External-emission dominance in pion-proton bremsstrahlung*

B. M. K. Nefkens and D. I. Sober†
Department of Physics, University of California, Los Angeles, California 90024
(Received 26 April 1976)

The differential cross section for $\pi^{\pm}p \to \pi^{\pm}p\gamma$ in the Δ resonance region can be described in detail by an "external-emission dominance" calculation in which the matrix element is based on the first term in the soft-photon expansion of Low. The matrix element is gauge- and Lorentz-invariant and has no free parameters.

We report that the measurements of the differential cross section for pion-proton bremsstrahlung, $\pi^{\pm}p \to \pi^{\pm}p\gamma$, in the region of the $\Delta(1232)$ resonance[1-3] can be satisfactorily described by a simple calculation based on the first term in the soft-photon expansion of Low,[4] while more sophisticated calculations including higher terms have failed.[1] We shall refer to this calculation as "external-emission dominance" (EED). The following expression is used:

$$|M_{\pi^{\pm}p\gamma}|^2 = -e^2 A^{\mu}A_{\mu}\,|M_{\pi^{\pm}p}(\bar{s},\bar{t})|^2\,, \tag{1}$$

with

$$A^{\mu} = \mp\frac{P_1^{\mu}}{P_1\cdot K} - \frac{P_2^{\mu}}{P_2\cdot K} \pm \frac{P_3^{\mu}}{P_3\cdot K} + \frac{P_4^{\mu}}{P_4\cdot K}\,. \tag{2}$$

P_1 and P_3 are the 4-momenta of the incident and outgoing pion, P_2 and P_4 are the 4-momenta of the initial and the final proton, and K is the 4-momentum of the photon. (The corresponding lower-case letters represent 3-momenta.) The upper signs in Eq. (2) correspond to π^+p bremsstrahlung and the lower to π^-p. $e^2 = 4\pi\alpha$ and α = the fine-structure constant. $|M_{\pi p\gamma}|^2$ and $|M_{\pi p}|^2$ are the squares of the invariant amplitudes[5] for $\pi p \to \pi p\gamma$ and $\pi p \to \pi p$, respectively, averaged over initial spins; each must be multiplied by the appropriate density of states and phase-space factors to give the respective differential cross section. Thus, the radiative cross section is equal to the elastic scattering *cross section* multiplied by purely kinematic factors; no parameterization of the elastic amplitude is needed. Specifically, in the center-of-mass system, we obtain

$$\frac{d\sigma(\pi p \to \pi p\gamma)}{d\Omega_\pi d\Omega_\gamma dk} = \frac{e^2}{2(2\pi)^3}\frac{\sqrt{s}}{p_1}\frac{p_3{}^3 k}{p_3{}^2(\sqrt{s}-k) + E_3(\vec{p}_3\cdot\vec{k})} \times A^{\mu}A_{\mu}\frac{d\sigma(\pi p \to \pi p)}{d\Omega}(\bar{s},\bar{t})\,. \tag{3}$$

The elastic scattering cross section is evaluated at the average total energy squared $\bar{s}$ and the average 4-momentum transfer squared $\bar{t}$:

$$\bar{s} = \tfrac{1}{2}[(P_1+P_2)^2 + (P_3+P_4)^2]\,, \tag{4}$$

$$\bar{t} = \tfrac{1}{2}[(P_1-P_3)^2 + (P_2-P_4)^2]\,. \tag{5}$$

The numerical values of the elastic scattering cross section that we use have been calculated from the phase-shift analysis of Carter *et al.*[6]

The four terms on the right side of Eq. (2) can give rise to spectacular interference patterns in various angular distributions of the differential cross section for πp bremsstrahlung. The dots in Fig. 1 show $d^5\sigma/d\Omega_\pi d\Omega_\gamma dE_\gamma$ (in the lab system) as a function of α_γ for 80-MeV photons produced in $\pi^+p \to \pi^+p\gamma$ at an incident beam energy of 298 MeV with pions scattered at 50° and coplanar geometry; α_γ is the horizontal angle of the photon counter measured clockwise from the beam.[1] For orientation purposes, we have indicated by arrows in Fig. 1 the approximate horizontal angles of the scattered π^+ and recoil proton that were detected in the experiment. Insertion of Eq. (3) into a suitable Monte Carlo program allows one to calculate the expected cross section averaged over the acceptance of the counters. The rectangles in Fig. 1 are our Monte Carlo-averaged predictions using Eq. (1), and the crosses indicate the experimental results for the geometry of Refs. 1–3. The effects associated with a noncoplanar geometry are particularly evident for the measurements at $\alpha_\gamma = 320°$ and 360° of Fig. 1. The photon counters at these two positions are located about 56° below the horizontal plane.

The agreement between the predictions based on Eq. (1) and the experimental results is remarkably good over the entire measured angular range including the region of maximum destructive interference of the four terms near $\alpha_\gamma \approx 220°$ (where various theories[7,8] predict a large internal structure contribution). The photon spectra of Ref. 1 for $\pi^+p \to \pi^+p\gamma$ and $\pi^-p \to \pi^-p\gamma$ in this sensitive region are shown in Figs. 2(a) and 2(b), respectively. The solid curves indicate the predictions based on Eq. (1), averaged over the acceptance of the counters. Two photon spectra from Ref. 2, measured at different photon angles, are shown in Fig. 3. Equation (1) agrees with experiment to an accuracy

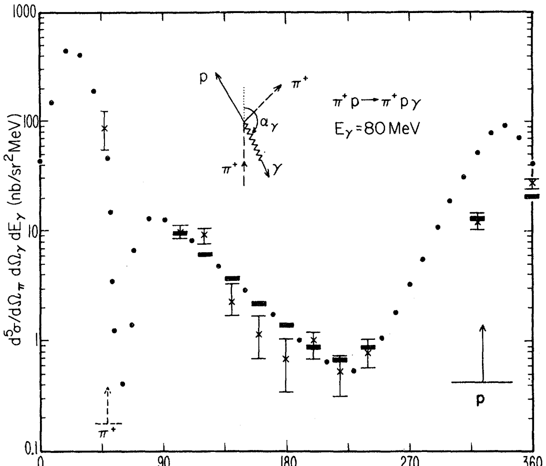

FIG. 1. Differential cross section for $\pi^+p \to \pi^+p\gamma$ with $E_\gamma = 80$ MeV for 298-MeV incident π^+. α_γ is the horizontal projection angle of the photon counters. ● is the EED $(\bar{s},\bar{t})$ prediction for coplanar geometry; ■ is the EED $(\bar{s},\bar{t})$ prediction for geometry of experiments (Refs. 1 and 2): $\bar{\Sigma}$ are the experimental results of Refs. 1 and 2.

of about 25% up to the highest measurable photon energy of 120 MeV.

The choice of the variables $\bar{s}$ and $\bar{t}$ at which we evaluate the elastic cross section is the same as that of Ref. 8, and is not unique. To show the sensitivity to these variables, we have evaluated Eq. (1) with $s=(P_1+P_2)^2$ and t fixed at the elastic scattering value for the pion angle. The results are shown by a dashed line in Figs. 2 and 3, and do not vary drastically from those using Eqs. (4) and (5). (Note that the falling trend of the photon spectrum is not affected.)

The validity of Eq. (1) with its $1/k$ behavior is fully expected at sufficiently low photon energies. The dominance of the first term of the soft-photon expansion in electron bremsstrahlung is well known.[9] Picciotto[10] uses an expression similar to our Eq. (1) to make a first estimate of $\pi^+p \to \pi^+p\gamma$. The surprising fact is that Eq. (1) works much better than the more complete soft-photon approximation or more sophisticated models in describing the high-energy part of the photon spectrum in π^+p and π^-p bremsstrahlung in the region of the $\Delta(1232)$ resonance. The prediction of the full soft-photon approximation (SPA) calculation of Ref. 1 is shown in Fig. 2 by the dotted line and exhibits a rise at high photon energy. This rise is due in part to higher terms in the expansion and in part to the fact that the elastic scattering matrix element is evaluated at two different values of momentum transfer in the terms corresponding to photon emission from the pion and from the proton. When two values of the elastic matrix element are used, the bremsstrahlung cross section is no longer simply proportional to an elastic cross section. Apparently the evaluation of the elastic matrix element at a single value of momentum transfer is a significant factor in the success of Eq. (1) at high-photon energy. However, we note that there is some evidence that at

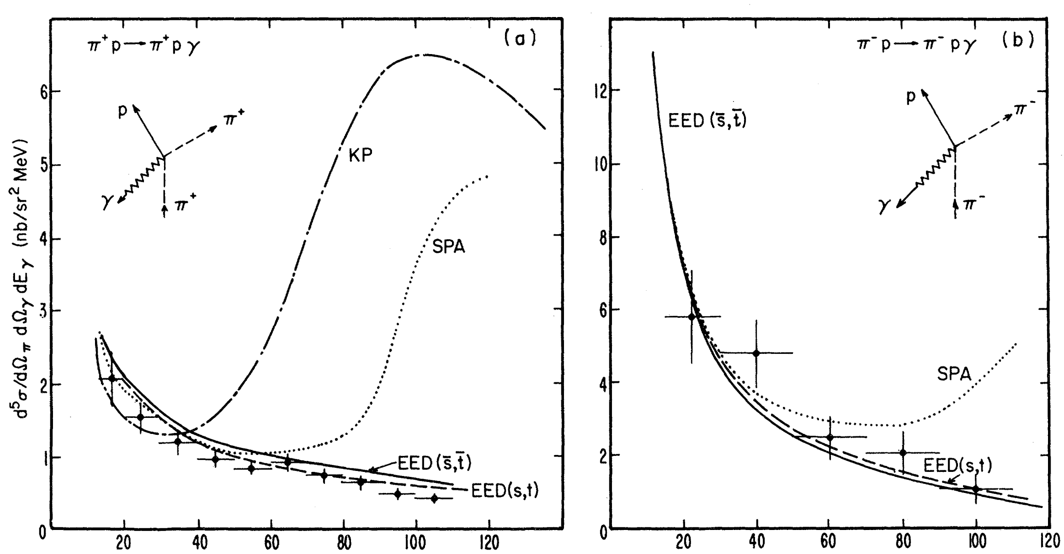

FIG. 2. Differential cross section in the lab system for $\pi^\pm p \to \pi^\pm p\gamma$ in the backward photon geometry of Ref. 1 for 298-MeV incident π^+. KP is the model calculation of Kondratyuk *et al*. (Ref. 7) using $\mu_\Delta = +2\mu_p$; SPA is the soft-photon approximation, namely the first two terms in a Low-type expansion, Ref. 1; EED is the prediction using external-emission dominance, Eq. (1); the definitions of $(\bar{s},\bar{t})$ and (s,t) are given in the text. The data points are from Ref. 1.

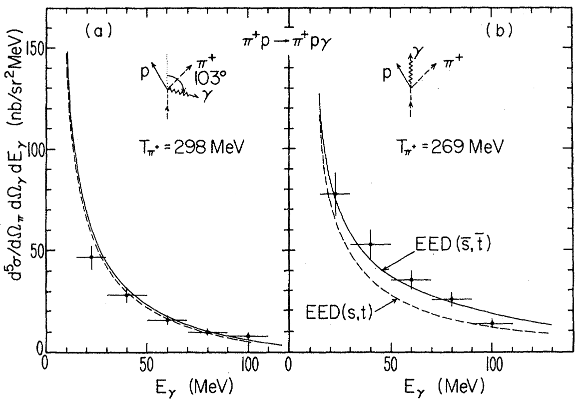

FIG. 3. Differential cross section for $\pi^+p \to \pi^+p\gamma$ at two other photon angles, Ref. 2. The solid and dashed curves are the EED $(\bar{s},\bar{t})$ and EED (s,t) calculations as in Fig. 2, averaged over the experimental acceptances. (a) 298-MeV incident π^+, photon detector at $\alpha_\gamma = 103°$, in the horizontal plane; (b) 269-MeV incident π^+, photon detector at $\alpha_\gamma = 0°$, 59° below the horizontal plane.

low photon energy the SPA may give slightly better agreement with the data at some photon angles.[3]

The detailed calculations of Refs. 7 and 8 fail dramatically in the region near $\alpha_\gamma \sim 220°$; an example is seen in Fig. 2. Recently, several models[11-13] have been proposed to explain the smoothly falling photon spectrum reported in Ref. 1. None of these models has yet made a quantitative prediction for the full angular distributions of π^+p and π^-p bremsstrahlung which would allow a more rigorous test of their validity.

Besides agreeing with experiment and being very simple, Eq. (1) is Lorentz-invariant, gauge-invariant, and has no free parameters. Equation (1) contains only the so-called "external" photon emission by the incoming and outgoing charged particles. The physical significance of the applicability of Eq. (1) is that πp bremsstrahlung is dominated by this external-emission contribution. Apparently, the contributions to photon emission due to the magnetic dipole moment of the proton, the contact term, and the internal emission term cancel one another for some unknown reason, or are unexpectedly small. This cancellation or smallness shows up dramatically in Fig. 1 in the region of the dip near $\alpha_\gamma \sim 220°$.

The validity of Eq. (1) at different incident energies is unknown at present. We speculate that deviations will occur when the incident energy is sufficiently high that virtual vector mesons must be considered, e.g. $\pi p \to \pi p \rho \to \pi p \gamma$. It has been argued that the radiative decay of mesons that proceeds via internal emission[14,15] such as $\omega \to \pi^0\gamma$ is evidence for vector-meson dominance (VMD). We speculate that high-energy hadron-hadron bremsstrahlung can be described completely in terms of VMD and external-emission dominance.

In conclusion, we note that the validity of Eq. (1) for incident pions of medium energy seriously reduces the prospects for using bremsstrahlung to measure off-mass-shell effects in the energy region of interest for nuclear physics or to determine the electromagnetic dipole moment of the $\Delta(1232)$.[7] On the other hand, the calculation of the external radiative corrections for accurate $\pi^\pm p$ experiments has become exceedingly simple.

---

*Work supported in part by ERDA.

†Present address: Department of Physics, Catholic University, Washington, D. C. 20064.

[1] D. Sober, M. Arman, D. Blasberg, R. Haddock, K. Leung, B. Nefkens, B. Schrock, and J. Sperinde, Phys. Rev. D 11, 1017 (1975).

[2] B. Nefkens *et al.*, in *Proceedings of the Fifth International Conference on High Energy Physics and Nuclear Structure*, edited by G. Tibell (Almqvist and Wiksell, Upsala, 1973), p. 51.

[3] K. C. Leung, M. Arman, H. C. Ballagh, P. F. Glodis, R. P. Haddock, B. M. K. Nefkens, and D. I. Sober, Phys. Rev. D 14, 698 (1976).

[4] F. E. Low, Phys. Rev. 110, 974 (1958).

[5] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[6] J. Carter *et al.*, Nucl. Phys. B58, 378 (1973).

[7] L. A. Kondratyuk and L. A. Ponomarev, Yad. Fiz. 7, 111 (1968) [Sov. J. Nucl. Phys. 7, 82 (1968)]; some corrections to this paper are given in A. Vanzha and M. Musakhanov, Dubna Report No. JINR-P2-6305, 1972 (unpublished).

[8] W. E. Fischer and P. Minkowski, Nucl. Phys. B36, 519 (1972); P. Minkowski (private communication).

[9] L. C. Maximon, Rev. Mod. Phys. 41, 193 (1969).

[10] C. Picciotto, Phys. Rev. 185, 1761 (1969).

[11] R. H. Thompson, Nuovo Cimento 16A, 290 (1973).

[12] B. Bosco *et al.*, Phys. Lett. 60B, 47 (1975).

[13] Q. Ho-Kim and J. P. Lavine, Phys. Lett. 60B, 269 (1976).

[14] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Lett. 8, 261 (1962).

[15] J. J. Sakurai, *Currents and Mesons* (Univ. of Chicago Press, Chicago, 1969).