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Perturbative recipe for quark-gluon theories and some of its applications*

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We explore the possibility that the intercepts of the nonstrange meson and baryon Regge trajectories are simply related to quark spin. This possibility arises if one adopts a multiperipheral bootstrap for these (ordinary) trajectories and also a perturbative recipe relating a process with a given number of particles to quark-gluon diagrams. The possible implication of the scheme for the Pomeron are also discussed.

I. INTRODUCTION

In a recent letter[1] we have speculated that a simple correspondence exists between quark-gluon diagrams and Regge exchanges leading to prediction of some of the intercepts and properties of the various exchanges. The two-gluon model for the Pomeron was independently suggested by Low[2] and its space-time picture extensively developed.

The purpose of the present paper is to clarify our basic approximation scheme, to explain in more detail the results of Ref. 1, to add some more applications, and, finally, to try to relate the present attempts to the various recent developments in more basic approaches to the quark-gluon theory, in dual S-matrix theory and more semiphenomenological approaches like the MIT bag model.[3]

The plan of the paper is the following: Section II describes the various motivations and the approximation scheme. Sections III and IV describe the calculations of leading meson and baryon trajectories. Section V describes some further results on multiplicities, and exotic and nonleading trajectories which we believe to be more model-dependent and less reliable. In Sec. VI we consider application of the simple rearrangement recipe for ψ physics and find no difficulties there as well. The last two sections, VII and VIII, contain the scheme for the Pomeron and some concluding remarks.

II. MOTIVATION AND THE APPROXIMATION SCHEME

Most of the known hadronic spectrum can be described as $\bar{q}q$ and qqq bound states, where q is a spin-$\frac{1}{2}$ fermion transforming like a fundamental representation of the classification ("flavor") group—SU(3) or SU(4). Local quark current algebra and the observed scaling in eN and νN deep-inelastic scattering strongly favor pointlike spin-$\frac{1}{2}$ fractionally charged (or Han-Nambu) quarks. Further support comes from various ratios [e.g., $-\mu_n/\mu_p, \sigma_{\text{tot}}(mB)/\sigma_{\text{tot}}(BB)$] being consistent with SU(6)-symmetric wave functions or simple quark counting. In order to reconcile the symmetric three-quark model for baryons with quark Fermi statistics, another SU(3) (color) was introduced with the requirement that all physical states be color singlet.[4] Color explains also the zero-triality rule and helps us to understand the hadronic e^+e^- cross section and $\pi^0 - 2\gamma$ decay.[5]

Non-Abelian gauge theories are the only renormalizeable theories which are "asymptotically free",[6] thus offering an explanation of Bjorken scaling and, one hopes, also an explanation for the pattern of its violation. Even more intriguing is the complementary phenomenon of increasing coupling constant at large distances. It was widely conjectured, proved in two-dimensional models,[7] and indicated in the four-dimensional-lattice[8] version of the theory, that this provides a mechanism for quark confinement. The postulate of existence of only color-singlet physical states may thus find its explanation within the model itself. All this encouraged many theoreticians to adopt the following gluon-quark Lagrangian as a viable candidate, one hopes, for the Lagrangian of strong interactions:

$$\mathcal{L}_{\text{strong}} = \bar{q}^{ai}(\gamma^\mu D_\mu - m_a)q^a{}_i + \tfrac{1}{2}G_{\mu\nu i}{}^j G^{\mu\nu}{}_j{}^i \cdots , \quad (1)$$

where ij (a) are color (flavor) indices, respectively, $G_{\mu\nu}{}^i{}_j$ are the gauge field strengths, and D_μ, the covariant derivative

$$D_\mu q^a{}_i = \partial_\mu q_i{}^a + gA_i{}^j{}_\mu q^a{}_j , \quad (2)$$

includes the coupling g.

The quark masses m_a are taken very small for the nonstrange quarks. The theory therefore has an extra $SU(2)_{\text{chiral}}$ symmetry. The quark masses may represent weak-interaction effects so that the pure hadronic Lagrangian is scale-free and contains only one dimensionless coupling constant.

This extremely succinct formulation at the quark-gluon level contrasts with the many diverse concepts employed in S-matrix phenomenology, such as Regge behavior, duality, and multiperipheralism. To explain all these concepts and predict all hadronic parameters from the Lagrangian is an extremely ambitious undertaking. We would like to note that if we start from the basic premises of S-matrix theory, analiticity and unitarity, and demand that these (and, in particular, the resulting high-energy bound) be perturbatively implemented in field theory, we are forced to renormalizeable theories. The choice of non-Abelian gauge theories is then a slight further restriction due to the requirement of asymptotic freedom.

Thus, without workable approximation schemes the content of the Lagrangian may not be much more than that of some fundamental S-matrix theory or a shorthand for all known symmetries. Various nonperturbative variational, semiclassical,[9] and larger-N (Ref. 10) [for color SU(N)] approximations are being developed. In the following we adopt a more pragmatic approach and assume the following:

(i) The S-matrix multi-Regge description (for high-s small-transverse-momenta region) and the quark-gluon description are simultaneously valid.

(ii) Planar duality and a simple $\bar{q}q$ (qqq) picture of bosons and baryons are correct.

(iii) Gluon exchanges can be treated perturbatively except when we are using such exchanges in order to bind $\bar{q}q$ or qqq to color-singlet states.

Clearly (iii) is the strongest and most ill-defined among our assumptions. It means that in trying to generate a given hadronic S-matrix process in terms of quarks and gluons, the lowest-order diagrams consistent with the initial and final states will be chosen. The familiar Zweig rule suppressing a $\phi\to\rho\pi$ annihilation diagram [Fig. 1(a)] relative to a lowest-order (one-gluon) planar $\rho\to\pi\pi$ decay [Fig. 1(b)] is clearly consistent with assumption (iii). Figure 1(b) serves also to illustrate the distinction between the nonperturbative binding gluons in the initial and final ρ and $\pi\pi$ states and the perturbative (heavily drawn) gluon which plays no obvious binding role but is essential to achieve the extra $\bar{q}q$ pair in the final state.

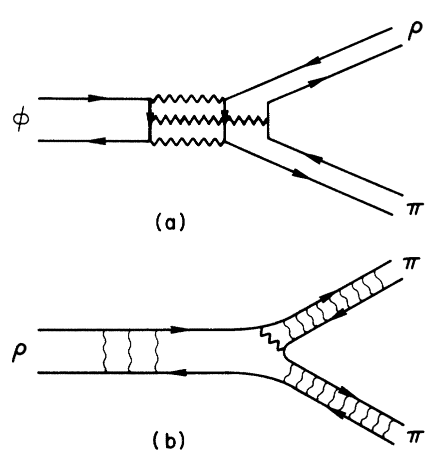

FIG. 1. The quark-gluon diagrams for $\phi\to\rho\pi$[(a)] and $\rho\to\pi\pi$ [(b)] decays.

Similar separation of perturbative from nonperturbative aspects of a gluon theory occur in other approaches. Thus in the MIT bag model gluon exchanges between quarks inside the bag and their effect on hadron masses are treated perturbatively—the bag itself being the reflection of the nonperturbative confining force. Here we do not employ any detailed space-time picture but instead make the more formal assumptions (i) and (ii).

III. LEADING-MESON-TRAJECTORY INTERCEPTS

For roughly parallel Regge trajectories (with a slope $\alpha'\approx$ GeV^{-2}) the intercepts $\alpha_i(0)$ fix the hadronic masses. It has been speculated that consistency requirements will restrict the intercepts in the framework of dual models. This is indeed the case for the covariant string model,[11] where, unfortunately, the values 2 and 1 are bigger than the phenomenologically (and theoretically) preferred $\alpha_P(0)=1$, $\alpha_M(0)=\frac{1}{2}$, where α_M indicates the leading (ρ-f^0-ω-A_2) meson trajectory.

The Pomeron intercept $\alpha_P(0)=1$ has a simple geometrical interpretation in terms of scattering from a fixed (grey or black) disk, and it was conjectured[12] that $\alpha_M(0)=\frac{1}{2}$ has a similar geometric interpretation. This may actually be realized in the string model where the Pomeron (ordinary Reggeon) trajectory may correspond roughly to intersecting (touching) string configurations.

All this suggests that quantized values for some of the intercepts may not be spurious and attempts to calculate them from a quark-gluon picture may be worthwhile. Let us consider building the meson trajectory ("f^0" exchange) from a multi-Regge multiple-production process [Fig. 2(a)]. To be

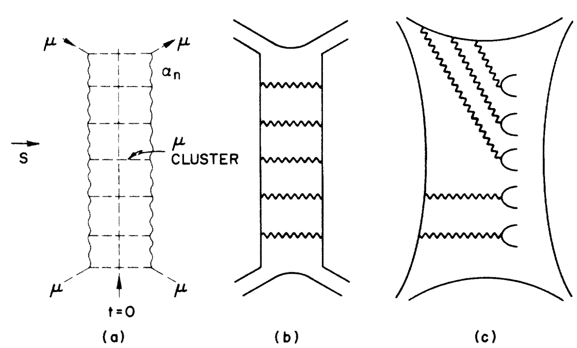

FIG. 2. (a) The multi-Regge diagram. (b) The corresponding multigluon ladder. (c) A bremsstrahlung diagram.

specific we will imagine working in the planar dual-bootstrap framework of Veneziano and coworkers,[13] i.e., we assume that the planar f^0 trajectory can be generated self-consistently from planar diagrams. Thus the couplings $G_{\rm pl}{}^2$ of the exchanged Reggeons [the same $\alpha_M(t)$] in Fig. 2(a) to the produced mesonic clusters need not be the same as the $G_{\rm full}{}^2$, the full coupling strength including nonplanar configuration which is relevant to generating the Pomeron [in particular Lee[14] and Veneziano[15] found that

$$G_{\rm full}{}^2 = 2G_{\rm pl}{}^2 \tag{3}$$

arises naturally within the model and in a weak-coupling limit this leads to $\alpha_P(0)=1$]. In a weak-coupling (Chew-Pignotti) approximation the relevant cross section for n-cluster emission is

$$\frac{(G^2 \ln s)^n}{n!}\, s^{2\alpha_M(0)-1}. \tag{4}$$

We note that the coupling G^2 is not a pure trilinear coupling like $g_{N\bar{N}\pi}{}^2/4\pi \cong 14$, but that it includes factors from integrating over transverse momenta. They still are dimensionless quantities determining the density per rapidity interval of the produced clusters and, in particular, the energy dependence of their average multiplicity

$$\bar{n}_{\rm clusters} = {\rm const} + G^2 \ln s. \tag{5}$$

Summing Eq. (4) over all n we find

$$\alpha_M^{\rm out}(0) = 2\alpha_M(0) - 1 + G^2 \tag{6}$$

so that in particular if a bootstrap condition is imposed

$$G_{\rm pl}{}^2 = 1 - \alpha_M(0). \tag{7}$$

For our purpose the bootstrap condition and the detailed weak-coupling approximation will not be important.

Going over to the quark-gluon picture next we realize that the "planar part" of the meson-meson amplitude corresponds to the planar duality diagrams,[16] which originally served just as a mnemonic to the Chan-Paton scheme, or for having nonexotic "resonances" and Reggeon exchanges.

We would like to use planar diagrams like these with gluons exchanged between the quarks [leaving flavor SU(3) quantum numbers to flow along the quarks on the periphery] interpreted as ordinary Feynman diagrams in order to investigate the dynamics of these processes as well.

The fact that planar Feynman diagrams may play a dominant role, in a large N [of $SU(N)_{\rm color}$] limit was pointed out by 't Hooft[10] and later utilized by him[17] to obtain the $\bar{q}q$ spectrum in the one-dimensional case from a Bethe-Salpeter equation. Our main assumption (iii) and the requirement of correspondence in this case to the multi-Regge diagram make the gluon ladder diagrams, Fig. 2(b), the relevant planar diagrams. *A priori* the same number of $\bar{q}q$ hadronic clusters could be produced in the same order from other planar but nonladder diagrams. Thus we could have bremsstrahlung-type diagrams [Fig. 2(c)] with a finite fraction of the gluons emitted from the horizontal on-going quark lines. Since these gluons have to rearrange into a multiperipheral hadronic final state the total rest mass carried by them is very high, which implies that the quark lines, and, in particular, the first one which emerges directly from the incident hadron, will be very much off shell. This would lead to very strong damping since we know from precocious scaling that quarks inside hadrons are not very far off shell.[18] The correspondence between the $(n-1)$-gluon ladder diagram (for the specific case of $n=3$) and the multi-Regge process for n-cluster production is illustrated in Figs. 3(a) through 3(d). Each gluon splits into a $\bar{q}q$ pair. These pairs are in a color-octet state so that they cannot transform directly to physical clusters. The transformation into a physical state occurs via a recombination (for the $\bar{q}q$ color-singlet part) between a quark originating from one gluon and an antiquark originating from a neighboring gluon, and, in particular, the original $\bar{q}$ and q which came from the initial state participate in forming the upper (fastest) and lowest (slowest) meson clusters in the final state. The combination into a $\bar{q}q$ bound state involves a nonperturbative many-gluon exchange. We also dress up the original perturbative quark-gluon diagram by allowing many exchanges (also nonperturbatively) between neighboring $\bar{q}q$ lines in the t channel; thus, finally, we reproduce an S-matrix picture with Regge exchanges and particles in all channels.

The assumption that we make here is that the computation of the contribution to the forward imaginary amplitude can be made at the perturbative quark-gluon level by taking the discontinuity of ladder diagrams 2(b); i.e., the dressing-up process leaves it essentially invariant. This

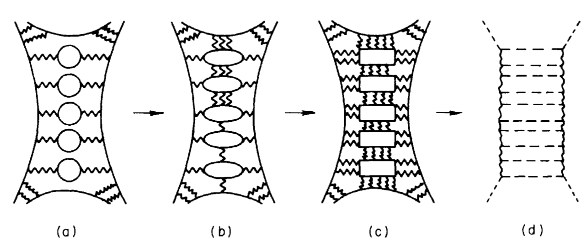

FIG. 3. The dressing up of the gluon ladder to form the multi-Regge process.

is reminiscent of the procedure used in the quark-parton model to compute in the quark basis the imaginary part of the forward Comptom amplitude. The important extra assumption (iii) used here allows us to make this correspondence for each n-particle state. The $(n-1)$-gluon [or the n-$(\bar{q}q)$-pair] ladder gives the following contribution in the weak-coupling limit:

$$\mathrm{Im} A_n = s^{2S_q - 1} \frac{[g^2(\ln s)]^n}{n!} = \frac{[g^2(\ln s)]^n}{n!}. \qquad (8)$$

We note that the detailed wave functions of the colliding particles are not important for our present purpose though they would appear, e.g., in comparing ρ versus ρ' scattering and in absolute determination of the cross sections. The question of interest here is only the high-energy behavior of the diagrams and it is fixed by the spin-$\frac{1}{2}$ exchanged fermion and our assumption of multiperipheral-type mechanism. The s^{2S_q-1} factor reflects the iterated exchange of fermions with $S_q=\frac{1}{2}$. $(g^2 \ln s)^n/n!$ is the standard Poisson form for independent n-gluon emission over a rapidity interval $\ln s$ and an average multiplicity

$$\bar{n}_{\text{gluons}} \sim \text{const} + g^2 \ln s\,. \qquad (9)$$

If we allow the quarks to Reggize (by including self-energy diagrams which we have so far neglected), then $S_q=\frac{1}{2}$ is to be replaced by some trajectory intercept $\alpha_q(0)$. However, we will consider in the following only the nonstrange mesons and quarks. Since the latter are effectively massless, $\alpha_q(0)=\frac{1}{2}$ anyway.

Summing (8) over all n (in the $\ln s \to \infty$ limit) yields an output trajectory

$$\alpha_M^{\text{out}}(0) = 2S_q - 1 + g^2\,. \qquad (10)$$

Our correspondence between the gluon and cluster emissions requires in particular that their average number will be the same, so that comparing Eqs. (5) and (9) and letting $\ln s \to \infty$ we find

$$g^2 = G^2 \qquad (11)$$

as a minimal requirement for the consistency of our approach. Using this and comparing the output trajectory computation in both pictures Eq. (6) and Eq. (10), we find

$$\alpha_M(0) = S_q = \tfrac{1}{2}\,, \qquad (12)$$

a result which does not actually depend on the bootstrap approach or on the particular value of $\alpha_M^{\text{out}}(0)$ resulting from the calculation.

If we demand, however, that $\alpha_M^{\text{out}}(0)$ is also consistently at $\alpha_M(0)=\frac{1}{2}$, we conclude

$$g^2 = G^2 = \tfrac{1}{2}\,. \qquad (13)$$

Note, however, that unlike (13) the result on the trajectory intercept does not depend on the weak coupling, i.e., the no-correlation approximation. If correlations between the emitted particle clusters and/or quark pairs occur, then Eq. (6) and Eq. (10) should be changed to

$$\alpha^{\text{out}} = 2\alpha_M - 1 + G^2 + F(G^2)\,, \qquad (6')$$

$$\alpha^{\text{out}} = 2S_q - 1 + g^2 + f(g^2)\,, \qquad (10')$$

where the various (short-range) cluster $(\bar{q}q)$ correlations are given by the derivatives at $x=0$ of $F(x)$ $[f(x)]$, respectively. Our assumption on the correspondence between gluon and particle clusters implies not only the equality of average numbers $(g^2=G^2)$ but of all moments of the distribution as well, and hence also

$$F(G^2) = f(g^2)\,, \qquad (14)$$

and $\alpha_M(0)=\frac{1}{2}$ can be obtained from comparing (6′) and (10′). An alternative approach of using specific exclusive n-gluon (or n-cluster) processes has been used by us earlier.[1] A particular advantage of this is that by going to high energies, multi-Regge diagrams [Fig. 2(a)] with the leading trajectory can be selected. Note that in a pure bootstrap approach monleading trajectories do indeed pose a severe problem, since all nonleading, e.g., pion, trajectories have to be included as well. We will return to the problem of nonleading trajectories later on (Sec. V).

As will be seen later, we can interpret the Pomeron as diagrams involving gluon exchanges in the t channel. The supression of ordinary $(\bar{q}q)$ trajectories does reflect then in our diagrammatic approach the supression (due to spin-$\frac{1}{2}$ propagators) of diagrams in which the $\bar{q}q$ lines have to propagate all the way along the t channel and are not allowed at any point to annihilate into gluons. It is precisely diagrams of this kind which can convey charge and, in general, flavor quantum numbers between the colliding particles. Thus we have a specific realization of the suggestion[19] that the experimentally observed short-range charge correlations supress the "unitarity overlap" construction of charge-exchange amplitudes.

IV. THE LEADING BARYON TRAJECTORY

Simple arguments suggest that the $(3q)$ baryons are heavier than $\bar{q}q$ mesons. This does not necessarily involve the intrinsic quark mass, which is essentially zero for the nonstrange quarks presently considered, but it could involve the kinetic energy of the confined quarks. To see how the present approach contains a quark-counting rule which supresses the leading baryon trajectory versus the leading meson trajectory, let us, in analogy to Sec. III, consider a process involving baryon exchanges, i.e., the annihilation into n-mesonic clusters of Fig. 4(a).

It behaves like

$$A^{P}_{\bar{B}B\to n\,\text{clusters}} = s^{2\alpha_B(0)-1}\frac{[(G'^2(\ln s)]^n}{n'}, \tag{15}$$

where A^P indicates a planar approximation that is the part generated from the planar Feynman quark-gluon diagrams, as indicated in Fig. 6(b). These diagrams behave like

$$A^{P}_{\bar{B}B\to\,\text{gluon}} \approx s^{4S_q-3}(g'^2 \ln s)^n/n! \tag{16}$$

The basic assumption on gluon–quark-pair–cluster correspondence forces $g'^2 = G'^2$ on us again, and hence

$$\alpha_B(0) = 2S_q - 1 = 0. \tag{17}$$

Note that again this prediction does not depend on the actual value of the sum of $\overline{q}\overline{q}qq$ ladders,

$$\sum_n A_n^{PL}(\bar{B}B\to\cdots) = s^{4S_q-3+g'^2} = s^{-1+g'^2}. \tag{18}$$

Also $\alpha_B(0)=0$ need not depend on a specific weak-coupling approximation, in the same way that $\alpha_M(0) = \frac{1}{2}$ does not.

As in Sec. III, $\alpha_B(0)=0$ is the prediction for the intercept of the leading baryon trajectory which will be identified with the completely spin-aligned (like the ρ meson) Δ state. Indeed, if we take the trajectories with slope $\alpha' \approx 1$ (GeV^{-2}), then $\alpha_\Delta(0) \approx \frac{3}{2} - \alpha' m_\Delta{}^2 \approx 0$ and $\alpha_N(0) = \frac{1}{2} - \alpha' M_N{}^2 \approx -\frac{1}{2}$ is non-leading. Note that in this case it is difficult to utilize the data on backward πN scattering because of the possible much-larger coupling of N exchanges.

We can also compute $\alpha_B(0)$ by considering backward baryon scattering. The multi-Regge diagrams considered are shown in Fig. 5(a) and yield an output trajectory

$$\alpha_{\text{out}} = \alpha_B(0) + \alpha_M(0) - 1 + \tilde{G}^2, \tag{19}$$

where again $\tilde{G}^2$ is some effective coupling to a mesonic cluster. Only diagrams in which the meson and baryon trajectory continue all along the sides of the trajectory are considered so that no baryonic clusters are produced in the s channel. Such diagrams correspond, according to our recipe, to the planar ladders of Fig. 5(b) with the quark lines at the periphery. These sum up to yield a power behavior

$$\alpha_{\text{out}} = 3S_q - 2 + \tilde{g}^2, \tag{20}$$

where $\tilde{g}$ is the effective coupling relevant to this case. Equating (19) and (20) with $\tilde{g}^2 = \tilde{G}^2$ as a correspondence requirement we find

$$\alpha_B(0) + \alpha_M(0) = 3S_q - 1 = \tfrac{1}{2}, \tag{21}$$

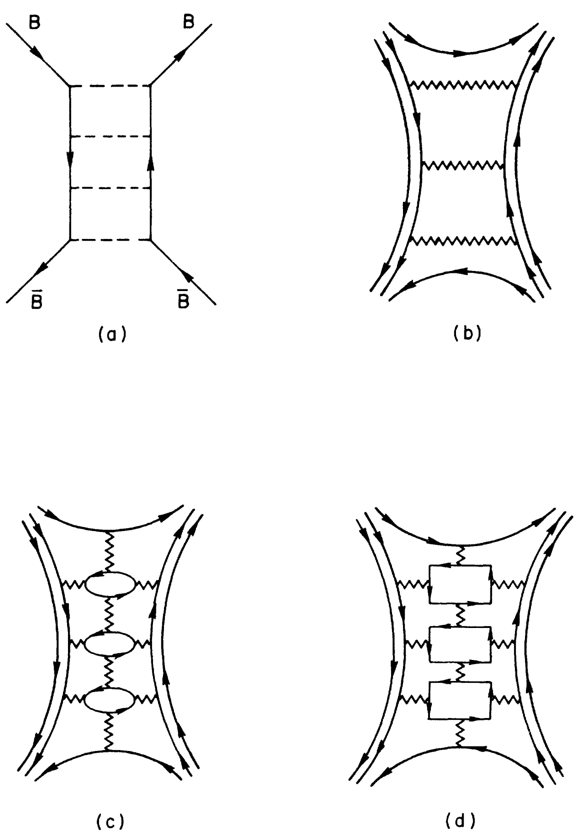

FIG. 4. (a) $\bar{B}B$ annihilation into meson clusters. (b), (c), (d) Generation of annihilation S-matrix diagram from the quark-gluon diagrams.

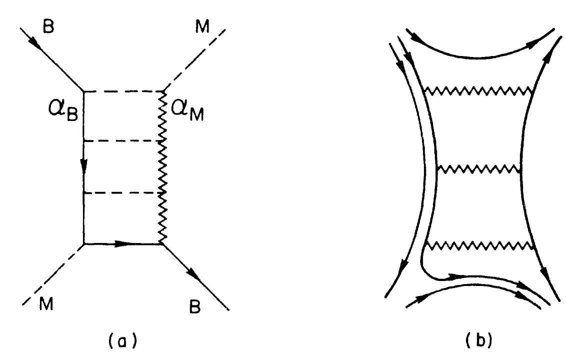

FIG. 5. (a) Multi-Regge diagram for backward mB scattering. (b) The corresponding quark diagram.

consistent with our earlier finding,

$$\alpha_M(0)=\tfrac{1}{2}, \quad \alpha_B(0)=0.$$

V. COUPLINGS AND NONLEADING TRAJECTORIES

In all our considerations so far, the requirement that the output trajectory obtained by summing the gluon ladders — or equivalently, the multi-Regge diagrams — must be at the appropriate value of $\alpha_M(0)$ or $\alpha_B(0)$ was not used. If a weak-coupling limit is indeed appropriate we find from $\alpha_M(0)=\frac{1}{2}$ ($\alpha_B=0$) and Eq. (20) that

$$G_{\rm pl}{}^2 = g^2 = \tilde{g}^2 = \tfrac{1}{2}. \tag{22}$$

$G_{\rm pl}{}^2=\frac{1}{2}$, which is actually a prediction of the multiperipheral bootstrap itself, Eq. (7), means that the logarithmic growth of the cluster multiplicity with energy for the part of the cross section dual to Reggeon exchange is $\frac{1}{2}\ln s$. This is hard to test because first we have to know the average multiplicity of charged particles per cluster[20] and relevant charge distributions like $\sigma(mP\to n \text{ charged}) - \sigma(\bar{m}P\to n \text{ charged})$. If Eq. (3) is correct, then $G_{\rm full}{}^2=1$ and the growth of cluster numbers for the full physical process is $\approx \ln s$.

The result $g^2=\tilde{g}^2$ means that the effective gluon coupling is the same for making the baryon $q\cdots qq$ bound state and the meson q-$\bar{q}$ bound state. It can be naturally explained via the non-Abelian nature of the couplings and the demand that the $3q$ system in Fig. (5) combine to a t-channel color singlet. Thus the qq pair on the left of Fig. 5(b) must couple to a $\bar{3}$ of $SU(3)_{\rm color}$. The exchanged relatively soft gluons couple, to the total $SU(3)_{\rm color}$ ($=\bar{3}$) content of the qq system and therefore the same coupling $\tilde{g}^2=g^2$ is expected as in the $\bar{q}q$ ladder for the meson trajectory. It is this feature of being able to view baryons as $(q\text{-}qq)=(3,\bar{3})$ composites — at least for soft exchanges — which accounts for the similarity between the baryons and mesons in many approaches independently of their detailed dynamics. Thus, for example, in a recent calculation of Regge slopes in the context of the bag model, equal slopes for the baryon and meson trajectory result precisely from the fact that the same flux pattern results in an elongated q-qq as in a $q\bar{q}$ configuration.[21] This feature would be completely absent if the basic gluon interaction were to be Abelian coupling to, say, quark number — not to mention the fact that it is very difficult to envision binding the (same charge) quarks to form baryons in the first place.

Returning now to the planar annihilation $\overline{qq}\cdots qq$ ladders, fig. 4(b), their sum yields an output exotic $qq,\overline{qq}$ trajectory[22] with intercept

$$\alpha^{\rm exotic}_{\overline{qq},qq}(0)=4S_q-3+g'^2. \tag{23}$$

If we use the above argument to infer that $g'^2=\tilde{g}^2=g^2=\frac{1}{2}$, then we predict $\alpha^{\rm exotic}_{\overline{qq},qq}(0)=-\frac{1}{2}$. This yields a very-low-lying $m^2=\frac{1}{2}$ exotic 0^+ particle, which is somewhat unsatisfactory.[22] Also, we cannot identify the total annihilation cross section with just these planar contributions because we would then have

$$\sigma(\bar{N}N\to \text{mesons}) \underset{s\to\infty}{\simeq} s^{\alpha^{\rm exotic}(0)-1} \approx s^{-3/2}, \tag{24}$$

which seems to be too fast a falloff even with the limited experimental data available. A simple way to avoid this difficulty is to realize that, just as in nonannihilation processes, the major contribution comes from nonplanar diagrams and in particular that annihilation may contribute to the Pomeron exchange,[23] a point of view consistent with our Pomeron.

It is difficult to isolate in the present scheme the contribution of nonleading meson $(\pi, A_1, \ldots)$ or baryon $(N,\ldots)$ trajectories. The masslessness of the pion—a necessary consequence of the spontaneously broken $SU(2)_{\rm chiral}$ symmetry—means that $\alpha_\pi(0)=0$, a result which when translated to a statement about the asymptotic behavior of a sum of ladder diagrams seems very artificial. We find, nonetheless, the following numerical coincidence rather amusing: Let us assume that the antiparallel addition of spins in forming the singlet state π reflects in the high-energy behavior of the corresponding sum of ladders via

$$\begin{aligned}\alpha_\pi^{\rm out} &= S_{q1}-S_{q2}+g^{*2}-1\\ &= g^{*2}-1,\end{aligned} \tag{25}$$

where g^{*2} is the relevant effective gluon coupling for this case of binding a fermion-antifermion into a pseudoscalar. This is to be compared with

$$\begin{aligned}\alpha_\rho^{\rm out} &= S_{q1}+S_{q2}+g^2-1\\ &= 2S_q-1+g^2\end{aligned} \tag{10$''$}$$

our expression for the spin-aligned leading ρ trajectory, and g^2 is the relevant coupling for binding via a vector exchange to a vector in the crossed (s) channel. The long-range Coulomb part of the interactions is insensitive to whether we have a singlet or a triplet $\bar{q}q$ state. The main role of these long-range interactions is to achieve confinement by compensating for the presumably infrared infinite self-energy of the quarks allowing us to use finite-mass quarks. If the remaining short-range interactions are approximated by a local four-fermion interaction pure vector in the s (gluon-exchange) channel, then from the comparison of the VV and VP Fierz coefficient we find $(g^*)^2 = 2g^2$, so that if we use $g^2 = \frac{1}{2}$, as suggested from all our earlier weak-coupling results, then

$$\alpha_\pi(0) = S_{q1} - S_{q2} + g^{*2} - 1 = g^{*2} - 1 = 0 \qquad (26)$$

The Δ-N relation within the planar scheme and the $(qq, q) \to (\overline{3}, 3) \sim (\overline{q}, q)$ approximation of the baryons is analogous to the ρ-π relation. Again, the nonleading nucleon is obtained from the completely aligned spin configuration of the Δ by flipping the spin of a q with respect to the qq combined antisymmetrically in $SU(3)_{color}$ to $\overline{3}$ and symmetrically in spin to $S_{qq} = 1$. The experimental and (partial conservation of axial-vector current + duality)[24]-motivated relation

$$m_\Delta{}^2 - m_N{}^2 \simeq m_\rho{}^2 - m_\pi{}^2$$

is very suggestive in this respect.

VI. IMPLICATIONS OF THE SCHEME FOR ψ DECAYS

For the purpose of this section we will assume that the recently discovered ψ particles are $c\overline{c}$ charmed-quark states bound via gluon exchanges like the normal $\overline{q}q$ states.

In Ref. 1 we speculated that the ψ's may be essentially gluonic states which would explain their $I = 0$ [and SU(3) singlet] nature and conceivably also their smaller sizes and hadronic cross sections. It is difficult, however, to understand (because of the neutrality of the gluons in the present non–Han-Nambu scheme) the large couplings to the photon. A model of this type was considered by Brower and Primack.[25] At any rate, such purely gluonic nonplanar states are likely to dominate in the hadronic ψ decays.[26]

The $\overline{c}c$ assumption and a reasonable confining potential explain various features of the ψ, ψ', and the new even-C states although many puzzles and, in particular, the nonobservation of charmed particles are detrimental to this scheme. The hadronic ψ-particle decay proceeds in these models via three gluons.[27] In particular, our lowest-order recipe separates the rearrangement diagrams into three mesonic clusters in a nonplanar tubelike configuration,[26] Fig. 6(a), or, *a priori*, also into two baryonic clusters, Fig. 6(b). However, the $SU(3)_{color}$ factor for the last diagram,

$$\psi \to F(3g^c) \to \overline{B}B = f^{abc}[\epsilon_{ijk}\epsilon_{\pi su}(\lambda_a)^{i\pi}(\lambda_b)^{js}(\lambda_c)^{ku}], \qquad (27)$$

vanishes because the term in the square brackets is completely symmetric uncer the exchange of gluon color indices whereas the antisymmetric f^{abc} coefficient is needed to insure the coupling of the three-gluon octet to an odd-charge-conjugation color-singlet ψ 1^{--} state. The vanishing of the diagram corresponding to Fig. 6(b) is indeed very welcome for our rearrangement scheme since we do not have an anomalously high inclusive $\overline{p}$ to π^- ratio at the ψ or ψ' in agreement with the data. Our recipe suggests also that two- (ordinary) meson decays of ψ and ψ' will be suppressed and the branching ratio of $\psi \to \rho\pi$ of $\simeq 1\%$ does not contradict this expectation. Note that this argument is not valid for the even-C χ (p wave) and η_c state which can couple into two gluons and thus "naturally" decay into two mesonic clusters.

A fine point of some theoretical interest involves the question of interference between the direct hadronic decays of the ψ and the amplitude to decay via an isoscalar photon. It is usually neglected since the direct contribution (2–3 keV) of the isoscalar photon $|A_s^\gamma|^2$ is small. However, the interference with the hadronic amplitude A_H which leads in the standard estimate to $\Gamma^{\text{direct}}_{\psi\to\text{hadrons}} = 45$ keV could in principle amount to $\pm 2\sqrt{45}(\sqrt{3}) \approx \pm 20$ keV. Such a maximal interference will be achieved only is we have the same sign in each decay mode. Since a direct A_ψ^H amplitude does not correspond as A_ψ^γ does to a planar $\overline{q}q$ configuration, maximal interference seems very unlikely.

VII. THE POMERON

In the present section we would like to explore the consequences which follow if the Pomeron is identified with two- (and conceivably more) gluon

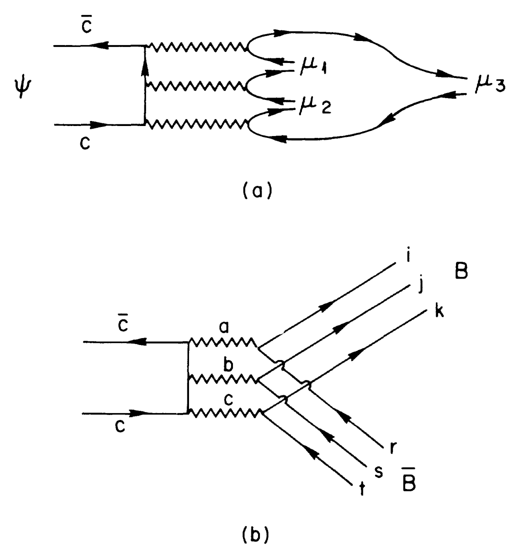

FIG. 6. The lowest-order diagrams (a) and (b) for ψ decay into 3 mesons and into $\overline{B}B$, respectively.

exchanges. This section is not as strongly tied to the correspondence assumption (iii) in Sec. II as our earlier considerations for the ordinary Regge trajectories. Thus some of the general conclusions [e.g., $\alpha_P(0)=1$] may be on a much more firm ground than our earlier results. However, for the same reason, we also find a much less complete picture of the intermediate s-channel states dual to the Pomeron. Indeed, in the present approach the well-known S-matrix difficulties of understanding multiperipheral dynamics with Pomeron exchanges[28] are simply translated into our ignorance on the dynamics or even existence of purely gluonic states.[29]

The paper by Low[2] contains, beside a detailed space-time picture of the "Born" (two-gluon) Pomeron, many of our[1] qualitative remarks so we will not expand the present discussion of these too much and we will concentrate mainly on possible further evidence for the perturbative scheme.

In a completely general way we can divide the quark-gluon diagrams for $A+B\to A+B$ forward scattering into those which can be separated in the $t(A\bar{A}\to B\bar{B})$ channel by cutting gluon lines and those which cannot. We suggest identifying the first class of diagrams with Pomeron exchange.[30] One then has the following consequences:

(a) No isospin or SU(3) quantum numbers are exchanged with the Pomeron. This, in particular, implies that asymptotically

$$\sigma_{\mathrm{tot}}(\pi^- p)\approx\sigma_{\mathrm{tot}}(\pi^+ p), \tag{28a}$$

$$\sigma_{\mathrm{tot}}(K^- p)\approx\sigma_{\mathrm{tot}}(K^+ p)\sim\sigma_{\mathrm{tot}}(\pi p). \tag{28b}$$

The latter relation seems to be violated by SU(3) breaking.

(b) The s-channel states $\bar{q}q$, $\bar{q}q$ for MM and $\bar{q}q$, qqq for MB scattering are exotic and our basic assumption [(iii) in Sec. II] suggests that we attempt to represent the Pomeron in terms of the lowest-order 2-, 3-, ... gluon exchanges Figs. 7(a) and 7(b). Also, since the existing meson spectrum can very largely be explained in terms of $\bar{q}q$ bound states and no evidence for pure gluon bound states exists, we will, unlike in our treatment of ordinary Reggeons, neglect the multiperipheral gluon ladders. If no bound $B_\mu B^\mu$ state exists, our recipe would not require us to treat the gluon-pair system in the t channel nonperturbatively by summing infinite ladders. Nonetheless, this remains a problem whose solution is very unclear, reflecting the difficulties within the multiperipheral S-matrix approach with Pomeron t-channel iterations, to which we will return later.

(c) In the Abelian QED-type case investigated in detail by Cheng and Wu[31] the lowest-order process would be one-gluon exchange leading in a weak-coupling limit to a real, charge-conjugation odd elastic amplitude. To suppress this contribution, as required by experiment, strong absorption due to the dominant inelastic "towers" has to be invoked and a simple perturbative approach seems impossible. It is a nice feature of the non-Abelian case that one-gluon exchange is forbidden because the scattered particles are color singlets.

(d) The two-gluon Born Pomeron yields a constant cross section and an amplitude behaving like

$$F_{AB\to AB}(s,t=0)=\mathrm{const}\times is. \tag{29}$$

In general, the asymptotic behavior of the N-gluon exchange will correspond to an output trajectory with

$$\alpha_P=NS_B-(N-1)=1\quad(S_B=\text{gluons, spin}=1). \tag{30}$$

For the $N=2$ case, the only coupling to a color singlet is $B^i B_i$, which yields a charge-conjugation-even exchange and the ensuing requirement

$$F_{AB\to AB}(s,t=0)=F_{\bar{A}B\to\bar{A}B}(s,t=0) \tag{31}$$

implies a dominantly imaginary amplitude in this two-gluon approximation, and Eq, (29) ensues. In the Abelian case the leading real $s\ln s$ terms for qq or $\bar{q}q$ scattering cancel when we add the crossed and uncrossed two-gluon diagrams. Such a cancellation does not occur for the present nonabelian case because of the noncomutativity of λ_{color} matrices. However, the remaining real term has precisely the odd charge conjugation (and the octet nature) typical of the one-gluon exchange so that it does not contribute (it cancels between the diagram in which the quarks antiquarks in the colliding particles exchange gluons).

(e) We would like to discuss the relation with *the Pomeranchuk theorem*. A basic feature of any theory with vector exchanges is the possibility of real odd-charge-conjugation exchanges persisting at very high energies, which in a general way would correspond to a violation of the Pomeranchuk "theorem" which is based on the premise of complete dominance of the even (A^+) imaginary amplitude. We have seen that the most dangerous lowest-order term is absent and that to order g^4 we have an imaginary even amplitude. However,

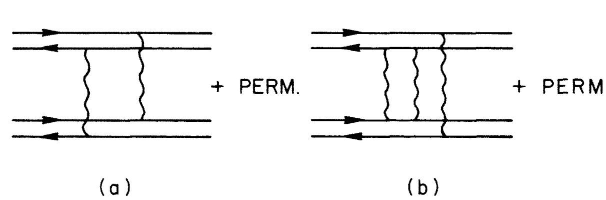

FIG. 7. The lowest-order two- and three-gluon diagrams for the Pomeron.

in order g^6 (three-gluon exchange) we might have odd charge-conjugation exchanges corresponding to f_{abc} coupling of the exchange gluons to a color singlet. This is precisely the state via which $\psi \to \overline{P}P$ and $\psi \to \overline{K}K$ decays proceed [indeed, in the exact SU(3) limit (28b) the Pomeranchuk theorem holds for K scattering and $\psi \to \overline{K}K$ is forbidden by SU(3) G parity]. If we could boldly neglect the effect of continuation from $t=0$ to $t=9.6=M_\psi^2$ in the amplitude ratio we might relate

$$\left[\frac{\mathrm{Re}(K^+N)-\mathrm{Re}(K^-N)}{\mathrm{Re}(NN)-\mathrm{Re}(\overline{N}N)}\right]^2 \approx \frac{\Gamma'(\psi \to K\overline{K})}{\Gamma'(\psi \to \overline{P}P)}, \tag{32}$$

where in Γ' the one-photon contribution has been subtracted away.

The arguments presented in Sec. VI suggest that the Landshoff diagram for large-angle scattering, if indeed interpreted as a three-gluon exchange, has no odd-charge-conjugation piece. It is conceivable that also in the case of the Pomeron the odd-charge-conjugation process contributes only in a higher order in g^2 relative to the even charge conjugation than naively estimated, if the nonplanar diagrams dominate.[32] [See the discussion at the end of consequence (g).] The experimental ratio $10\% = \mathrm{Re}A(PP)/\mathrm{Im}A(PP)$ might be acommodated, particularly if we take into account the fact that some absorption of the odd-charge-conjugation amplitude does take place.

(f) While the discontinuity of the perturbative gluon-exchange diagram may be adequate for describing $\mathrm{Im}F_{AB\to AB}(s,t=0)$, such diagrams do not specify the intermediate states. This may have some analogy to the quark "handbag" diagrams in deep-inelastic scattering. In both cases the physical intermediate states may be generated only after some complicated long-time scale evolution into bound color-singlet states and only a completeness sum simplifies the description to the impulsive gluon-exchange picture.

A naive unitarity cut between the gluons of Fig. 7 (shown in Fig. 8) yields an upper and a lower cluster which are color octets since the initial particles are singlets of color. This suggests that the intermediate states dual to the Pomeron are not diffractively produced excited A^*B^* or AB states. In this Born approximation we expect, therefore, pionization multiparticle intermediate states with no large rapidity gaps to allow for local color neutralization in rapidity. This conforms to the conventional S-matrix approach in which one builds a Born Pomeron first as the shadow of all multiperipheral intermediate states with short-range correlations.[33] The diffractive component of the intermediate states corresponds here as it would there to iterations of the basic two-gluon exchange, as indicated in Fig. 8(b).

In the quark-gluon perturbative approach we expect diagram 8(b) to indeed be smaller than the Born diagram 7(a). It involves the basic gluon coupling to four more orders and also two more loop integrations with a transverse cutoff. This is precisely what is also involved in adding two gluons in the planar ladder. In both cases the effective g^2 involves also the effects of projection on the color-singlet states in t (or s) channels. For the two gluons only the $(1\sqrt{8})B^iB_j$ is chosen and in the case of gluon exchanges the requirement that quarks emerging from neighboring gluons will combine to a singlet again yields a $1\sqrt{3}$ suppression for each gluon and $1/\sqrt{9}$ for two gluons. Thus, very naively, we may expect

$$\begin{aligned}\sigma_{\mathrm{el}}+\sigma_{\mathrm{dif}} &\approx g^4\sigma_{\mathrm{tot}}\\ &\approx \tfrac{1}{4}\sigma_{\mathrm{tot}}\,.\end{aligned} \tag{33}$$

The ratio of σ_{el} to σ_{tot} clearly involves a very detailed picture of the hadronic opacity which we do not attempt to construct here.[34]

(g) To generate the intermediate states corresponding to the Born two-gluon diagram, let us "dress up" the two gluons by $n\ \overline{q}q$ bubbles [Fig. 9(a)].

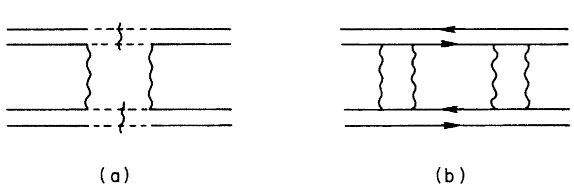

FIG. 8. A naive discontinuity of diagram 7(a). The diffractive state is shown in (b).

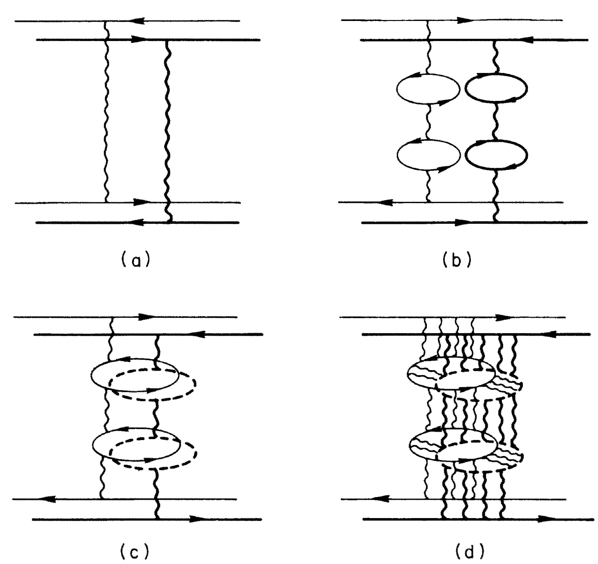

FIG. 9. Generating the multiparticle states which correspond to the Pomeron.

For simplicity let us consider the symmetrical case with equal number of bubbles. We can then form color-singlet $\bar{q}q$ states propagating in the t channel by pairing both $\bar{q}_i q_i$ on both sides of the bubbles, thus obtaining, finally, the cylindrical nonplanar topology suggested in the dual model for the Pomeron [see Fig. 9(b)-9(e)]. It also has physical $\bar{q}q$ states in the s channel which topologically have the distinction of being in the front or in the back of the "cylinder." It is clear that we are assuming throughout the discussion that a locally planar configuration with a simple topology does dominate.[13,35] Thus we cannot claim to really predict this type of Pomeron from the colored gauge theories. However, it is amusing that we do have that kind of a "cylinder Pomeron" as the result of the simplest dressing-up of our two-gluon Born Pomeron. Furthermore, it is hard to recognize any ordinary multi-Reggeon exchanges in the final dressed form. Thus we may not necessarily have to adopt the weak-coupling multiperipheral bootstrap result, Eq. (3) of Lee and Veneziano, which in particular has the rather strong prediction of a twice as large $\ln s$ coefficient in the average multiplicity for Pomeron processes versus Reggeon processes.[36] To the extent that the Pomeron dressing-up does not change the original energy behavior, no specific multiplicities are necessary to ensure the consistency of $\alpha(0)=1$.

It is amusing to note that this simple construction of the S-matrix picture for the Pomeron works only for the nonplanar configuration where each gluon of the Born Pomeron is exchanged between a quark in one of the mesons and an antiquark in the other. If we try to repeat the same construction for the planar Born term in which both gluons are exchanged between the same pair of lines in the two mesons, then not all $q\bar{q}$ will be automatically paired into low-mass $q\bar{q}$ composites, but one quark and antiquark from each of the incident particles will remain unpaired.

The significance of this fact is not completely clear to us. However, it is worthwhile noting the following points:

(i) The nonplanar structure arises naturally if we are interested in making the Pomeron out of ordinary Regge trajectories (the nonplanar Mandelstam cut).

(ii) The planar diagrams do not contribute to diffractive production $A+B\rightarrow A^*+B$, etc., thus if the elastic and diffractive production scattering are to be treated on the same footing then only the nonplanar diagrams are to be retained.

(iii) If we keep only the nonplanar diagrams, difficulties with the Pomeranchuk theorem in order g^6 do not arise.

(h) If we would have considered in analogy to the $\bar{q}q$ ladder also gluon ladders [Fig. 10], we would have obtained an output trajectory above 1:

$$\alpha_P^{\text{out}}(t)=1+g_P{}^2(H(t)), \tag{34}$$

where we normalize $H(0)=1$ and the Pomeron would also have a slope of

$$\alpha_P'(t=0)=g_P{}^2(H'(0)). \tag{35}$$

We cannot offer at this point any satisfactory explanation as to why the effective coupling $g_P{}^2$ is phenomenologically so small. However, assuming that the rise of cross sections at the CERN ISR does reflect an effective trajectory with intercept slightly above 1,[37] a very small $\alpha_{\text{eff}}-1\approx0.05$ is suggested, and that may be correlated with the small experimental Pomeron scope.

The difficulty with intercepts above 1 is not restricted to gauge theories but occurs also in the ordinary multiperipheral approach when one iterates Pomeron exchanges in the t channel. In gauge theories very remarkable cancellations between diagrams occur, which may in fact reduce the asymptotic behavior of the gluon ladders. It may thus be that gauge theories will not only offer a simple Born "Pomeron" but eventually cure the more profound difficulties with the Pomeron in the S-matrix approach.

(i) Traditionally there have been two different approaches to the question of diffractive high-energy scattering which schematically we call the "algebraic" and the "geometric" approaches.

The first approach, which could embody the quark-counting rules,[38] emphasizes the algebraic structure of the residue[39] of a presumably factorizing Pomeron pole. We have, in particular

$$\sigma_{\text{tot}}(AB)\propto\beta_A\beta_B \tag{36}$$

and thus

$$\sigma_{\text{tot}}(AA)\sigma_{\text{tot}}(BB)=\sigma_{\text{tot}}{}^2(AB).$$

A simple way to envision how such a result would come about is the analogy with Coulomb scattering. The universal coupling of a massless photon makes

$$\sigma_{\text{Coulomb}}(AB)\approx Q_AQ_B\sigma_C, \tag{37}$$

where Q_A (Q_B) is the total charge of the A (B) system. Unfortunately, the unshielded Coulomb cross

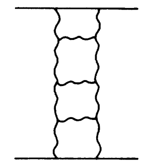

FIG. 10. Gluon ladders.

section σ_{Coulomb} is infinite because of the masslessness of the photon. This difficulty is obviously not unrelated to the factorization. The alternative approach views the scattering as resulting from the overlap of some hadronic matter distributions.[40] In an extreme black-disk situation

$$\sigma_{AB} \approx (r_A + r_B)^2, \quad \sigma_{AA} \approx (2r_A)^2, \quad \text{and } \sigma_{BB} \approx (2r_B)^2$$

and thus

$$\sigma_{AB} = \tfrac{1}{2}(\sqrt{\sigma_{AA}} + \sqrt{\sigma_{BB}})^2, \tag{38}$$

so that Eq. (33) is not, in general, satisfied and factorization and the algebraic approach are lost.[41] In the present picture we view hadrons more like neutralized charge (color) distributions which in particular have no permanent color moments. Thus the scattering results from a mutual polarization and to the extent we can use that analogy at all we expect to have van der Waals (or rather Casimir-Podolsky) fast-falling potentials and no long-range divergences. The strict factorization is lost but may partially be regained by the following considerations:

The $q_\alpha q_\beta q_\gamma$ nucleon can behave as a 3-$\overline{3}$ color dipole in three different ways: $q_\alpha q_\beta - q_\gamma$, $q_\alpha q_\gamma - q_\beta$, $q_\beta q_\gamma - q_\alpha$. However, because of the double counting of the quarks (or, alternatively, the appropriate normalization of the baryon state) we have only $\frac{3}{2}$ as many dipole units to excite in a baryon as compared to one dipole unit in a meson. The contribution to the total cross section from the long-range impacts (larger than the typical quark distances in a single hadron) is expected to behave (on dimensional ground) as $\sigma_{AB} = d_A d_B$ and hence $\sigma_{mB}/\sigma_{BB} = \frac{2}{3}$ may be retrieved for that part.

VIII. SUMMARY AND CONCLUSIONS

The simple perturbative recipe that we have suggested for correlating quark-gluon diagrams with physical processes seems to have several remarkable successes and no obvious shortcomings. It fits very nicely with many different theoretical frameworks, such as planar bootstrap, massive quarks, and bag models, and also simple naive quark counting. In the fixed-angle limit we have a gradual transition from the ladder picture to the constituent-interchange model from which the simple counting rule of Brodsky and Farrar and Matveev, Muradyan, and Tavkhelidze[42] can be abstracted.[43] It may well be that the approaches based on S-matrix dual bootstrap[44] or the purely field-theoretical models which will be appropriate four-dimensional generalizations of the 't Hooft model[14] will yield by themselves satisfactory explanation of the hadronic parameters. However, using either of these approaches may be very difficult. Essentially what we suggest here is a possibility of some short cut by demanding consistency between (admittedly, rather crude and simplified versions of) the field-theoretic and S-matrix approaches.

We would like to close this discussion with a short list of many of the problems which still remain unresolved.

(i) Another striking regularity of trajectories with just one strange quark,

$$\alpha_{K^*}(0) \approx \tfrac{1}{4},$$

and two λ quarks, (39)

$$\alpha_\varphi(0) \approx 0,$$

is completely unexplained by the present approach.[45,46]

(ii) The whole question of explaining in a satisfactory way nonleading trajectories and in particular their role in the bootstrap is left largely untouched. This is, however, a difficulty with most ordinary bootstrap models as well.

(iii) In spite of our ability to predict the leading baryon trajectory there is no really satisfactory treatment of this essentially nonplanar problem.

(iv) Extraction of effective coupling constants such as that describing cluster production density depends on some weak-coupling approximation and is quite unreliable.

(v) There may arise difficulties with the Pomeranchuk theorem.

Notwithstanding all these, we do hope that our approach will be helpful for elucidating some of the puzzling features of hadron dynamics. We find the possibility that the quantized values of leading intercepts reflect simply the spins of the elementary constituent fields sufficiently intriguing to make the pursuit of this approach worthwhile.

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