Einstein Lagrangian as the translational Yang-Mills Lagrangian*

Y. M. Cho†

The Enrico Fermi Institute, and The Department of Physics, University of Chicago, Chicago, Illinois 60637

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The gauge theory of translation with a Yang-Mills–type Lagrangian quadratic in the field strengths is shown to be precisely Einstein's theory of gravitation and the corresponding gauge transformation is identified as the general coordinate transformation. The gauge potentials of the translation group are interpreted as the nontrivial part of the vierbein fields and the gauge field strengths are given in terms of the anholonomity of the local orthonormal basis one starts with.

I. INTRODUCTION

Gauge theories of the Yang-Mills type and Einstein's theory of gravitation have a common feature: the self-interaction of the fields. Then one is led to ask whether Einstein's theory itself is a gauge theory. Of course this is an old question and many people[1-3] have suggested that Einstein's theory can be viewed as the gauge theory of the four-dimensional translation group T_4. Unfortunately certain features seem not to have been fully clarified so far, and it is precisely these features that bear out the complete relationship between the Yang-Mills and Einstein theories.

In this paper we show that if one only applies the gauge principle (this includes a Yang-Mills—type Lagrangian quadratic in the field strengths) for the group of translation T_4 of space-time, the gauge theory that one obtains is unique and becomes precisely Einstein's theory of gravitation. In this T_4 gauge formalism of Einstein's theory the translational gauge potentials are identified as the nontrivial part of the vierbein fields and the gauge field strengths are given in terms of the commutator coefficients (i.e., the anholonomity) of the local orthonormal basis one starts with.

To prove that the unique gauge theory of the translation is Einstein's theory, it is important to observe that although the gauge group T_4 is Abelian, it is not an internal-symmetry group and acts on space-time itself. Fortunately the geometric meaning of gauge theories has been well understood by now in terms of the bundle picture.[4-6] The power of this bundle picture has been appreciated by Cho and Freund[5,6] in unifying gauge theories with gravitation and also recently by Wu and Yang.[7] In the following we will first prove our claim in a formal way constructing the eight-dimensional bundle of the translation group T_4 over space-time and then will give a precise physical meaning to this translational bundle. For the details about the bundle formalism of gauge theories we refer the reader to Ref. 5.

II. THE GAUGE THEORY OF THE TRANSLATION GROUP

Let us assume that the structural group G of our bundle P is T_4 with four commuting generators ξ_α ($\alpha=1,2,3,4$),

$$[\xi_\alpha, \xi_\beta]=0, \quad (1)$$

and that the base manifold M is the four-dimensional space-time with an orthonormal basis at each point, i.e., four orthonormal vector fields e_i ($i=1,2,3,4$) with the commutation relations

$$[e_i, e_j]=T_{ij}{}^k e_k. \quad (2)$$

Of course the basis independence of a theory is one of the basic principles in physics, and one can choose any other basis if one wants to, but for obvious reasons the local orthonormal basis (2) is the natural one to start with in our problem. Notice that this orthonormal basis is not in general a coordinate basis since the basis vectors do not commute. If we introduce a coordinate basis ∂_μ ($\mu=1,2,3,4$) with

$$[\partial_\mu, \partial_\nu]=0,$$

then e_i can be written in terms of the vierbein fields h_i^μ,

$$e_i=h_i^\mu \partial_\mu,$$

and correspondingly we have

$$T_{ij}{}^k=(\partial_i h_j^\mu-\partial_j h_i^\mu)h_\mu^k. \quad (3)$$

Here $\partial_i=h_i^\mu\partial_\mu$ is the directional derivative in the direction of e_i and h_μ^k are the inverse vierbein fields

$$h_\mu^i h_k^\mu=\delta_k^i, \quad h_i^\mu h_\nu^i=\delta_\nu^\mu.$$

Observe that due to the commutation relation (2) of the basis e_i, the directional derivatives ∂_i do not commute either.

At this point we would like to emphasize that all the above expressions are just a matter of a formalism and we have *not* assumed that our space-time is curved. Eventually we will create a curvature by introducing gauge fields associated with T_4 symmetry, but we start with a *flat* space-time and so far our h_i^μ remain trivial.

Now, given a connection form[8] $\omega = \omega^\alpha \xi_\alpha$ in the bundle P, the gauge potentials B_i^α are as usual given by the connection coefficients of $\tilde{e}_i$, the lift of e_i into a four-dimensional gauge-defining submanifold σ, i.e., a cross section of P:

$$\omega^\alpha(\tilde{e}_i) = \kappa B_i^\alpha, \tag{4}$$

where we have introduced a dimensional constant κ (of dimension of a length) to give the canonical dimension to the gauge potentials B_i^α. This κ will serve as the coupling constant for the gauge group T_4 and will be related to the gravitational constant later on.

With $\hat{e}_i$ ($i=1,2,3,4$) as the horizontal lift of e_i and ξ_α^* ($\alpha=1,2,3,4$) as the fundamental vector fields which are vertical, we clearly have

$$\begin{aligned} &[\xi_\alpha^*, \xi_\beta^*] = 0, \\ &[\xi_\alpha^*, \hat{e}_k] = 0, \\ &[\hat{e}_i, \hat{e}_j] = T_{ij}{}^k \hat{e}_k - \kappa G_{ij}{}^\alpha \xi_\alpha^*, \end{aligned} \tag{5}$$

where G_{ij}^α are the vertical components of the commutator coefficients of the horizontal lift vector fields $\hat{e}_i$. The first two equations come from the definition and the third is due to the fact that the projection of $[\hat{e}_i, \hat{e}_j]$ down to the base manifold is the same as $[e_i, e_j]$. Notice that because of the Abelian character of the ξ_α^*'s the group action on the bundle space is really a translation and there is no "rotation" whatsoever. Mathematically this means that the holonomy group of P is T_4 and not the Lorentz group.

Let us recall that in the bundle picture the gauge field strengths are given by the vertical commutator coefficients of two horizontal vector fields, i.e., by $G_{ij}{}^\alpha$. To find the gauge field strengths in terms of potentials B_i^α, notice that from Eq. (4) and from the definition of ω^α,

$$\begin{aligned} &\omega^\alpha(\hat{e}_i) = 0, \\ &\omega^\alpha(\xi_\beta^*) = \delta_\beta^\alpha, \end{aligned}$$

it follows that

$$\hat{e}_i = \tilde{e}_i - \kappa B_i^\alpha \xi_\alpha^*,$$

and

$$\begin{aligned} [\hat{e}_i, \hat{e}_j] &= [\tilde{e}_i - \kappa B_i^\alpha \xi_\alpha^*, \tilde{e}_j - \kappa B_j^\beta \xi_\beta^*] \\ &= T_{ij}{}^k \tilde{e}_k - \kappa(\partial_i B_j^\alpha - \partial_j B_i^\alpha)\xi_\alpha^* \\ &= T_{ij}{}^k \hat{e}_k - \kappa[(\partial_i B_j^\alpha - \partial_j B_i^\alpha) - T_{ij}{}^k B_k^\alpha]\xi_\alpha^*. \end{aligned} \tag{5'}$$

Thus one has

$$G_{ij}{}^\alpha = (\partial_i B_j^\alpha - \partial_j B_i^\alpha) - T_{ij}{}^k B_k^\alpha . \tag{6}$$

Now following Kibble's suggestion[2] we interpret the gauge potentials B_i^μ as the nontrivial part of the vierbein fields

$$h_i^\mu = \delta_i^\mu + \kappa B_i^\mu . \tag{7}$$

This means that *we now have created the curvature of space-time by introducing the gauge potentials B_i^μ for T_4 and making the vierbein fields h_i^μ nontrivial.* Notice that the decomposition of h_i^μ into δ_i^μ and B_i^μ is basis-dependent since δ_i^μ is not invariant under a rotation of the local orthonormal basis e_i.

From Eqs. (3), (6), and (7) one finds that

$$\begin{aligned} G_{ij}{}^\alpha &= (\partial_i B_j^\alpha - \partial_j B_i^\alpha) - T_{ij}{}^k B_k^\alpha \\ &= \frac{1}{\kappa} T_{ij}{}^k (h_k^\alpha - \kappa B_k^\alpha) \\ &= \frac{1}{\kappa} T_{ij}{}^k \delta_k^\alpha . \end{aligned} \tag{6'}$$

The gauge field strengths G_{ij}^α are thus determined by the commutation coefficients $T_{ij}{}^k$ of the orthonormal basis vectors that one starts with. We would like to emphasize here that once the connection ω (i.e., the gauge potentials in physical terms) is given, the gauge field strengths G_{ij}^α are uniquely determined from the geometrical structure of the bundle and are not something that one can define otherwise as sometimes suggested.[3]

Gauge transformations in this picture are changes of bundle cross sections.[4,5] If we change the cross section σ to σ' by a four-translation $\theta^\alpha(x)$ ($\alpha=1,2,3,4$) in the four-dimensional fiber space [geometrically $\theta^\alpha(x)$ are simply a set of transition functions that relate σ' to σ], we clearly have

$$\tilde{e}_i' = \tilde{e}_i + (\partial_i \theta^\alpha)\xi_\alpha^*$$

and

$$B'^\alpha_i = \frac{1}{\kappa}\,\omega^\alpha(\tilde{e}_i') = B_i^\alpha + \frac{1}{\kappa}\partial_i\theta^\alpha . \tag{8}$$

From Eq. (7) this means that under the gauge transformation we have

$$\begin{aligned} h_i^\mu \to h'^\mu_i &= h_i^\mu + \partial_i \theta^\mu \\ &= h_i^\alpha(\delta_\alpha^\mu + \partial_\alpha \theta^\mu) \\ &= h_i^\alpha X_\alpha^\mu , \end{aligned}$$

where

$$X_\alpha^\mu = \delta_\alpha^\mu + \partial_\alpha \theta^\mu ,$$

and now *the gauge transformation is unambiguously identified as a general coordinate transformation in the coordinate basis* ∂_μ:

$$\begin{aligned} \partial_\mu \to \partial_\mu' &= (\delta_\mu^\alpha + \partial_\mu \theta^\alpha)\partial_\alpha \\ &= X_\mu^\alpha \partial_\alpha . \end{aligned} \tag{9}$$

Notice that gauge transformations (or equivalently general coordinate transformations) do not change the orthonormal basis e_i: They change the coordinate basis ∂_μ. Also notice that under a local gauge transformation one has

$$\begin{aligned} G_{ij}{}'^{\alpha} &= (\partial_i B_j'^{\alpha} - \partial_j B_i'^{\alpha}) - T_{ij}{}^k B'^{\alpha}_k \\ &= G_{ij}{}^{\alpha} + \frac{1}{\kappa}(\partial_i\partial_j - \partial_j\partial_i)\theta^{\alpha} - \frac{1}{\kappa} T_{ij}{}^k \partial_k \theta^{\alpha} \\ &= G_{ij}{}^{\alpha}, \end{aligned} \tag{10}$$

i.e., $G_{ij}{}^{\alpha}$ are invariant under gauge transformations as expected for an Abelian gauge group.

Once we have the gauge field strengths $G_{ij}{}^{\alpha}$, we can write in the manner of Yang and Mills the most general Lagrangian quadratic in these $G_{ij}{}^{\alpha}$. Using Eq. (6′) we have

$$\begin{aligned} \mathcal{L} &= \sqrt{-g}\, G_{ij}{}^{\alpha} G_{kl}{}^{\beta}(a\eta^{ik}\eta^{jl}\eta_{\alpha\beta} + b\eta^{ik}\delta^l_\alpha\delta^j_\beta + c\eta^{ik}\delta^j_\alpha\delta^l_\beta) \\ &= \frac{1}{\kappa^2}\sqrt{-g}\,(aT_{ijk}T_{ijk} + bT_{ijk}T_{ikj} + cT_{ijj}T_{ikk}), \end{aligned}$$

where

$$\sqrt{-g} = \det(h^i_\mu),$$

a, b, and c are for the time being arbitrary constants, and we have used the *flat* metric $\eta_{\alpha\beta}$ for the fiber space. Notice that in our formalism we do *not* need a Riemannian metric *a priori*. The crucial question now is whether the constants a, b, and c are really arbitrary. To answer this question let us point out that the above Lagrangian is basis dependent as it is obtained using Eq. (6′). Now, if the theory is going to have any meaning at all, it should not depend upon which orthonormal frame one starts with. This means that if one chooses a different set of e_i's, the Lagrangian should differ only by a total divergence. We now show that this consistency requirement removes all the arbitrariness in a, b, and c.

Notice that under an infinitesimal change of orthonormal frame, one has

$$h^\mu_i(x) \to h'^\mu_i(x) = h^\mu_i(x) + \omega_{ik} h^\mu_k(x), \tag{11}$$

where $\omega_{ik}(x) = -\omega_{ki}(x)$ are six infinitesimal functions so that

$$\begin{aligned} \delta\mathcal{L} &= \sqrt{-g}\,(2aT_{ijk}\delta T_{ijk} + 2bT_{ijk}\delta T_{ikj} + 2cT_{ijj}\delta T_{ikk}) \\ &= \sqrt{-g}\,[2aT_{ijk}(\partial_i\omega_{jk} - \partial_j\omega_{ik}) + 2bT_{ijk}(\partial_i\omega_{kj} - \partial_k\omega_{ij}) \\ &\qquad + 2cT_{ijj}\partial_k\omega_{ki}] \\ &= \sqrt{-g}\,[(4a-2b)T_{ijk}\partial_i\omega_{jk} - 2bT_{ijk}\partial_k\omega_{ij} \\ &\qquad + 2cT_{ijj}\partial_k\omega_{ki}] \\ &= \sqrt{-g}\,[(4a-2b)T_{ijk}\partial_i\omega_{jk} - (2b+c)T_{ijk}\partial_k\omega_{ij}] \\ &\quad - 2c\partial_\mu(h^\mu_i\sqrt{-g}\,\partial_k\omega_{ki}). \end{aligned} \tag{12}$$

Here the last equality comes from the following identity:

$$\partial_\mu(h^\mu_i\sqrt{-g}\,\partial_k\omega_{ki}) = -\sqrt{-g}\,(\tfrac{1}{2}T_{ijk}\partial_k\omega_{ij} + T_{ijj}\partial_k\omega_{ki}).$$

Now in the last line of Eq. (12) the third term is explicitly a total divergence. But each of the first two terms cannot be made into a total divergence and one is forced to choose $a:b:c = 1:2:-4$ to satisfy the consistency requirement. So the Lagrangian should have the form

$$\mathcal{L}_{T_4} = \frac{1}{\kappa^2}\sqrt{-g}\,(\tfrac{1}{4}T_{ijk}T_{ijk} + \tfrac{1}{2}T_{ijk}T_{ikj} - T_{ijj}T_{ikk}). \tag{13}$$

We would like to emphasize that *any other linear combination in $\mathcal{L}$ does not yield a meaningful theory.*

Now it is readily seen that $\mathcal{L}_{T_4}$ is (again up to a divergence) precisely Einstein's Lagrangian. This completes our argument that the four-dimensional translational gauge theory with Lagrangian quadratic in the field strengths is precisely Einstein's theory of gravitation. The fact that the Lagrangian (13) is equivalent to Einstein's one is of course well known. But the geometrical meaning of this Lagrangian does not seem to have been fully understood. *We now understand it as the translational gauge formalism of Einstein's theory of gravitation.*

At this point one may wonder whether we have required a Lorentz gauge invariance by imposing the independence of the theory under the local Lorentz transformation (11) of the orthonormal basis e_i. Even so, however, one does not need to introduce Lorentz gauge fields in one's theory if one has only scalars and internal gauge fields as one's source fields.[9] This is so because scalars are singlets under the Lorentz transformation and also the internal gauge fields do not couple directly to the gauge fields of the Lorentz group owing to the gauge invariance of the internal symmetry. In any event it should be made clear that the independence of the theory under the local Lorentz transformation (11) is a consistency condition that one has to require for one's theory.[10] In the presence of spinor source fields, of course, this consistency condition naturally leads us to introduce the gauge fields of the Lorentz group to the theory and one obtains the Einstein-Cartan[11] theory of gravitation as has been argued by Kibble.[2,12] In this case the translational gauge group is replaced by the Poincaré group.[9]

III. PHYSICAL INTERPRETATION

We now wish to make clear how in the presence of source fields the translation group T_4 acts on them. By doing so we will give a precise physical meaning to our bundle of translation group. Remember that we have treated our bundle just like a principal fiber bundle of an internal-symmetry group except for one crucial difference, i.e., the identification of Eq. (7). This equation interlocks the fiber space of the translation group T_4 with space-time and allows us to speak of our T_4 as a space-time symmetry rather than an internal symmetry. We will first justify this basic equation and clarify the meaning of the translation group T_4.

Let us consider a scalar field $\phi(x)$ as the source field for simplicity and start with an action integral written in the usual coordinate basis of a global Minkowski frame:

$$I = \int (\tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}m^2\phi^2)d^4x. \tag{14}$$

Clearly under a global translation of the coordinate

$$x^\mu \to x'^\mu = x^\mu + \epsilon^\mu, \tag{15}$$

where ϵ^μ are four infinitesimal parameters, one has

$$\delta'\phi \equiv \phi(x') - \phi(x) = \epsilon^\mu\partial_\mu\phi(x). \tag{16}$$

This suggests that one may view the generators ξ_μ of T_4 as the coordinate derivatives ∂_μ. For the moment let us take this point of view. Now under a local translation one has to introduce a covariant derivative to keep the action integral (14) invariant. But since the covariant basis is the one in which the components of the metric remain flat, it is quite natural to identify ∂_i as the covariant derivative of ∂_μ. Then one is led to the following equation:

$$\begin{aligned}\partial_i &= h_i^\mu\partial_\mu \\ &= (\delta_i^\mu + \kappa B_i^\mu)\partial_\mu.\end{aligned} \tag{17}$$

In short, interpreting ξ_μ as ∂_μ and identifying the local orthonormal basis as the covariant basis one easily obtains Eq. (7). In fact this rather intuitive interpretation has been given by Kibble.[2]

Now one can easily write down the action integral which is invariant under a local translation and independent of a choice of a local orthonormal basis. Including the kinetic term of the translational gauge fields one has[13]

$$\begin{aligned}I = \int \sqrt{-g}\Big[&\tfrac{1}{2}\eta^{ik}\partial_i\phi\partial_k\phi - \tfrac{1}{2}m^2\phi^2 \\ &+\frac{1}{\kappa^2}(\tfrac{1}{4}T_{ijk}T_{ijk} + \tfrac{1}{2}T_{ijk}T_{ikj} - T_{ijj}T_{ikk})\Big]d^4x.\end{aligned} \tag{18}$$

Clearly the Lagrangian (18) describes Einstein's theory of gravitation in the presence of the scalar source field $\phi(x)$ provided

$$\kappa^2 = 16\pi G, \tag{19}$$

where G is the gravitational constant. Thus the coupling parameter κ of the group T_4 is indeed related to the gravitational constant.

Now we will give another interpretation, i.e., the one for our bundle formalism of the translational gauge theory, which allows us to keep the complete parallelism between a gauge theory of an internal symmetry and that of the space-time symmetry T_4. Notice that in the above interpretation we started from the usual coordinate basis of a global Minkowski frame. But clearly one should be able to start with any other basis as well. Indeed it may be more desirable to construct the theory in a basis-independent way. So let us start from the beginning with a local orthonormal frame e_i and write down the action integral (14) as

$$I = \int (\tfrac{1}{2}\eta^{ik}\partial_i\phi\partial_k\phi - \tfrac{1}{2}m^2\phi^2)\sqrt{-g}\,d^4x. \tag{14$'$}$$

In this case one is led to have the following covariant derivative D_i for the bundle of T_4:

$$D_i = \partial_i + B_i^\mu\xi_\mu. \tag{20}$$

This is dictated by the geometry of the bundle[5] since the covariant derivative of ∂_i in the bundle formalism is given by the horizontal lift $\hat{\partial}_i$ of ∂_i. Also in this picture the group actions are interpreted to transform the field components along the fiber space keeping the *physical* space-time points invariant. This means that under the translation (15) one should have

$$\begin{aligned}\delta\phi &= \delta\theta^\mu\xi_\mu^*\phi = \delta\theta^\mu\frac{\partial}{\partial\theta^\mu}\phi \\ &= \phi'(x') - \phi(x) = 0,\end{aligned} \tag{21}$$

where θ^μ ($\mu = 1, 2, 3, 4$) are the fiber-space coordinate variables as before and we have identified the generators ξ_μ^* as $\partial/\partial\theta^\mu$, which is again dictated by the geometry of the bundle.[5] Notice the difference between this equation and Eq. (16). Thus in this bundle picture ξ_μ^* annihilate the source fields as the fields remain invariant at each physical space-time point and do not depend upon the fiber-space variables. Observe here that this fiber-space dependence of the source fields has been derived, not assumed, from what one means by the translational invariance. The way the gauge fields of T_4 couple to source fields is then given by Eq. (7):

$$\begin{aligned}D_i\phi &= (\partial_i - \kappa B_i^\mu\xi_\mu^*)\phi \\ &= \partial_i\phi = (\delta_i^\mu + \kappa B_i^\mu)\partial_\mu\phi.\end{aligned} \tag{22}$$

Thus the gauge fields of T_4 couple to source fields *as if* the generators of T_4 were the coordinate derivatives ∂_μ and one arrives at the action integral (18) as before due to Eq. (7).

One can choose either of the interpretations above. The first one emphasizes too much the usual global Minkowski coordinate frame but gives an intuitively clear meaning to the translational symmetry, whereas the other has the merit of treating the theory in a basis-independent way and allows us to keep the parallelism between the gauge theory of the space-time symmetry T_4 and that of an internal symmetry, with the identification of Eq. (7).

IV. CONCLUSIONS

We have shown that the gauge theory of the four-dimensional translation group is unique and becomes precisely the vierbein formalism of Einstein's theory of gravitation as far as one chooses the Lagrangian to be the lowest possible combinations, quadratic in field strengths. The gauge potentials of the translation group are interpreted as the nontrivial part of the vierbein fields and the corresponding gauge transformations are shown to be the general coordinate transformations.

Yang[14] has recently proposed a GL(4) gauge theory of gravitation of a Yang-Mills quadratic type which differs from Einstein's theory and may conflict with observation.[15] A Yang-Mills–type gauge theory of gravitation which gives Einstein's theory is, as we have seen, the gauge theory of the translation group T_4.

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†Present address: Department of Physics, New York University, New York, New York 10003.

[1]See, e.g., S. L. Glashow and M. Gell-Mann, Ann. Phys. (N.Y.) 15, 437 (1961); R. P. Feynman, *Lectures on Gravitation* (Caltech, Pasadena, Calif., 1963).

[2]T. W. B. Kibble, J. Math. Phys. 2, 212 (1961).

[3]See, e.g., K. Hayashi and T. Nakano, Prog. Theor. Phys. 38, 491 (1967); G. D. Kerlick, thesis, Princeton Univ., 1975 (unpublished); J. M. Nester, report, 1974 (unpublished). See also, F. A. Kaempffer, Phys. Rev. 165, 1420 (1968).

[4]A. Trautman, Rep. Math. Phys. 1, 29 (1970).

[5]Y. M. Cho, J. Math. Phys. 16, 2029 (1975).

[6]Y. M. Cho and P. G. O. Freund, Phys. Rev. D 12, 1711 (1975).

[7]T. T. Wu and C. N. Yang, Phys. Rev. D 12, 3845 (1975). See also L. N. Chang, K. I. Macrae, and F. Mansouri, *ibid.* 13, 235 (1976).

[8]Our notation here is the same as the one in Ref. 5.

[9]Y. M. Cho, Phys. Rev. D (to be published).

[10]There appear different opinions in the literature with which we do not agree. See, e.g., K. Hayashi, Nuovo Cimento 16, 639 (1973); Gen. Relativ. Gravit. 4, 1 (1974).

[11]É. Cartan, C. R. Acad. Sci. 174, 593 (1922); Ann. École Normale 40, 325 (1923); 41, 1 (1924).

[12]Kibble's work followed after the pioneering work of Utiyama. See R. Utiyama, Phys. Rev. 101, 1597 (1956). See also D. W. Sciama, *Recent Developments in General Relativity* (Pergamon, New York, 1962).

[13]In this paper our signature of the metric is $(+,-,-,-)$.

[14]C. N. Yang, Phys. Rev. Lett. 33, 445 (1974).

[15]R. Pavelle, Phys. Rev. Lett. 34, 1114 (1975).