Δ^+ pole parameters from a dispersion-theory fit to photopion production

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We use our multichannel dispersion-relation formulation of photoproduction to obtain excellent fits to low-energy γp reactions and $\gamma n \to \pi^- p$ data. As expected, these fits are quite sensitive to the values of the Δ^+ mass. We obtain a Δ^+ pole position $m - i\Gamma/2 = 1208 - 53i$ MeV. Predictions of the Δ^- parameters are made.

Very accurate experiments by Carter *et al.*[1] have determined the P_{33} phase shifts for $\pi^+ p$ and $\pi^- p$ scattering. From these results, the S-matrix pole parameters for the Δ^{++} and Δ^0 have been determined.[2,3] The pole parameters were shown[2] to be considerably more model-independent than the usual Breit-Wigner position and width parameters. These results make it important, in testing theoretical models of electromagnetic mass splitting in the baryon decuplet, to have a determination of the Δ^+ parameters.

We have previously developed[4] a phenomenological model of photoproduction and have obtained good fits to the low-energy data for the γp reactions

$$\gamma p \to \pi^0 p\,, \tag{1a}$$

$$\to \pi^+ n\,, \tag{1b}$$

and the less-well-known reaction

$$\gamma n \to \pi^- p\,. \tag{2}$$

The essential ingredients of this model, based on a multichannel ND^{-1} description of the strong πN partial-wave scattering, are the following: The usual $\gamma N \to \pi N$ Born terms are unitarized, and, via the rescattering integrals, the inelastic channels influence the multipole amplitudes even at low energy. Parameters are introduced to describe the photoproduction Born terms for the phenomenological inelastic channels (used in the previous strong fit). We found, as one would expect, that the fits were sensitive to the details of the P_{33} phase shifts for the Δ^+.

In the present paper, we analyze more recent data compiled by Donnachie,[5] and determine the Δ^+ resonance parameters: In our two-channel ND^{-1} fit[2] to the P_{33} data for the Δ^{++}, one input pole was used for the left-hand cut of the form

$$\begin{pmatrix} g_{\pi N,\pi N} & g_{\pi N,BN} \\ g_{\pi N,BN} & g_{BN,BN} \end{pmatrix} \Big/ (W - W_0)\,.$$

We assume that the main changes for the Δ^+ and Δ^0 are possible corrections to the diagonal coupling in the πN channel. Thus we take $g_{\pi N,\pi N} \equiv g_{\Delta^+}$ for Δ^+ and g_{Δ^0} for Δ^0 as parameters to be varied in the fit to the photoproduction data. Appropriate kinematical factors are used both in the strong ND^{-1} solutions for Δ^+ and Δ^0 and in fitting the reactions (1a), (1b), and (2).

We fitted some 725 data points from threshold to $E_\gamma = 400$ MeV. Our 10-parameter fit (eight multipole photoproduction Born terms for the inelastic channels coupling to the S and P πN states, g_{Δ^+}, and g_{Δ^0}) gave a χ^2 of 1.1/point. An illustration of this excellent fit is shown in Fig. 1. Furthermore, we note that the fit to 675 γp data points yielded a χ^2 of 1.0/point.

As was done in Ref. 2 for the Δ^{++}, we extrapolate the explicit ND^{-1} solution for the Δ^+ to its pole in the second sheet of the complex energy plane. We find a pole position

$$M^+ \equiv m^+ - i\Gamma^+/2 = 1208 - 53i \text{ MeV}\,. \tag{3}$$

We estimate the errors in (3) to be 2 MeV. As can be seen from the χ^2 of our fit, errors in the data for reaction (1) must be decreased in order to get a better determination of Δ^+. In contrast, the position of Δ^{++},

$$M^{++} = 1211 - 50i \text{ MeV}\,, \tag{4}$$

is very accurately determined to about 0.5 MeV.[4,6] The value M^0 has been determined to be[3]

$$M^0 = 1211 - 53i \text{ MeV} \tag{5}$$

from a fit to the $\pi^- p$ data.[1]

The data for the γn process (2) are less well determined than the data for the γp processes (1).

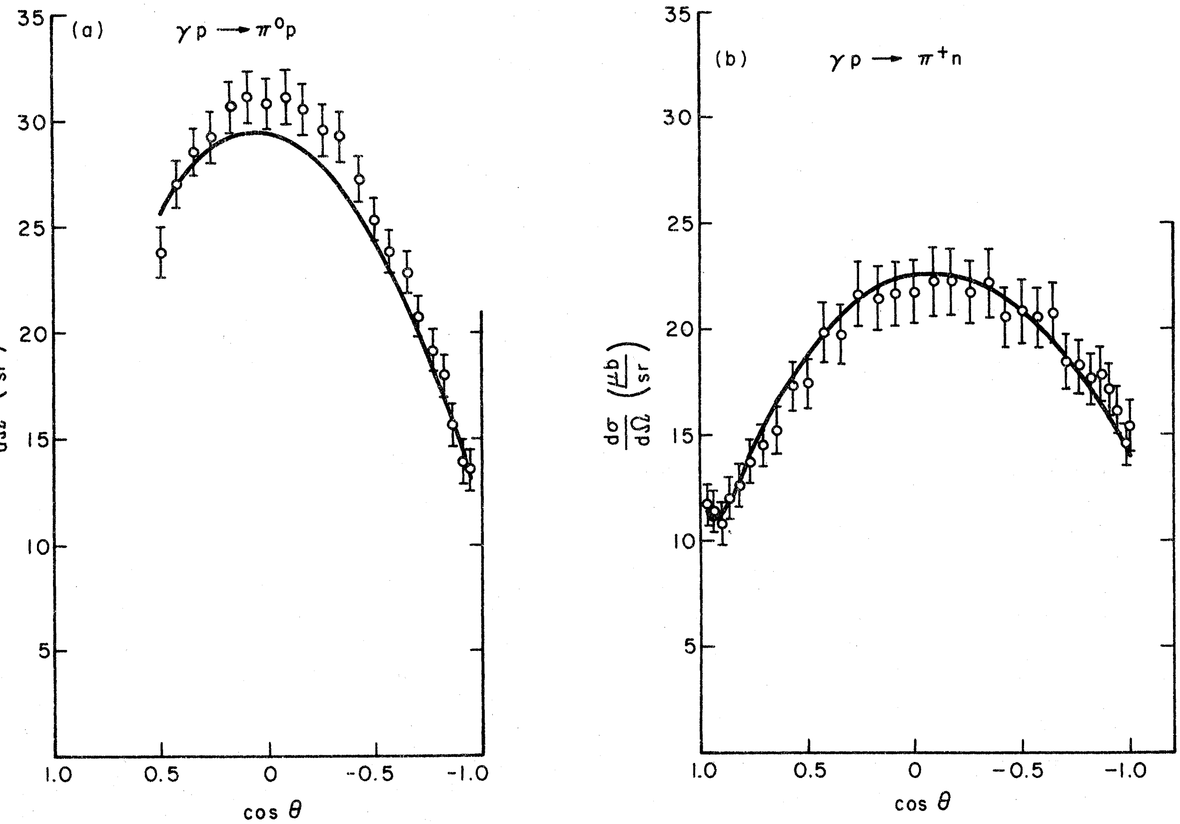

FIG. 1. As an example, we show the fit to the $\gamma p \to \pi^+ n$ and $\gamma p \to \pi^0 p$ data at $E_\gamma = 320$ MeV. Our fit to 725 data points for reactions (1) and (2) (from threshold to $E_\gamma = 400$ MeV) compiled by Donnachie (Ref. 5) had a χ^2 of 1.1/pt. The values of our ten fitted parameters (see Ref. 4 for notation and details of our multichannel dispersion calculation) are $g_{\Delta^+} = 84.80$, $g_{\Delta 0} = 95.8$, $\gamma(M_{1+}^{(3)}) = 0.1156$, $\gamma(E_{1+}^{(3)}) = 0.002\,65$, $\gamma(M_{1-}^{(1)}) = 0.003\,58$, $\gamma(E_{0+}^{(1)}) = 0.1501$, $\gamma(E_{0+}^{(0)}) = 0.003\,73$, $\gamma(M_{1-}^{(0)}) = 0.013\,74$, $\gamma(E_{0+}^{(3)}) = 0.004\,72$, $\gamma(M_{1-}^{(3)}) = 0.012\,28$. (Tabulations of our multipoles can be obtained from Shaw.)

We found that our fit was insensitive to the Δ^0 parameters. The resulting Δ^0 mass M^0 from the present photoproduction fit is consistent with the value (5) determined from a fit to $\pi^- p$ data, although the error is many times larger.

Our method of parameterizing the electromagnetic changes in the P_{33} phases produces a particular correlation between the shifts in the real and imaginary parts of the pole position. A more general method would be to vary both $g_{\pi N, \pi N}$ and $g_{\pi N, BN}$ for Δ^+ (and Δ^0). However, the additional parameters cannot improve our fit ($\chi^2 = 1.0$/pt) and hence cannot be independently determined from these data.

The most general mass formula for the Δ charge (Q) states (consistent with the extablished isovector nature of the photon) is[7]

$$M = A + BQ + CQ^2 . \qquad (6)$$

Equation (6) must, of course, be satisfied separately by the real (m) and imaginary ($\Gamma/2$) parts of the pole position M. Using the determined values (3)–(5) for M^+, M^{++}, and M^0, we obtain from (6)

$$M^- = 1217 - 50i \text{ MeV} . \qquad (7)$$

There are many theoretical calculations of mass differences. However, we will only compare the above results with the exact-SU(3) predictions which give a result similar to (6) for the ($J = \frac{3}{2}^+$) decuplet

$$M = a + bQ + cQ^2 \qquad (8)$$

in which b and c are constants for the entire decuplet.[8] Then using resonance parameters[9] for $\Sigma(1385)$ and $\Xi(1530)$ given by the Particle Data Group[6] along with (8), we obtain the Δ mass splittings from the Δ^{++}. This is summarized in Table I along with the resonance parameters of Δ^- determined in the peripheral production experiment of Gidal *et al.*[10] Clearly, more accurate measurements of the Δ^- parameters would be extremely interesting.

TABLE I. Δ mass splittings. The numbers listed first for each Δ splitting are the values corresponding to (3)–(5) and (7). The SU(3) predictions are in parentheses ( ) and the number in square brackets for the $\Delta^- - \Delta^{++}$ entry is from Ref. 10. (Note that the quantities in brackets and parentheses refer to resonance parameters.)

	m (MeV)	Γ (MeV)
$\Delta^+ - \Delta^{++}$	−3	6
	(2)	(3.5)
$\Delta^0 - \Delta^{++}$	0	6
	(4)	(7)
$\Delta^- - \Delta^{++}$	6	0
	(6)	(10.5)
	[7.9 ± 6.8]	[25 ± 23]

Theoretical work on electromagnetic mass differences has concentrated on the behavior of the real parts (which is of course the entire mass difference for stable particles). These calculations are inherently difficult (although for the same reason quite interesting) since they involve contributions from high-mass intermediate states. On the other hand, the electromagnetic width differences have been ignored by theorists. At first sight, one might expect that the splittings in the Γ's are simply explained in terms of (a) the kinematics due to the mass differences of the decay products, and (b) the contributions of the γ-hadron channels. However, we note that these effects are too small to explain the substantial splittings in Γ for the Δ's. This is also clearly true for the $\Sigma(1385)$ for which[9] $\Gamma(\Sigma^-) - \Gamma(\Sigma^+) = 7$ MeV. Thus we stress that these splittings in Γ depend on the details of the dynamics and are of as significant interest theoretically as the real parts of the electromagnetic splittings.

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