Second-class currents and very light quarks

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We consider induced second-class nucleon currents in a light-quark model generated by unequal u and d quark masses. The effect produced by gluon vertex corrections is strikingly large without confinement effects. Restricting the quarks to the limited spatial region of a physical nucleon changes the energy scale from the quark mass to the **quark**-bound-state energy and thereby reduces the second-class form factors well below their first-class counterparts.

I. INTRODUCTION

The results of recent angular correlation measurements in the β decay of some light nuclei have opened a new chapter in the tortuous history of second-class weak interactions. The experiments of Sugimoto *et al.* on the mass-12 system and the experiment of Calaprice *et al.* on the mass-19 system support the existence of second-class currents.[1,2] In contrast, the experiment of Nathan *et al.* on the mass-8 system does not find a comparable second-class effect, in agreement with older experiments on this system.[3-5] The experimental situation is, therefore, in a very unsettled state.[6]

It is well known that second-class currents are alien to conventional quark models with degenerate u and d quark masses.[7] However, it is not clear whether an appreciable second-class current can be induced in a model in which $(m_d - m_u)/(m_d + m_u)$ is of order unity. Such a mass relation has been suggested by Leutwyler[8] and can also be accommodated in the MIT bag model.[9] In this paper we examine the second-class currents induced by gluon-exchange effects in a model with low, unequal quark masses. In Sec. II, we consider induced second-class current matrix elements taken between free-quark states. In Sec. III, we consider the modifications due to confining both quarks and gluons to the interior of a physical nucleon. Section IV summarizes our results. In the main body of the text, we shall neglect the Cabibbo admixing of the d and s quarks in weak interactions. We shall comment on the induced second-class current effects in $|\Delta S|=1$ β decays in the summary.

II. SECOND-CLASS CURRENTS WITH FREE QUARKS

The weak charged-current matrix element between u and d free-quark states can be expressed as

$$\langle u|j_\beta(0)|d\rangle = \bar{u}[\gamma_\beta(f_V + f_A\gamma_5) + (q_\beta/2m)(f_S + f_P\gamma_5) + i\sigma_{\beta\alpha}(q^\alpha/2m)(f_M + f_{II}\gamma_5)]d\,, \quad (1)$$

where u and d are the 4-spinor functions, and $q = p_u - p_d$ is the momentum transfer. The G parities of first-and second-class vector (axial-vector) currents are, by definition, even (odd) and odd (even), so that in the limit of equal quark mass only f_S and f_{II} are second-class form factors.

In this section, we consider the form factors in a weak-interaction theory of the conventional type, in which the quarks are coupled to the charged intermediate-vector-boson field by

$$L_w(x) = g_w\bar{\psi}_u(x)\gamma_\beta(1+\gamma_5)\psi_d(x)W^\beta(x)\,. \quad (2)$$

The strongest quark interaction is assumed to be with a massless Yang-Mills field (gluon) whose coupling is

$$L_s(x) = g_s\bar{\psi}(x)\lambda_i\gamma_\beta\psi(x)A_i^\beta(x)\,, \quad (3)$$

where i is the color SU(3) index. The strong coupling constant is assumed to satisfy $g_s^2/4\pi \cong 0.5$ at a subtraction point characterized by the mass scale $\mu^2 \lesssim 1$ GeV^2 in accordance with the ideas of asymptotic freedom[10] or the MIT bag model,[9] so that it makes sense to treat L_s perturbatively. We shall distinguish between m_u and m_d but will otherwise ignore electromagnetic interactions.

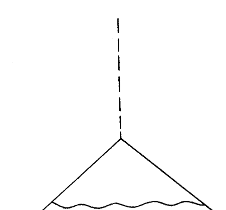

FIG. 1. Feynman diagram for the gluon-exchange correction to $d \to u + w^-$. The wavy line is a gluon.

To first order in g_w and zero order in g_s, $f_V = f_A = 1$ and the remaining form factors in Eq. (1) vanish. To second order in g_s, we have the gluon-exchange correction shown in Fig. 1. The second-class form factors generated by this diagram can be calculated without renormalization difficulties by standard methods. After Feynman parameterization and some algebraic manipulation, the integrals for f_S and f_{II} can be written as

$$\frac{f_j}{2m} = -\tfrac{32}{3} i g_s^2 \int dZ_1 dZ_2 dZ_3 \frac{d^4k}{(2\pi)^4} \times \delta(1 - Z_1 - Z_2 - Z_3) F_j H^{-3}, \quad (4)$$

where

$j = S$ and II,

and

$$H = k^2 + q^2 Z_2 Z_3 - m^2(1 - Z_1)^2 - 2m\Delta m Z_3(1 - Z_1) - (\Delta m)^2 Z_3(1 - Z_1) + i\epsilon,$$

$$F_S = 2m[2Z_2(Z_1 - 2) + (1 - Z_1)^2 + (1 - Z_1)] + 2\Delta m[2Z_2^2 + Z_2(3Z_1 - 4) + (1 - Z_1)^2 + (1 - Z_1)],$$

$$F_{II} = mZ_1(1 - Z_1 - 2Z_2) + \Delta m Z_1(1 - Z_1 - Z_2),$$

$$m = m_u, \quad m + \Delta m = m_d.$$

To first order in $\Delta m/2m$, we find that

$$f_S(0)/2m = -(4\alpha_s/9\pi m)(\Delta m/2m), \quad (5a)$$

$$f_{II}(0)/2m = -(4\alpha_s/3\pi m)(\Delta m/2m), \quad (5b)$$

where $\alpha_s \equiv g_s^2/4\pi$. Further details and expressions to all orders in $\Delta m/m$, together with $\Delta m \neq 0$ corrections to first-class form factors, are given in the Appendix.

In addition to f_{II}, the gluon-exchange mechanism therefore generates a nonzero f_S, which violates the conserved-vector-current (CVC) hypothesis. This is a consequence of our assumption that $m_u \neq m_d$, and is experimentally acceptable, since the present experimental tests of CVC are essentially restricted to the first-class current.[7]

We now turn to the analogous nucleon form factors, g_s and g_{II}, associated with neutron β decays through the $n \to p$ second-class current matrix element

$$\langle p | j_\beta^{2\text{nd}}(0) | n \rangle = (2M)^{-1} \bar{u}_p (q_\beta g_s + i\sigma_{\beta\alpha} q^\alpha g_{II}) u_n, \quad (6)$$

where u_n and u_p are free-nucleon spinors and M is the nucleon mass. Ignoring quark-confinement and gluon-exchange effects between different quarks within a nucleon, the nucleon form factors are given by

$$g_S(0) = -\frac{4\alpha_s}{9\pi}\left(\frac{M}{m}\right)\frac{\Delta m}{2m}, \quad (7a)$$

$$g_{II}(0) = -\frac{4\alpha_s}{3\pi}\frac{M}{m}\frac{\Delta m}{2m}. \quad (7b)$$

If we use quark masses like those suggested by Leutwyler,[8] say $m_u = 4$ MeV, $m_d = 6$ MeV, and if we use $\alpha_s = 0.5$, then $g_{II}(0) = +3g_s(0) = -11$. Such a value of the tensor form factor, g_{II}, is comparable to the weak-magnetism term, as suggested by the experiments of Refs. 1 and 2.

III. EFFECTS OF CONFINEMENT

In this section we estimate the effects of confining quarks and gluons to a nucleon of finite size. The estimates are made within the framework of the MIT bag model, but we believe they are substantially model-independent. The effects are of two types. The first is the generation of a second-class current to zero order in α_s, which owes its existence not only to $m_u \neq m_d$, but also to the mixing of upper and lower components in the transition matrix element and reflects the relativistic nature of light quarks. The same phenomenon allows zero-mass quarks to produce a nonvanishing nucleon magnetic moment to zero order in α_s.[11] The second type of confinement effect consists of those modifications of the order α_s terms of Eq. (7) that result from quark and gluon confinement.

We begin by reminding the reader that the nucleon magnetic-moment form factor, $g_M(0)$, can be expressed in the Breit frame as

$$i\frac{g_M(0)}{2M} U_f^\dagger \vec{\sigma} U_i (2\pi)^3 \delta^3(0) = \int \langle p_f | \vec{r} \times \vec{V}(\vec{r}, 0) | p_i \rangle d^3r, \quad (8)$$

where V_ν is the vector current field operator, $|p_i\rangle$ and $|p_f\rangle$ are initial and final zero-momentum nucleon states, and U is a two-component spinor. The analogous expression for the second-class-current tensor coefficient, $g_{II}(0)$, is

$$i\frac{g_{II}(0)}{2M} U_f^\dagger \vec{\sigma} U_i (2\pi)^3 \delta^3(0) = \int \langle p_f | \vec{r} A_0(\vec{r}, 0) | p_i \rangle d^3r, \quad (9)$$

where A_ν is the axial-vector current operator.

In the rigid sphere approximation to the MIT bag model, the lowest-energy single-quark wave function is given in the rest frame by[9]

$$q(r) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i\left(\dfrac{\omega + m}{\omega}\right)^{1/2} j_0(kr) U \\ -\left(\dfrac{\omega - m}{\omega}\right)^{1/2} j_1(kr) \vec{\sigma} \cdot \hat{r} U \end{pmatrix}, \quad (10)$$

where

$$N^{-2}=j_0(kR)^2R^3[2\omega(\omega R-1)+m]/\omega(\omega-m)\,. \tag{11}$$

U is a 2-spinor, and j_0 and j_1 are spherical Bessel functions. R is the bag radius, and ω is the single-quark energy which is related to k by

$$\omega=(k^2+m^2)^{1/2}\,. \tag{12}$$

The eigenvalue equation for the wave number is

$$\tan kR=kR[1-mR-R(k^2+m^2)^{1/2}]^{-1}\,. \tag{13}$$

The wave function has been normalized to $\int q^\dagger q\,d^3r=1$. Thus, to zero order in α_s

$$\begin{aligned}\frac{g_{\rm II}^{(0)}(0)}{2M}&=\int r_i q^\dagger(r)\gamma^0\gamma_5 q(r)\,d^3r\\ &=\frac{N^2}{3}\frac{\Delta m}{\omega}\int_0^R r^3 j_0(kr)j_1(kr)\,dr\\ &=\frac{\Delta m}{\omega}\frac{N^2}{3}\frac{R^4}{4\beta^3}\left(\cos 2\beta+2-\frac{3}{2\beta}\sin 2\beta\right),\end{aligned} \tag{14}$$

where we have introduced

$$\beta\equiv kR\,.$$

In the above, we have ignored the difference between the energy of a quark when it is in a neutron bag as opposed to a proton bag. The second line of Eq. (14) shows the cross term between upper and lower components of the nucleon. For $m\ll\omega=400$ MeV, $R=(200\text{ MeV})^{-1}$, we find $g_{\rm II}(0)\simeq\Delta m/\omega$, so that for Δm equal to a few MeV, $g_{\rm II}$ is very small. This zero-order second-class effect is, therefore, not of any practical interest at the present time.

For the calculation of confinement effects on order-α_s terms we can neglect the non-Abelian nature and it will be convenient to treat the quark-gluon interaction in the radiation gauge, where we can write the interaction Hamiltonian as[12]

$$\begin{aligned}H_s&=H_s^{(1)}+H_s^{(2)}\,,\\ H_s^{(1)}&=-(4\pi\alpha_s)^{1/2}\int\bar\psi(x)\vec\gamma\lambda_i\psi(x)\cdot\vec A_i(x)\,d^3x\,,\\ H_s^{(2)}&=\frac{\alpha_s}{2}\int\frac{\psi^\dagger(x)\lambda_i\psi(x)\psi^\dagger(y)\lambda_i\psi(y)}{|\vec x-\vec y|}\,d^3x\,d^3y\,.\end{aligned} \tag{15}$$

The associated old-fashioned, time-ordered perturbation diagrams are of two kinds: α_s modifications of a single quark within the bag, and modifications involving more than one of the three quarks. Representative examples are shown in Figs. 2 and 3. Self-energy insertions have been omitted, since we are using the "physical" quark mass.

We single out Fig. 2(a) as the lowest-energy intermediate-state contribution to the single-quark vertex correction, when intermediate quarks and gluons are in their lowest modes, and the bag within which they are contained is at rest. This contribution to $g_{\rm II}(0)$ is given by

$$g_{\rm II}^{2(a)}(0)U_f^\dagger\frac{\vec\sigma}{2M}U_i=\tfrac{16}{3}g_s^{\,2}\int\vec y\,\bar q_u(z)\gamma^i q_{u\lambda}(z)\bar q_{u\lambda}(y)\gamma^0\gamma_5 q_{d\rho}(y)\bar q_{d\rho}(x)\gamma^j q(x)\,G_m^i(z)G_m^{j*}(x)k_0^{-1}(k_0+E_W)^{-1}d^3x\,d^3y\,d^3z\,, \tag{16}$$

where E_W is the W-boson energy and k_0 is the energy of the lowest gluon mode in the cavity. The gluon wave functions are[13]

$$\begin{aligned}\vec G_m(\vec r)&\propto\vec\nabla\times\vec r\,Y_{1m}(\theta,\phi)j_1(kr)\\ &=N_m[\hat\theta im-\hat\phi(|m|\cos\theta-\delta_{m0}\sin\theta)]j_1(kr)e^{im\phi}\end{aligned}$$

and

$$N_m^{-2}=2k_0\frac{4\pi}{3}(1+|m|)[j_1(kR)^2-j_0(kR)j_2(kR)]R^3 \tag{17}$$

so as to normalize to

$$i\int G_m^{j*}(r,t)\frac{\overleftrightarrow{\partial}}{\partial t}G_n^j(r,t)\,d^3r=\delta_{mn}\,,$$

where

$$G_m^j(r,t)=G_m^j(r)e^{-ik_0t}\,.$$

Here the smallest wave number is $k\equiv k_0=2.73/R$. Since the $\vec y$-dependent factors in Eq. (16) are identical to those appearing in the evaluation of

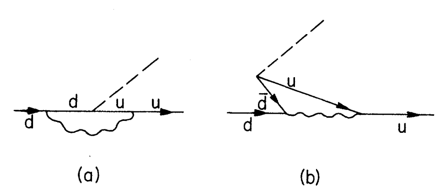

FIG. 2. Single-quark modifications to order α_s.

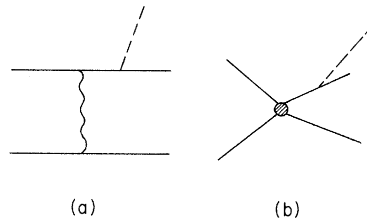

FIG. 3. Modifications involving quark-quark correlations to order α_s. The blob in (b) is the instantaneous Coulomb interaction, $H_s^{(2)}$.

$g_{II}^{(0)}(0)$ [see Eq. (14)], we may write

$$g_{II}(0)U_f^\dagger\vec{\sigma}U_i = \tfrac{16}{3}g_{II}(0)^{(0)}g_s^2 \times \int \overline{q}_u(z)\gamma^j q_{u\lambda}(z)U_\lambda^\dagger\vec{\sigma}U_\rho\overline{q}_{d\rho}(x)\gamma^k q_d(x) \times G_m^j(z)G_m^{k*}(x)k_0^{-1}(k_0+E_W)^{-1}d^3x\,d^3z\,. \tag{18}$$

We can make an order-of-magnitude estimate of the integral by noting that the quarks are highly relativistic, so that $\int\overline{q}\vec{\gamma}q\,d^3x\sim 1$. The normalization factor for $\vec{G}_m$ implies that $|\vec{G}_m|\sim(4\pi k_0R^3)^{-1}$. Thus,

$$g_{II}^{2(a)}(0)\sim g_{II}^{(0)}(0)\frac{5\alpha_s}{(k_0R)^3} \sim g_{II}^{(0)}(0)\frac{\alpha_s}{4}\,. \tag{19}$$

This estimate is certainly within an order of magnitude, and provided that the α_s expansion of $g_{II}(0)$ converges, makes it clear that setting $g_{II}(0)=g_{II}^{(0)}(0)$ does not give a misleading picture of confinement effects. To strengthen this conclusion, we note that a factor of $g_{II}^{(0)}(0)$ will arise from the $q\overline{q}W$ vertex in every diagram, including those in Fig. 3, in which only ground-state quarks are included; inclusion of higher modes of excitation will produce a similar factor at that vertex proportional to Δm. A similar reduction of the scalar form factor, $g_s(0)$, will obviously also result from confinement.

In order to convince the reader (and ourselves) that this result is essentially model-independent, we attempt to modify the free-quark vertex correction of Eq. (4) without the introduction of explicit bound states. Examination of the k integration in Eq. (4) shows that the major contribution comes from small Euclidean k^2 values. But confinement of the gluons to a finite spatial region implies that Euclidean k^2 can be no smaller than the square of the lowest gluon bound-state energy, k_0^2. We have evaluated Eq. (4) for $f_{II}(0)$ with this lower limit for k^2. The modification to Eq. (5b) is essentially to replace m by k_0. Since gluons and light quarks confined to a radius R will each have an energy of order R^{-1}, the numerical result obtained in this way is about the same as that of the α_s bag-model contribution. More specifically, we find that

$$g_{II}^{(\alpha_s)}(0)=+\frac{\alpha_s}{9\pi}\frac{M\Delta m}{k_0^2}\,.$$

Translated into coordinate space, this estimate recognizes that the radius of the nucleon, R, places an upper limit on the wavelength of gluons that can communicate with quarks confined to this region.

IV. CONCLUSION

We have shown that light-quark models of the nucleon in which the u-d quark mass difference is taken into account predict second-class form factors of order $M\Delta m/m^2$ (M = nucleon, m = quark mass), if confinement is totally ignored. The first effect of confinement is to replace the quark mass by the much larger single-quark energy, which reduces the second-class form factors to values well below those of the first-class form factors. The lesson to be learned is that free-quark results are totally misleading when the quarks are, in fact, confined and highly relativistic. Finally, we note that in $|\Delta S|=1$ baryon β decays the relevant Δm is $m_s - m_{u,d}$, so that here $\Delta m/\omega$ is of order unity. We expect that $g_{II}(|\Delta s|=1)/g_V\sim g_S(|\Delta s|=1)/g_V\sim 1$ as a consequence of a mechanism analogous to that described here.

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APPENDIX

In this appendix we give details associated with the free-quark vertex correction of Eq. (4). As in the analogous vertex correction in QED,[14] the vertex diagram can, except for a color factor of $\lambda_i^2=\frac{16}{3}$, be written as

$$\Lambda_\mu(p,p')=(-ig_s)^2\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-\lambda^2+i\epsilon}\gamma_\nu\frac{i}{p\!\!\!/'-k\!\!\!/-m+i\epsilon}\gamma_\mu(1+\gamma_5)\frac{i}{p\!\!\!/-k\!\!\!/(m+\Delta m)+i\epsilon}\gamma^\nu$$
$$=-2ig_s{}^2\int d^4k\,dZ_i\,\delta(1-Z_1-Z_2-Z_3)\gamma_\nu(p\!\!\!/'-k\!\!\!/+m)$$
$$\times\gamma_\mu(1+\gamma_5)(p\!\!\!/-k\!\!\!/+m+\Delta m)\gamma^\nu[k^2-\lambda^2Z_1+q^2Z_2Z_3-m^2(1-Z_1)^2-2m\Delta mZ_3(1-Z_1)$$
$$-(\Delta m)^2Z_3(1-Z_1)+i\epsilon]^{-3}. \tag{A1}$$

λ is the gluon mass to handle the infrared divergence. Separating off the logarithmic infinity, we have

$$\Lambda_\mu(p',p)=\frac{\alpha_s}{2\pi}\int dZ_i\,\delta(1-Z_1-Z_2-Z_3)\left[\gamma_\mu(1+\gamma_5)\left(-\tfrac{3}{2}+\ln\frac{\mathfrak{M}^2}{D_0}+\ln\frac{D_0}{D}\right)-\frac{1}{2}\frac{N_\mu}{D}\right], \tag{A2}$$

where

$$D_0=m^2(1-Z_1)^2+\lambda^2Z_1+\Delta m(2m+\Delta m)Z_3(1-Z_1),$$
$$D=D_0-q^2Z_2Z_3, \tag{A3}$$
$$N_\mu=\gamma_\nu[p\!\!\!/'(1-Z_2)-p\!\!\!/Z_3+m]\gamma_\mu(1+\gamma_5)[p\!\!\!/(1-Z_3)-p\!\!\!/'Z_2+m+\Delta m]\gamma^\nu, \tag{A4}$$

and $\mathfrak{M}$ is a high-energy cutoff on k^2. Since Λ_μ is to be multiplied by free-quark spinors, N_μ can be written as

$$N_\mu=\gamma_\mu\gamma_5[2(1-Z_2)(1-Z_3)(-q^2+m^2+m\Delta m+(\Delta m)^2)-2(m+\Delta m)^2(1-Z_3)Z_3$$
$$-2(1-Z_2)Z_2m^2+2Z_2Z_3m(m+\Delta m)+2m(m+\Delta m)]$$
$$+4p_\mu\gamma_5[mZ_2(1-Z_3)+(m+\Delta m)Z_3{}^2]-4p'_\mu\gamma_5[(m+\Delta m)Z_3(1-Z_2)+mZ_2{}^2]$$
$$+\gamma_\mu[2(1-Z_2)(1-Z_3)(-q^2+3m^2+3m\Delta m+(\Delta m)^2)-2(m+\Delta m)^2(1-Z_3)Z_3$$
$$-2Z_2(1-Z_2)m^2-2Z_2Z_3m(m+\Delta m)-2m(m+\Delta m)]$$
$$+4p_\mu[Z_2(1-Z_3)m-Z_3{}^2(m+\Delta m)]+4p'_\mu[Z_3(1-Z_2)(m+\Delta m)-Z_2{}^2m]. \tag{A5}$$

The Gordon identity and its axial-vector counterpart are (free-quark spin factors understood)

$$\tfrac{1}{2}[(p\!\!\!/'-p\!\!\!/),\gamma_\mu]=-(p'_\mu+p_\mu)+(2m+\Delta m)\gamma_\mu, \tag{A6}$$
$$\tfrac{1}{2}[(p\!\!\!/'-p\!\!\!/),\gamma_\mu]\gamma_5=-(p'_\mu+p_\mu)\gamma_5-\Delta m\gamma_\mu\gamma_5. \tag{A7}$$

N_μ can therefore be rewritten as

$$N_\mu=\gamma_\mu[-2(1-Z_2)(1-Z_3)q^2+2(2-(1-Z)^2-2(1-Z_1))(m+\Delta m)m+2Z_1\Delta m^2]$$
$$+\gamma_\mu\gamma_5[-2(1-Z_2)(1-Z_3)q^2+2((1-Z_3)Z_1+1+Z_2Z_3-Z_2(1-Z_2))(m+\Delta m)m+2((1-Z_3)Z_1+Z_1Z_2)\Delta m^2]$$
$$+[q\!\!\!/,\gamma_\mu][-Z_1(1-Z_1)m-Z_1(1-Z_1-Z_2)\Delta m]$$
$$-2q_\mu\gamma_5[(4Z_2{}^2-4Z_2(1-Z_1)+(1-Z_1)^2+(1-Z_1))m+(2Z_2{}^2-Z_2(4-3Z_1)+(1-Z_1)^2+(1-Z_1))\Delta m]$$
$$+2q_\mu[(2Z_2(Z_1-2)+(1-Z_1)^2+(1-Z_1))m+(2Z_2{}^2+Z_2(3Z_1-4)+(1-Z_1)^2+(1-Z_1))\Delta m]$$
$$+[q\!\!\!/,\gamma_\mu]\gamma_5[Z_1(1-Z_1-Z_2)\Delta m+Z_1(1-2Z_2-Z_1)m]. \tag{A8}$$

The renormalized vertex is obtained by performing one subtraction at $q^2=0$. To first order in q, we therefore obtain after integrating over the Z_i

$$\Lambda_\mu^{\rm ren} = q_\mu \gamma_5\left(-\frac{\alpha_s}{4\pi}\right)\left\{\frac{1}{\Delta m} - \frac{1}{(2m+\Delta m)}\left[3-2\left(\frac{m}{\Delta m}\right)^2\right] - \frac{2(m+\Delta m)}{(2m+\Delta m)^2(\Delta m)^3}\left[3m(\Delta m)^2+2m^2(m+\Delta m)\right]\ln\left(1+\frac{\Delta m}{m}\right)\right\}$$
$$+[\not{q},\gamma_\mu]\frac{\alpha_s}{4\pi}\left[\frac{1}{2(2m+\Delta m)}+\frac{m(m+\Delta m)}{\Delta m(2m+\Delta m)^2}\ln\left(1+\frac{\Delta m}{m}\right)\right]$$
$$+q_\mu\frac{\alpha_s}{4\pi}\left\{\frac{1}{2m+\Delta m}-\frac{2}{(2m+\Delta m)^2\Delta m}\left[5m^2+6m\Delta m+\tfrac{3}{2}(\Delta m)^2\right] + \frac{4(m+\Delta m)m}{(2m+\Delta m)^3(\Delta m)^2}\left[5m^2+5m\Delta m+\tfrac{3}{2}(\Delta m)^2\right]\ln\left(1+\frac{\Delta m}{m}\right)\right\}$$
$$+[\not{q},\gamma_\mu]\gamma_5\left(-\frac{\alpha}{4\pi}\right)\left[\frac{1}{2\Delta m}-\frac{m}{(\Delta m)^2}\frac{1+\Delta m/m}{2+\Delta m/m}\ln\left(1+\frac{\Delta m}{m}\right)\right]. \tag{A9}$$

To first order in $\Delta m/m$,

$$\Lambda_\mu^{\rm ren} = \frac{\alpha_s}{4\pi m}\left\{\frac{7}{3}\left(1-\frac{\Delta m}{2m}\right)g_\mu\gamma_5+\frac{1}{2}\left(1-\frac{\Delta m}{2m}\right)[\not{q},\gamma_\mu]-\frac{1}{3}\frac{\Delta m}{2m}q_\mu-\frac{1}{2}\frac{\Delta m}{2m}[\not{q},\gamma_\mu]\gamma_5\right\}. \tag{A10}$$

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$$\frac{m_d-m_u}{m_d+m_u}=\frac{m_{K^0}{}^2-m_{K^+}{}^2}{m_\pi{}^2}\simeq 0.2,$$

which can be deduced from M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

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[14]We follow the procedures of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), p. 166.