorentz frame, and the inner product, after the t' integration, becomes exactly the three-dimensional inner product of nonrelativistic quantum mechanics. If P and P' represent different velocities, one wave function should appear Lorentz-contracted in the rest frame of the other. The Lorentz-contraction properties of these harmonic-oscillator wave functions have been extensively discussed in the literature.[12]

The crucial difference between the present formalism and the conventional nonrelativistic harmonic oscillator is the presence of the t variable in Eq. (6). This is the time difference between the two bound quarks. The presence of this variable is inevitable in a covariant formulation of quantum mechanics. Quantum mechanics deals with uncertainties in space separation. Relativity deals with the linear mixture of space and time. Therefore, the uncertainty in time separation should be taken into account in any attempt to combine quantum mechanics with relativity.[13] Indeed, this time separation enabled us to construct a covariant theory of Lorentz contraction which is urgently needed in high-energy hadronic physics.[14]

Solutions of the harmonic-oscillator wave functions have well-defined space-reflection properties. Here we have, in addition, the time coordinate. Equation (9) restricts the solutions of the covariant-harmonic-oscillator equation to the ground state in the t' coordinate. Since the ground-state solution is even under the operation $t' \to -t'$, all the solutions are even under this operation. This means that the causality among the constituent particles in the hadron rest frame cannot be imposed on this permanently bound system, and that we have to take into account both cases when the constituent quarks interact with other particles, such as gluons. This point will be discussed again in Sec. III in connection with formulating a covariant bound-state perturbation theory.

III. PERTURBATION DUE TO FEYNMAN PROPAGATION FUNCTION

Let us go back to Eqs. (4), (5), and (6). While $f(X)$ represents the plane-wave solution of the hadron, $\psi(x)$ describes the internal motion of the constituent particles. It is this $\psi(x)$ function which determines the discrete mass spectrum through the localization condition on probability distribution. We now consider adding a small perturbation to the oscillator equation of Eq. (6). This perturbation will cause a shift in the harmonic-oscillator mass spectrum. Let us now consider possible forms of this perturbing potential. Among many possibilities, we choose the Feynman picture of perturbation due to the exchange of a meson. For the exchange of a single meson, we can consider

$$\delta V(x) = -iG^2\Delta_F(x), \tag{11}$$

where G is the strength of the coupling of the meson to the quark. $\Delta_F(x)$ is the Feynman propagation function and $i\Delta_F(x)$ describes the meson propagation from one quark to the other. The constant G^2 has no dimension. $\Delta_F(x)$ has the dimension of m^2. Thus the above $\delta V(x)$ can be added to the binding potential $-\frac{1}{2}\omega^2 x_\mu x^\mu$ in Eq. (6).

The energy levels and normalization conditions on the wave function are well known. The first-order mass shift becomes

$$\delta m^2 = -iG^2 \int d^4x\, \psi_n^*(x)\Delta_F(x)\psi_n(x). \tag{12}$$

We now introduce the momentum wave function defined as

$$\phi_n(q) = \left(\frac{1}{2\pi}\right)^2 \int d^4x\, e^{iq\cdot x}\psi_n(x). \tag{13}$$

In terms of this momentum wave function, the mass shift of Eq. (12) becomes

$$\delta m^2 = iG^2\left(\frac{1}{2\pi}\right)^4 \int d^4q\, d^4q' \,\frac{\phi_n^*(q)\phi_n(q')}{(q-q')^2-\mu^2+i\epsilon}, \tag{14}$$

where μ is the mass of the meson being exchanged.

We note that the momentum wave function depends on the velocity of the hadron. However, the Feynman propagator in the integrand of Eq. (14) is Lorentz-invariant. δm^2 is therefore independent of the frame in which the above integral is evaluated.

Although δm^2 of Eq. (14) is covariant and carries a physical interpretation, the perturbing potential of Eq. (11) is not Hermitian, and δm^2 is complex. This forces us to take the real part of Eq. (14). The physics of this operation is not difficult to understand. Taking the real part means replacing the Feynman propagator in Eq. (14) by

$$\frac{1}{2}\left[\frac{i}{(q-q')^2-\mu^2+i\epsilon} - \frac{i}{(q-q')^2-\mu^2-i\epsilon}\right]. \tag{15}$$

The propagator with the opposite sign of $i\epsilon$ means that the sense of time is reversed. We are considering here a system of permanently bound particles. As was mentioned in Sec. II, we are dealing with the system where the sign of the relative time can never be detected. Therefore, the proper-time symmetrization, such as the symmetrization of Eq. (15), is required for the harmonic-oscillator system.

Furthermore, this symmetrization is expected from the following aspect of nonrelativistic quantum mechanics. For scattering problems, we use the representation of running waves to describe the process, and the Feynman propagator is a representation of the running wave. For bound-state problems, the standing wave is the appropriate form, and we construct the standing wave by superposing two running waves traveling in opposite directions. The representation of Eq. (15) is therefore a "standing wave" Feynman propagation function.

After this replacement, the perturbation formula of Eq. (14) takes a very simple form:

$$\delta m^2 = \pi G^2\left(\frac{1}{2\pi}\right)^4 \int d^4q\, d^4q'\, \phi_n^*(q)\phi_n(q')\delta((q-q')^2-\mu^2). \tag{16}$$

The above integral is not difficult to evaluate when the meson mass μ vanishes. For the ground-state harmonic oscillator, δm^2 becomes

$$\delta m^2 = \left(\frac{G^2}{4\pi}\right)\left(\frac{\omega}{\pi}\right). \tag{17}$$

At this point, we may raise the question whether the above modification of the Feynman propagator was necessary in view of the existing nonrelativistic procedures.[15] We are familiar with the nonrelativistic procedure of obtaining weak perturbing potentials from one-meson-exchange Feynman amplitudes by means of a Fourier transformation. This is possible only when the energy transfer vanishes. When the mass of the final-state particle is different from that of the initial state, the energy transfer does not vanish, and the denominator of the propagator vanishes in the region of integration if the meson mass is sufficiently small. Therefore, the question of taking the real part exists even in nonrelativistic cases. Ignoring the energy component of the denominator, under the pretext of nonrelativistic approximation, makes an infinite quantity finite. We therefore have to conclude that there are no satisfactory solutions to this problem in the nonrelativistic approach.

Another point we have to note is that Eq. (16) is a perturbation formula for the (mass)^2. Since the invention of the Gell-Mann–Okubo (GMO) mass formula, whether we should use the linear mass or (mass)^2 has been an unsettled question. The (mass)^2 has been favored since the linearity in

Regge trajectories became apparent.[16] Although the linear mass was used for the original GMO mass formula for baryons, the latest indications are that the (mass)^2 formula is numerically more accurate even for the baryonic case.[17] We shall discuss practical applications of Eq. (16) in the following section.

IV. APPLICATIONS TO HADRONIC MASS SPECTRA

In this section, we shall discuss possible applications of Eq. (16) to the physics of current interest. What we have done in the preceding section is in close pace with the quark-confinement picture and with the perturbation formalism based on this picture.[18]

The ultimate goal of the confinement program, which is based on asymptotic freedom and infrared slavery, is to get a binding potential which is nonsingular at the origin and becomes infinite for large distances. The harmonic-oscillator potential we use in this paper indeed satisfies these requirements. At this time, it is not clear how accurately the confinement program will determine the binding potential. Even if the program produces an accurate potential, and if this potential does not take any simple form, soluble models such as the oscillator and linear potentials will play a major role in understanding the physics of this presumably more accurate confinement potential.

There are speculations on the linear potential among those supporting the confinement idea.[19] The linear potential is also soluble and contains many desirable features. The eigenvalue obtained in this approach is the linear mass. This is numerically close to getting the (mass)^2 eigenvalues using the harmonic-oscillator potential.[20] Therefore, there is no basic conflict between the linear and oscillator potentials.

In Sec. III, we explained the reason for choosing the (mass)^2 formula and consequently the harmonic-oscillator potential. However, the primary advantage of using the harmonic oscillator is the fact that its formalism is covariant and accepts a covariant probability interpretation. This point was also stressed in the preceding sections. All the physical systems of current interest are relativistic systems.

Among many possible applications of the relativistic perturbation formula, the most exciting recent development has been the symmetry-breaking mechanism proposed by De Rújula *et al.*[18] The most striking aspect of their work is that the symmetry-breaking mechanism is caused by one-gluon exchange between the constituent quarks and that this mechanism can explain the GMO mass relation and other interesting mass relations. As in all other papers on the mass formula, the weakness of their work is that their treatment is basically nonrelativistic and that there is an intrinsic difficulty which we mentioned in Sec. III in connection with deriving the Breit-Fermi formula for unequal quark masses.

The key term in the work of De Rújula *et al.* is the spin correlation term with unequal quark masses in the denominator. In this section, we derive this spin-correlation term using the perturbation formalism given in the preceding section.

Let us first discuss the baryonic case. The baryon consists of three quarks, and we can assign the four-momenta to these quarks[9]

$$\begin{aligned} p_1 &= \tfrac{1}{3}P + \tfrac{1}{6}q - \frac{1}{2\sqrt{3}}k\,, \\ p_2 &= \tfrac{1}{3}P + \tfrac{1}{6}q + \frac{1}{2\sqrt{3}}k\,, \\ p_3 &= \tfrac{1}{3}P - \tfrac{1}{3}q\,, \end{aligned} \tag{18}$$

where P is the total four-momentum of the baryon. q and k are two independent internal four-momenta for the three-quark system.

In terms of these momenta, the spatial part of the ground-state wave function takes the form

$$\begin{aligned} \psi(q,k) = \left(\frac{1}{\pi\omega}\right)^2 &\exp\left\{-\frac{1}{2\omega}\left[-q_\mu q^\mu + 2(q\cdot P)^2/P^2\right]\right\} \\ &\times \exp\left\{-\frac{1}{2\omega}\left[-k_\mu k^\mu + 2(k\cdot P)^2/P^2\right]\right\}. \end{aligned} \tag{19}$$

The above expression takes a particularly simple form in the hadron rest frame:

$$\psi(q,k) = \left(\frac{1}{\pi\omega}\right)^2 \exp\left\{-\frac{1}{2\omega}(\vec{q}^{\,2} + q_0^{\,2} + \vec{k}^2 + k_0^{\,2})\right\}. \tag{20}$$

Let us now consider spins. If there were no internal quark motion, the spin wave function in the hadron rest frame would be a Dirac spinor with positive-energy component only, which is

$$\left(\frac{\gamma_0 + 1}{2}\right)\chi_i\,, \tag{21}$$

where χ_i is the static Dirac spinor with appropriate spin index for the ith quark. If the quarks have the internal momenta, the spin wave function for the ith quark will take the form

$$\left(\frac{\gamma_1 p_i + m_i}{2m_i}\right)\chi_i\,, \tag{22}$$

where $i = 1, 2, 3$. The internal momenta p_i are given in Eq. (18). m_i is the mass of the ith quark. We should note here that the spatial wave function does not depend on the quark mass, and that the dependence on this mass comes solely through the spin considerations. In the symmetry-breaking picture of De Rújula *et al.*, this quark mass plays the decisive role.

With these preparations, we can now consider the mass shift due to the exchange of a massless gluon between the first and second quarks. We take the gluon propagator to be

$$\frac{-ig_{\mu\nu}}{(p_1'-p_1)^2+i\epsilon}\,. \tag{23}$$

The mass shift then becomes

$$\begin{aligned}\delta m^2 = -\pi G^2 N\left(\frac{1}{2\pi}\right)^4 \int d^4q\, d^4k\, d^4k'\,\delta((p_1-p_1')^2)\exp\left[-\frac{1}{\omega}(\vec{q}^{\,2}+q_0^{\,2})-\frac{1}{2\omega}(\vec{k}^2+k_0^{\,2}+\vec{k}'^2+k_0'^{\,2})\right]\\ \times\left[\bar{\chi}_1\left(\frac{\gamma\cdot p_1+m_1}{2m_1}\right)\gamma_\mu\left(\frac{\gamma\cdot p_1'+m_1}{2m_1}\right)\chi_1\right]\left[\bar{\chi}_2\left(\frac{\gamma\cdot p_2+m_2}{2m_2}\right)\gamma^\mu\left(\frac{\gamma\cdot p_2'+m_2}{2m_2}\right)\chi_2\right]\\ \times\left[\bar{\chi}_3\left(\frac{\gamma\cdot p_3+m_3}{2m_3}\right)\chi_3\right],\end{aligned} \tag{24}$$

where

$$p_1'=\tfrac{1}{3}P+\tfrac{1}{6}q-\frac{1}{2\sqrt{3}}k,\quad p_2'=\tfrac{1}{3}P+\tfrac{1}{6}q+\frac{1}{2\sqrt{3}}k\,.$$

The momentum relations for the gluon and the constituent quarks can be seen in Fig. 1. The constant N in the above expression is the normalization constant and takes the form

$$\frac{1}{N}=\int d^4q\,d^4k\exp\left[-\frac{1}{\omega}(\vec{q}^{\,2}+q_0^{\,2}+\vec{k}^2+k_0^{\,2})\right]\left[\bar{\chi}_1\left(\frac{\gamma\cdot p_1+m_1}{2m_1}\right)\chi_1\right]\left[\bar{\chi}_2\left(\frac{\gamma\cdot p_2+m_2}{2m_2}\right)\chi_2\right]\left[\bar{\chi}_3\left(\frac{\gamma\cdot p_3+m_3}{2m_3}\right)\chi_3\right]. \tag{25}$$

In Eq. (24), we assumed that the gluon is exchanged by quark 1 and quark 2 for convenience. Since the baryon wave function is totally symmetric, this particular choice is equivalent to all other possible choices. Now the remaining task is to evaluate the above integrals. This calculation is extremely lengthy but is straightforward. First, we write down the result of the normalization integral of Eq. (25)

$$\frac{1}{N}=\frac{(\pi\omega)^4}{8m_1m_2m_3}\left[(m_1+\tfrac{1}{3}M)(m_2+\tfrac{1}{3}M)(m_3+\tfrac{1}{3}M)-\tfrac{1}{36}\omega(M+m_1+m_2+m_3)\right]. \tag{26}$$

We note that the above result is totally symmetric under the exchange of quarks. Furthermore, the numerical vlaue of ω is approximately 1 (GeV)^2, and the mass of the quark is approximately one third of the nucleon mass. With these numerical values, we can ignore the second term in the square brackets, and

$$\frac{1}{N}=\frac{(\pi\omega)^4}{8m_1m_2m_3}(m_1+\tfrac{1}{3}M)(m_2+\tfrac{1}{3}M)(m_3+\tfrac{1}{3}M)\,. \tag{27}$$

In calculating Eq. (24), there are terms which are independent of the quark spin, those which are quadratic in the spin, and those which are quartic. The terms that are independent of the quark spin do not contribute to the mass differences. The terms which are quartic in the quark spin are numerically very small. Thus we retain only those terms which are quadratic in the quark spin. The algebra then becomes substantially simple, and the spin part of the integrand of Eq. (24) becomes

$$\frac{(m_1+\frac{1}{3}M)(m_2+\frac{1}{3}M)(m_3+\frac{1}{3}M)}{32m_1^{\,2}m_2^{\,2}m_3}\times\left[\frac{2}{27}\vec{q}^{\,2}+\frac{1}{9}\left(\frac{\vec{k}+\vec{k}'}{\sqrt{2}}\right)^2\right]\vec{\sigma}_1\cdot\vec{\sigma}_2\,, \tag{28}$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli spin matrices for quark 1 and quark 2, respectively. The integrations over q, k, and k' variables can now be per-

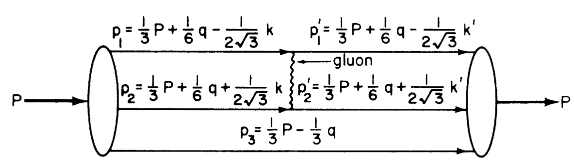

FIG. 1. Four-momenta of the quarks inside the baryon. They are distributed in such a way that the energy-momentum conservation is preserved at each vertex.

formed, and

$$\delta m^2 = \left(\frac{G^2}{4\pi}\right)\left(\frac{20}{3\pi}\right) \omega^2 \left(\frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{4 m_1 m_2}\right). \tag{29}$$

Now considering other pairs of quarks, we have

$$\delta m^2 = \left(\frac{G^2}{4\pi}\right)\left(\frac{20}{3\pi}\right) \omega^2 \sum_{i<j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{4 m_i m_j}. \tag{30}$$

This formula was used by De Rújula *et al.* in their work on hadronic masses.[18]

The spin consideration for mesons is very similar to the baryonic case,[4] and the mesonic mass formula will turn out to be of the form of Eq. (20).

Relativity \ Quantum Mechanics	Scattering	Bound State
Nonrelativistic	Born Series	$\delta E = (\phi, \delta V \phi)$
Relativistic	Feynman Diagrams	FIRST FORMULA

FIG. 2. The present status of perturbation theory. There is at present no accepted formula for relativistic bound states, and the first acceptable formula will play a key role for many problems of current interest.

V. CONCLUDING REMARKS

The purpose of this paper is not to derive the consequences of Eq. (30). This has already been done.[18] What is new in the present paper is the relativistic derivation of Eq. (30). The derivation may prove to be far more significant than justifying the work of De Rújula *et al.* As was stated in Sec. I, and as is indicated in Fig. 2, we do not have an accepted relativistic bound-state perturbation theory. The perturbation formula we presented in this paper is the strongest candidate since the invention of quantum mechanics for the first formula in a covariant bound-state perturbation theory.

ACKNOWLEDGMENT

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