Charge asymmetry in $e^+e^- \to \gamma +$ hadrons: New tests of the quark-parton model and fractional charge*

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We consider the process $e^+e^- \to \gamma + h + X$, where h is a hadron and γ is a hard photon, and show how it can be used to test the quark-parton model. Detailed formulas are given for the cross sections, which in the quark-parton model are products of cross sections for $e^+e^- \to \gamma\mu\bar{\mu}$ and quark breakup functions. We focus on the asymmetry between h and $\bar{h}$ production, and display sum rules and ratio tests which measure the quark charge, the quark Compton amplitude, and the large-x behavior of the quark breakup function. The asymmetry is calculated for the muon case, and is about 100% for the forward direction.

I. INTRODUCTION AND DISCUSSION

In this paper, we explore the predictions of the quark model for the order α^3 processes $e^+e^- \to \gamma + h + X$ (where γ is a hard photon, h is a hadron, and X is anything), particularly for the asymmetry of h versus $\bar{h}$ production. Measurements of this asymmetry, which is proportional to the cube of the quark charge, will test the spin structure of the quark current and the scaling behavior of the breakup of the quark into hadrons.

Our calculations rely on the following two assumptions commonly used in the parton model[1]:

(i) The electromagnetic current of hadrons is carried by quark currents; in effect, the quarks are elementary spin-$\frac{1}{2}$ fermions which are produced by virtual photons in the same manner as electrons or muons.

(ii) The quarks decay into hadrons. The hadrons have limited momentum transverse to the direction of the quark momentum, so there should be jets of hadrons in the quark direction, and the decay amplitude "scales" (unless the momentum of the hadron is very small) in that it is a function of the hadron longitudinal momentum only through its ratio to the quark momentum.

The breakup of the quark is described by the distribution $D_q^h(x)$ which is the probability $dN_q^h(x)/dx$ for the production of a hadron h with a fraction x of the quark momentum in its infinite-momentum frame. [The distribution $D_q^h(x)$ is identical to $\tilde{G}_{h/q}(x)$ in Ref. 2.] In a general frame x is the light-cone variable

$$x = \frac{E_h + |\vec{p}_h|}{E_q + |\vec{p}_q|} \quad . \tag{1}$$

One of the most striking confirmations of the predictions of the quark-parton model is the observation of leading (i.e., high-momentum) pions in $e^+e^- \to \pi + X$ with the angular distributions expected from the production and decay of elementary spin-$\frac{1}{2}$ fermions.[3] The results are particularly clear in this case because of the polarization of the e^+ and e^- beams at SPEAR. For $e^+e^- \to h + X$ the quark-parton model prediction, in the c.m., is

$$\frac{d\sigma}{d\Omega_h dx} = \sum_q \left[\frac{d\sigma}{d\Omega_q} D_q^h(x) + \frac{d\sigma}{d\Omega_{\bar{q}}} D_{\bar{q}}^h(x) \right] , \tag{2}$$

where $x = 2p_h \cdot q/q^2$, and

$$\frac{d\sigma}{d\Omega_q} = \frac{e_q^2}{e^2} \frac{\alpha^2}{4s} (1 + \cos^2\theta + P^2 \sin^2\theta \cos 2\phi) \tag{3}$$

is the cross section for $e^+e^- \to \mu\bar{\mu}$, with $e_\mu \to e_q$ and $\vec{p}_\mu \to \vec{p}_q$ which is parallel to $\vec{p}_h$. Similarly $d\sigma/d\Omega_{\bar{q}}$ is the $\mu\bar{\mu}$ cross section with $\vec{p}_{\bar{\mu}} = \vec{p}_{\bar{q}}$ parallel to $\vec{p}_h$. P is the polarization of either beam, and the sum includes all quarks whose production threshold has been passed.

Returning to the present work, we consider the process $e^+e^- \to \gamma + h + X$ (see Fig. 1). As already stated, we focus on the asymmetry for h versus $\bar{h}$ production. Equivalently, using C invariance, this is also the asymmetry of h with respect to the e^+ versus e^- direction, and it can be related by crossing to the difference between e^+ and e^- deep-inelastic bremsstrahlung.[4] Our motivation is to test the spin structure of the quark current, the behavior of the virtual Compton amplitude on a quark line, and the scaling laws for its breakup in a new domain. In the remainder of this section we will outline the main predictions for $e^+e^- \to \gamma$
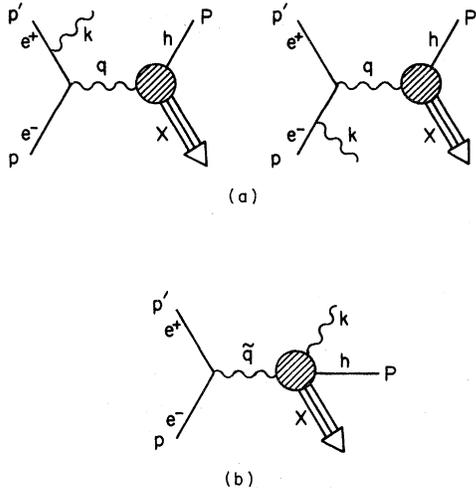
FIG. 1. The diagrams which contribute to radiative e^+e^- annihilation, $e^+e^- \to \gamma + h + X$.

$+h+X$ using intuitive arguments based on the quark-jet picture. The formal aspects, which are similar to those of Ref. 4, are discussed in Sec. II.

For $e^+e^- \to \gamma + h + X$ the simplest parton-model prediction is

$$\frac{d\sigma}{(d^3k/k_0)d\Omega_h dx} = \sum_q \left[\frac{d\sigma}{(d^3k/k_0)d\Omega_q} D_q^h(x) \right.$$
$$\left. + \frac{d\sigma}{(d^3k/k_0)d\Omega_{\bar q}} D_{\bar q}^h(x) \right] , \quad (4)$$

where $\vec{k}$ is the photon's momentum and $k_0 d\sigma/ d^3k d\Omega_q$ is the cross section for $e^+e^- \to \gamma\mu\bar\mu$ with $e_\mu \to e_q$ and $\vec{p}_\mu = \vec{p}_q$ which is parallel to $\vec{p}_h$. The cross section is given in detail in Sec. II and Appendix A. Note that for the muon case $|\vec{p}_\mu|$ is

kinematically determined by $\vec{k}$ and the angles of $\vec{p}_\mu$; when we go to the hadronic case and assume limited transverse momentum of the hadron relative to its parent quark, the only additional kinematic variable is $x = 2p_h \cdot q/q^2$. At fixed ratios of invariants, $k_0 s^2 d\sigma/d^3k/d\Omega_\mu$ is independent of s for $s \gg m_\mu^2$. Here $q^2 = (\tilde q - k)^2 = s - 2\tilde q \cdot k$.

The above scaling prediction [Eq. (4)] depends on being able to replace the diagrams in Figs. 1(a) and 1(b) by those in Figs. 2(a) and 2(b). This requires that the mass of the virtual photons be large. Also, Fig. 2(c) represents diagrams in which the photon is emitted from other quark lines or from hadron decay. However, these diagrams are suppressed by the hadronic vertex if the real photon has large transverse momentum relative to the quark jet. Hence, we can expect the scaling prediction to be valid when the photon is detected at large momentum relative to the hadron momentum.

The hadron asymmetry should be separate from the background $e^+e^- \to \gamma + h + X$ events in which the γ arises from hadron decay if π^0 decays can be excluded with high efficiency. The asymmetry of the background from radiative corrections or weak interactions (at $\sqrt{s} < 8$ GeV) should be less than $O(1\%)$ and is directly measurable in $d\sigma(e^+e^- \to hX) - d\sigma(e^+e^- \to \bar h X)$. Furthermore, for fixed ratios of invariants, the background is not scale-invariant and is predicted by the dimensional-counting rules[5] to vanish by two powers of $p_\perp$ faster than the scaling contribution. Thus, again we require the photon to be detected at large transverse momentum relative to the jet axis.

We can directly relate the hadron asymmetry to the corresponding $\mu-\bar\mu$ asymmetry at the same angle. The cross-section difference is

$$\Delta_h \equiv \frac{d\sigma(e^+e^- \to \gamma hX)}{(d^3k/k_0)d\Omega_h dx} - \frac{d\sigma(e^+e^- \to \gamma\bar h X)}{(d^3k/k_0)d\Omega_{\bar h} dx}$$

$$= \sum_q \frac{e_q^3}{e^3} \left\{ \frac{d\sigma}{(d^3k/k_0)d\Omega_\mu} [D_q^h(x) - D_q^{\bar h}(x)] + \frac{d\sigma}{(d^3k/k_0)d\Omega_{\bar\mu}} [D_{\bar q}^h(x) - D_{\bar q}^{\bar h}(x)] \right\} , \quad (5)$$

and using C invariance, $D_{\bar q}^{\bar h} = D_q^h$, etc. As will be seen, the asymmetry in the muon cross section is very large—nearly 100%—at $\cos\theta = \pm 1$. The ratio of the asymmetry in the hadron cross section to that in the muon cross section is

$$R_h^{(3)}(x) \equiv \frac{\Delta_h}{k_0 d\sigma/d^3k d\Omega_\mu - k_0 d\sigma/d^3k d\Omega_{\bar\mu}}$$

$$= \sum_q \frac{e_q^3}{e^3} [D_q^h(x) - D_{\bar q}^h(x)] . \quad (6)$$

This can be compared with the ratio of the order

α^2 hadron and muon cross sections,

$$R_h^{(2)}(x) = \frac{(d\sigma/d\Omega_h dx)(e^+e^- \to hX)}{(d\sigma/d\Omega_\mu)(e^+e^- \to \mu\bar\mu)}$$

$$= \sum_q \frac{e_q^2}{e^2} [D_q^h(x) + D_{\bar q}^h(x)] . \quad (7)$$

For definiteness we take the muon charge $e_\mu = e > 0$.

The critical prediction of the quark models is that both $R_h^{(2)}(x)$ and $R_h^{(3)}(x)$ are independent of s and the angles at which the hadron emerges. Further, the predicted form of the virtual Compton ampli-
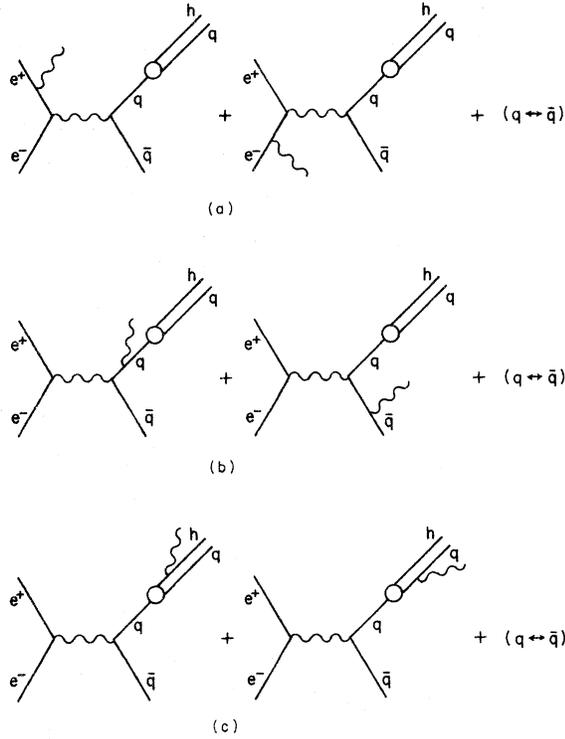
FIG. 2. The diagrams of Fig. 1 drawn in terms of the quark representation of the current. Figure 2(c) is expected to give small contributions if the component of $\vec{k}$ transverse to the quark (jet) direction is large.

tude on a quark (which assumed that quarks are elementary and require no form factors) is confirmed if $R_h^{(3)}(x)$ is independent of the photon kinematics in the scaling region. Several more specific predictions follow.

Valence-quark predictions. For $x \sim 1$ one may believe that a given quark can decay into a given hadron only if that quark is one of the valence quarks of the hadron.[2] Then, for example, for $\pi^+ \sim u\bar{d}$, $K^+ \sim u\bar{s}$, $p \sim uud$, we have

$$\frac{R_{\pi^+}^{(3)}}{R_{\pi^+}^{(2)}} = \frac{1}{e} \frac{e_u{}^3 D_u^{\pi^+} - e_d{}^3 D_d^{\pi^+}}{e_u{}^2 D_u^{\pi^+} + e_d{}^2 D_d^{\pi^+}} = \frac{3}{5} = \frac{R_{K^+}^{(3)}}{R_{K^+}^{(2)}} \tag{8}$$

and

$$\frac{R_p^{(3)}}{R_p^{(2)}} = \frac{1}{e} \frac{e_u{}^3 D_u^p + e_d{}^3 D_d^p}{e_u{}^2 D_u^p + e_d{}^2 D_d^p} = \frac{5}{9} , \tag{9}$$

where we assume that $D_s^{K^+} = D_u^{K^+}$ and $D_u^p(x) = 2D_d^p(x)$. However, if $\nu W_2^{(n)}/\nu W_2^{(p)} \to 1/4$ (see Ref. 6) as $x \to 1$, then one expects that $D_d^p/D_u^p \to 0$ (see Ref. 7) and $R_p^{(3)}/R_p^{(2)} = \frac{2}{3}$. The results are a direct test of fractional quark charge. It is also possible that, because of mass splittings, the $\bar{s}$ quark dominates the K^+ wave function, giving $R_{K^+}^{(3)}/R_{K^+}^{(2)} = e_{\bar{s}}/e_\mu = \frac{1}{3}$.
Predictions for the $R^{(3)}/R^{(2)}$ ratio when the electro-

magnetic current contains a color-octet contribution are given in Appendix B.

Quantum-number sum rules. We can define the effective multiplicity of hadron h from fragmentation of quark q as

$$n_q^h = \int_0^1 dx \, D_q^h(x). \tag{10}$$

Then

$$\int_0^1 R_h^{(2)}(x) \, dx = \sum_q \frac{e_q{}^2}{e^2} (n_q^h + n_{\bar{q}}^h)$$

$$= (n_h + n_{\bar{h}}) \sum_q \frac{e_q{}^2}{e^2} , \tag{11}$$

where

$$n_h + n_{\bar{h}} = \frac{1}{\sigma} \int_0^1 dx \, \frac{d\sigma}{dx} (e^+e^- \to hX)$$

is the hadron multiplicity in e^+e^- annihilation. The integral of the hadron asymmetry is

$$\int_0^1 dx \, R_h^{(3)}(x) = \sum_q \frac{e_q{}^3}{e^3} (n_q^h - n_q^{\bar{h}}). \tag{12}$$

Note that Eq. (12) is convergent because of the absence of the Pomeron contribution.

The simplest expression of quantum-number conservation is

$$\sum_h \lambda_h (n_q^h - n_q^{\bar{h}}) = \sum_{h,\bar{h}} \lambda_h n_q^h = \lambda_q , \tag{13}$$

where λ is any additively conserved quantity ($\lambda = Q, B, Y, I_z, \ldots$). Equation (13) would be valid if the quark were an ordinary decaying state. If the quarks are confined, Eq. (13) is only an ansatz, and expresses Feynman's hypothesis[8] that the quantum numbers of the quark are retained in the fragmentation region of the jet. The ansatz could be false if the quantum numbers are deposited at finite rapidity—thus giving a nonscaling contribution at $x \to 0$; cascade models of this sort are discussed in Ref. 9. Nevertheless, Eq. (13) remains an attractive possibility. It could also be testable in neutrino interactions $\nu p \to \mu^- X$ where a u-quark jet should dominate.

We may now write a sum rule for the hadron asymmetry,

$$\sum_h \lambda_h \int_0^1 dx \, R_h^{(3)}(x) = \sum_q \lambda_q \frac{e_q{}^3}{e^3} . \tag{14}$$

Specifically, for $\lambda_h = e_h/e$, the total charge asymmetry ratio is

$$\sum_h \frac{e_h}{e} \int_0^1 dx \, R_h^{(3)}(x) = \sum_q \frac{e_q{}^4}{e^4}$$

$$= \begin{cases} \frac{2}{3} & (q = u, d, s) \\ \frac{34}{27} & (q = u, d, s, c) \end{cases} \tag{15}$$

where we have included a factor of 3 for color. The values given above may be changed if the electromagnetic current contains pieces which are not color singlets, as in the Han-Nambu model. This is discussed in Appendix B. The above sum rule is particularly convenient in that it does not require identification of the particle, but only of the sign of the charge. If there are leptonic decays, the decay μ and e contributions to the charge asymmetry must also be included. Thus, the sum labeled h above should be a sum over all the long-lived charged particle states.

The convergence of the sum rule at small x is guaranteed by the absence of the Pomeron contribution. However, $D_q^h(x) - D_{\bar{q}}^h(x)$ could well have a Reggeon contribution $x^{-\alpha}$ at small x where α is of order of 0.5. Thus small-x data are required to evaluate the sum rule.

Heavy leptons. All of the above considerations also hold for the asymmetries due to the production and decay of an elementary Dirac heavy lepton L (see Ref. 10) in $e^+e^- \to \gamma L\bar{L}$. In particular, the charge asymmetry is scale-invariant and if $e_L = e$ it satisfies the sum rule

$$\sum_{h\in L} \frac{e_h}{e} \int_0^1 dx\, R_{h\in L}^{(3)}(x) = 1, \tag{16}$$

where the sum includes the hadrons and leptons μ, e in the decay of L. The sum of Eqs. (15) and (16) should be used if the heavy-lepton- and quark-generated events are not separated.

II. CALCULATIONS

The Feynman diagrams relevant to $e^+e^- \to \gamma + h + X$ are given in Fig. 1. Figure 1(a) will be referred to as a Bethe-Heitler process and Fig. 1(b) will be referred to as a Compton process. The momentum labels are given on the figure. The matrix elements for the diagrams are[11]

$$\mathfrak{M}_a(h) = ie^3 \bar{v}(p') \left(\gamma^\nu \frac{1}{\not{p}-\not{k}} \gamma^\mu + \gamma^\mu \frac{1}{\not{p}-\not{q}} \gamma^\nu \right) u(p) \frac{\epsilon_\mu}{q^2}$$

$$\times \int d^4 y\, e^{-iq \cdot y} \langle h, X | \mathcal{J}_\nu(y) | 0 \rangle$$

$$\equiv ie^3 \mathfrak{M}'_a(h) \tag{17}$$

and

$$\mathfrak{M}_b(h) = -e^3 \bar{v}(p') \gamma^\lambda u(p) \frac{\epsilon_\mu}{\tilde{q}^2}$$

$$\times \int d^4 x\, d^4 z\, e^{-i\tilde{q}z + ikx} \langle h, X | T^* \mathcal{J}_\lambda(z) \mathcal{J}_\mu(x) | 0 \rangle$$

$$= -e^3 \mathfrak{M}'_b(h). \tag{18}$$

Using the charge-conjugation operator, $|\bar{h}\rangle = c|h\rangle$, it follows that

$$\mathfrak{M}_a(\bar{h}) = -\mathfrak{M}_a(h)$$

and $\tag{19}$

$$\mathfrak{M}_b(\bar{h}) = \mathfrak{M}_b(h),$$

where the states X are charge-conjugated also. The difference in cross section between the processes involving h and $\bar{h}$ thus depends on the interference between diagrams 1(a) and 1(b).

The cross section for h is given by

$$d\sigma(h) = \frac{e^6}{2s} \frac{d^3k\, d^3P}{4(2\pi)^6 k_0 P_0} \sum_X (|\mathfrak{M}'_a|^2 + |\mathfrak{M}'_b|^2 - 2\,\text{Im}\,\mathfrak{M}'_a{}^* \mathfrak{M}'_b), \tag{20}$$

where $s = (p+p')^2 = \tilde{q}^2$ and the lepton masses have been neglected. The cross-section difference is proportional to the last term above, and can be written as

$$d\sigma(h) - d\sigma(\bar{h}) = \frac{\alpha^3}{2\pi^3 s^2 q^2} \frac{d^3k\, d^3P}{k_0 P_0} L^{\mu\nu\lambda}\, \text{Im}\, V_{\mu\nu\lambda}, \tag{21}$$

where the lepton trace is

$$L^{\mu\nu\lambda} = \tfrac{1}{4} \text{tr}\, \not{p}' \gamma^\lambda \not{p} \left(\gamma^\nu \frac{1}{\not{p}-\not{k}} \gamma^\mu + \gamma^\mu \frac{1}{\not{p}-\not{q}} \gamma^\nu \right) \tag{22}$$

and the hadron currents give

$$V_{\mu\nu\lambda} = \sum_X \int d^4x\, d^4y\, e^{iqy + ikx} \langle 0 | \mathcal{J}_\nu(y) | h, X \rangle$$

$$\times \langle h, X | T^* \mathcal{J}_\lambda(0) \mathcal{J}_\mu(x) | 0 \rangle \tag{23}$$

and the sum over X now includes a $(2\pi)^4 \delta^4(k+q-P-X)$.

The expressions given above are exact to the stated order of α, neglecting the lepton masses. The expression for $V_{\mu\nu\lambda}$ simplifies if we replace the physical hadrons in the closure sum by bare constituents (partons). We further assume that the charged constituents carry spin $\tfrac{1}{2}$ and that their charge is not necessarily an integral fraction of e (i.e., they are quarks). The observed hadron itself is a decay product of a quark.

We shall work in the kinematic region where q^2 and $\tilde{q}^2$ are large and where each of the three photons has a large transverse momentum relative to the direction of the observed hadron. The dominant contribution to $V_{\mu\nu\lambda}$ under these circumstances is given by Fig. 3. We assume, as stated earlier, that the decays of quarks into physical hadrons follow the same pattern as the fragmentation of hadrons into quarks: The decay products have limited transverse momentum with respect to the initial momentum of the parent quark, provided that the momentum is large. The dominant contribution in our kinematic region must have all
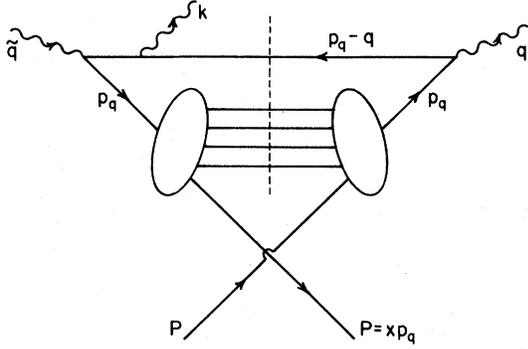
FIG. 3. The diagram for the leading contribution to $V_{\mu\nu\lambda}$. The diagram with k and $\tilde{q}$ connecting in the opposite order should also be included.

three photons interact with the same quark line. If some particular quark line had only one photon attached to it, it must then have a large transverse momentum in either the initial or final state and thus its contribution would be suppressed.

We might comment that the kinematic requirements are easy to satisfy at high energy if we do not place all the particles in the same plane. Consider the plane formed by the beam and the observed photon. All the photons must lie in this plane. The requirement on the transverse momentum of the photons is then satisfied simply if the observed hadron has a reasonable momentum out of this plane.

Using the function $D_q^h(x)$ to describe the breakup of the quark we obtain

$$V_{\mu\nu\lambda} = i^2 \sum_q \frac{e_q^3}{e^3} \int_0^1 \frac{dx}{x} [D_q^h(x) - D_{\bar{q}}^h(x)] \, \bar{u}(p_q) \left(\gamma_\lambda \frac{1}{\not{p}_q - \not{q} - \not{k}} \gamma_\mu + \gamma_\mu \frac{1}{\not{p}_q + \not{k}} \gamma_\lambda \right) \frac{1}{\not{p}_q - \not{q} + i\epsilon} \gamma_\nu u(p_q), \tag{24}$$

where x is the light-cone variable defined in (1). The appropriate discontinuity is also indicated in Fig. 3 and gives

$$\mathrm{Im} V_{\mu\nu\lambda} = \frac{\pi}{q^2} \sum_q \frac{e_q^3}{e^3} [D_q^h(x) - D_{\bar{q}}^h(x)] M_{\mu\nu\lambda}, \tag{25}$$

where we have obtained the relation

$$x = \frac{2P \cdot q}{q^2} \tag{26}$$

and

$$M_{\mu\nu\lambda} = \mathrm{tr} \, \not{p}_q \gamma_\nu (\not{p}_q - \not{q}) \left(\gamma_\lambda \frac{1}{\not{p}_q + \not{k}} \gamma_\mu \right.$$
$$\left. + \gamma_\mu \frac{1}{\not{p}_q - \not{q} - \not{k}} \gamma_\lambda \right). \tag{27}$$

The last expression is given in terms of the quark momentum p_q, which can be obtained from observable quantities by recalling the definition of x (in the scaling limit where masses can be ignored),

$$p_q = \frac{1}{x} P. \tag{28}$$

Note that $\tilde{q}^2 = (q + k)^2 = s$ is the square of the beam energy.

Inserting this result into Eq. (21) gives the formula for the cross-section asymmetry as

$$d\sigma(h) - d\sigma(\bar{h}) = \frac{d^3k \, d^3P}{k_0 P_0} \frac{\alpha^3}{2\pi^2 x s q^2}$$
$$\times \sum_q \frac{e_q^3}{e^3} [D_q^h(x) - D_{\bar{q}}^h(x)] |T_{\mathrm{int}}|^2, \tag{29}$$

with

$$|T_{\mathrm{int}}|^2 = \frac{L^{\mu\nu\lambda} M_{\mu\nu\lambda}}{s q^2}. \tag{30}$$

The complete formula for $|T_{\mathrm{int}}|^2$ is given in Appendix A, along with the corresponding formulas for the Bethe-Heitler and Compton cross sections, which do not contribute to the asymmetry.

The calculation for the asymmetry in $e^+ e^- \to \gamma \mu \bar{\mu}$ will be precisely the same if we substitute $p_\mu = p_q$. The sum over quarks is replaced by a single term for the muons and since the muon suffers no breakup the functions $D_q^h(x)$ are replaced by

$$D_\mu^{\mu^-}(x) = \delta(1 - x). \tag{31}$$

We let d^3P/P_0 become $E_\mu^2 x \, dx \, d\Omega_q$ so that

$$d\sigma(\mu^-) - d\sigma(\mu^+) = \frac{d^3k}{k_0} \, d\Omega_q \frac{\alpha^3}{2\pi^2 s q^2} E_\mu^2 |T_{\mathrm{int}}|^2, \tag{32}$$

and Eq. (6) for the ratio of hadronic to muonic asymmetry follows immediately.

Appendix A also gives complete formulas for the Bethe-Heitler and Compton contributions to the cross section. It is clearly of interest to know what fraction of the cross section is due to the asymmetry, and so for the leptons we have calculated the ratio

$$A = \frac{d\sigma(\mu^+) - d\sigma(\mu^-)}{d\sigma(\mu^+) + d\sigma(\mu^-)}. \tag{33}$$

This is plotted in Fig. 4 for values of the variables typical for SPEAR; it is seen that the asymmetry is close to 100% for a large range of angles near the forward directions.

The order of magnitude of the cross section for $e^+ e^- \to \mu^+ \mu^- \gamma$ can be estimated from the usual soft-photon formulas $(E_\gamma \ll \sqrt{s})$,
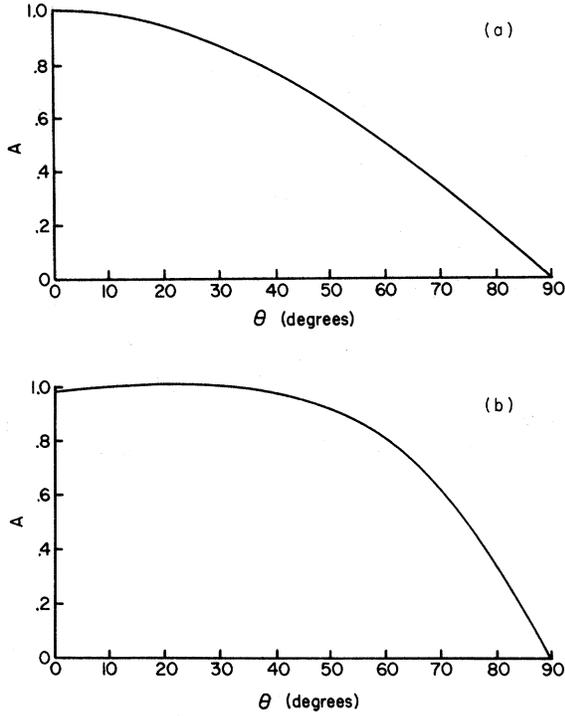
FIG. 4. The $\mu^+ - \mu^-$ asymmetry in $e^+e^- \to \gamma\mu^+\mu^-$ [Eq. (33)]. We have fixed $\sqrt{s} = 8$ GeV, $|\vec{k}| = 1$ GeV, and the photon direction perpendicular to the beams. The muon angles are measured relative to the incoming e^- direction and the $e^- - \gamma$ plane. In Fig. 4(a), the $e^- - \mu$ plane is perpendicular to the $e^- - \gamma$ plane ($\phi = 90°$); in Fig. 4(b), $\phi = 0°$.

$$\frac{d\sigma(\mu^+)}{d\Omega dE_\gamma / E_\gamma} - \frac{d\sigma(\mu^-)}{d\Omega dE_\gamma / E_\gamma}$$

$$= \frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-)\frac{8\alpha}{\pi}\ln\frac{1-\cos\theta}{1+\cos\theta},$$

where the angle $\theta \gg (m_\mu / E_\mu)^{1/2}$ is measured relative to the positron direction. Thus the signal is of order 1% and the μ^- is favored to emerge along the positron-beam direction. We shall give two examples of the actual cross section for a typical configuration at $\sqrt{s} = 8$ GeV, where the photon emerges at $90°$ to the beam axis with $E_\gamma = 1$ GeV ($q^2 = 48$ GeV²). For the large-asymmetry configuration with μ^+ along the e^- direction ($\theta_\mu \to 0$, $E_\mu = 3.43$ GeV),

$$d\sigma(\mu^+) = 2.1 \times 10^{-36} \text{ cm}^2 \frac{dE_\gamma}{E_\gamma} d\Omega_\gamma d\Omega_\mu ,$$

i.e.,

$$\frac{E_\gamma d\sigma(\mu^+)/dE_\gamma d\Omega_\gamma d\Omega_\mu}{d\sigma(e^+e^- \to \mu^+\mu^-)/d\Omega_\mu} = 0.013$$

and $d\sigma(\mu^-)$ is negligible. For the zero-asymmetry configuration $\theta_\mu = 90°$, $\phi_\mu = 90°$ (i.e., beam, photon, and muon mutually perpendicular, $E_\mu = 3.43$ GeV)

$$d\sigma(\mu^\pm) = 0.54 \times 10^{-36} \text{ cm}^2 \frac{dE_\gamma}{E_\gamma} d\Omega_\gamma d\Omega_\mu ,$$

i.e.,

$$\frac{E_\gamma d\sigma(\mu^\pm)/dE_\gamma d\Omega_\gamma d\Omega_\mu}{d\sigma(e^+e^- \to \mu^+\mu^-)/d\Omega_\mu} = 0.0067.$$

III. CONCLUSION

As we have shown in this paper, the hadron asymmetry in radiative e^+e^- annihilation, $e^+e^- \to \gamma hX$, can provide a sensitive test of the parton model and the electromagnetic interactions of the quark current. If the photon is not detected, the hadron asymmetry in $e^+e^- \to hX$ (from two-photon annihilation and radiative corrections) is small[12,13] [of order $8(\alpha/\pi)\ln(\tan\theta/2)$]. Also at SPEAR energies ($\sqrt{s} < 8$ GeV) weak-electromagnetic interference effects yield a small asymmetry (~1%).[13] However, once the hard photon is detected the electromagnetic asymmetry becomes maximal [~$O(\cos\theta)$].

In the scaling regime (large s with ratios of invariants fixed) the asymmetry is only due to the interference of the Compton and Bethe-Heitler diagrams and directly measures Compton scattering on a quark line. As discussed in Ref. 4, the existence of an elementary Compton current implies the existence of a fixed pole at $\alpha(t) = j = 0$ in the elastic Compton amplitude $\gamma h \to \gamma h$. The asymmetry thus provides a test of scaling and quark spin structure in a new timelike domain, determines the non-Pomeron part of the quark fragmentation distributions, and is sensitive to the cube of the quark charge. As we have shown, the parton-model formulas are particularly simple if the ratio $R_h^{(3)}(x)$ of hadron asymmetry in $e^+e^- \to \gamma hX$ to muon asymmetry in $e^+e^- \to \gamma\mu\bar{\mu}$ is measured. Various predictions based on fractional quark charge are given in Sec. I. In the case of the Han-Nambu model, the Compton amplitude receives extra contributions from the color-octet part of the photon, as discussed in Appendix B.

We have also proposed new sum rules based on an ansatz for quark-quantum-number conservation. The sum rule, Eq. (15), is particularly interesting, since it holds even for weak and electromagnetic decays, only long-lived charged particles need be detected, and particle identification is not required. The sum rule holds rigorously for heavy-lepton production.

The effects on the charge asymmetry as one

crosses new particle thresholds (charm, heavy leptons, color) should be especially illuminating. In any event, the charge asymmetry is important to study in order to understand the electromagnetic background to the exciting weak-interaction asymmetries which will be prominent at the next range $8 < \sqrt{s} < 40$ GeV of e^+e^- storage rings.

APPENDIX A: THE RADIATIVE ASYMMETRY CROSS SECTION

In this appendix we give the formulas for $e^+e^- \to \gamma + h + X$ for the case when the polarizations of the initial beams are not measured. However, the formulas as quoted in Sec. I still apply in general. The cross section is written

$$\frac{d\sigma(h)}{(d^3k/k)(d^3P/P_0)} = \frac{\alpha^3}{8\pi^2 x s q^2} \sum_{a=q,\bar{q}} D_a^h(x) \left(\frac{e_q^2}{e^2} |T_{BH}|^2 + 2\frac{e_q^3}{e^3} |T_{int}|^2 + \frac{e_q^4}{e^4} |T_c|^2\right) , \tag{A1}$$

where the expressions for $|T_i|^2$ will be given in terms of the invariants

$$p_q \cdot (p + p') = q^2 \tau,$$
$$p_q \cdot (p - p') = q^2 \tau',$$
$$k \cdot (p + p') = k_0 \sqrt{s} = q^2 \xi,$$
$$k \cdot (p - p') = q^2 \xi'. \tag{A2}$$

The $|T_i|$ are independent of x and are given by

$$|T_{BH}|^2 = \frac{4}{q^2} \{L_1^2(\xi' + \xi)[4\tau(\tau' - \tau) + 2(2\xi\tau + \xi'\tau - \xi\tau' - \tau' + 2\tau) - (\xi + \xi' + 1)]$$

$$+ 2L_1 L_2[+ 2(2\tau^2\xi + 2\tau\tau'\xi' + \tau^2 + \tau'^2) - 2(2\tau\xi^2 + \tau'\xi\xi' - \tau\xi'^2 + 3\tau\xi + \tau)$$

$$+ (2\xi + 1)(\xi + 1)] + L_2^2(\xi' - \xi)[4\tau(\tau' + \tau) + 2(-2\xi\tau + \xi'\tau - \xi\tau' - \tau' - 2\tau) + (\xi - \xi' + 1)]\}, \tag{A3}$$

$$|T_{int}|^2 = -\frac{2}{s} \{P_{11}[-4(\tau + \tau')(\tau - \tau')^2 + 4(3\tau^2\xi + \tau^2\xi' + 2\tau^2 - 2\tau\tau'\xi + \tau\tau'\xi' - \tau\tau' - \tau'^2\xi')$$

$$+ 2(-4\tau\xi^2 - 3\tau\xi\xi' - 5\tau\xi + \tau\xi'^2 - 2\tau\xi' + 2\tau + \tau'\xi^2 + \tau'\xi\xi' + \tau'\xi) + (2\xi^2 + 2\xi\xi' + 3\xi + \xi' + 1)]$$

$$+ P_{12}[-4(\tau + \tau')(\tau - \tau')^2 + 4(\tau^2 + \tau\tau'\xi' - \tau\tau' - \tau'^2\xi - 2\tau'^2\xi' - \tau'^2)$$

$$+ 2(2\tau\xi^2 + \tau\xi\xi' + 3\tau\xi + \tau\xi'^2 - \tau'\xi^2 - \tau'\xi\xi' - \tau'\xi - 2\tau'\xi'^2 - 2\tau'\xi')$$

$$+ (-4\xi^2 - 2\xi\xi' - 5\xi - 2\xi'^2 - \xi' - 1)]$$

$$+ P_{21}[4(\tau - \tau')(\tau + \tau')^2 + 4(-3\tau^2\xi + \tau^2\xi' - 2\tau^2 - 2\tau\tau'\xi - \tau\tau'\xi' - \tau\tau' - \tau'^2\xi')$$

$$+ 2(4\tau\xi^2 - 3\tau\xi\xi' + 5\tau\xi - \tau\xi'^2 - 2\tau\xi' + 2\tau + \tau'\xi^2 - \tau'\xi\xi' + \tau'\xi) + (-2\xi^2 + 2\xi\xi' - 3\xi + \xi' - 1)]$$

$$+ P_{22}[4(\tau - \tau')(\tau + \tau')^2 + 4(-\tau^2 - \tau\tau'\xi' - \tau\tau' + \tau'^2\xi - 2\tau'^2\xi' + \tau'^2)$$

$$+ 2(-2\tau\xi^2 + \tau\xi\xi' - 3\tau\xi - \tau\xi'^2 - \tau'\xi^2 + \tau'\xi\xi' - \tau'\xi - 2\tau'\xi'^2 + 2\tau'\xi')$$

$$+ (4\xi^2 - 2\xi\xi' + 5\xi + 2\xi'^2 - \xi' + 1)]\}, \tag{A4}$$

$$|T_C|^2 = \frac{8q^2}{s^2} \{D_1^2[2\tau(\tau'\xi' - \tau\xi) + (+2\tau\xi^2 - 2\tau'\xi\xi' + \tau\xi - \tau'\xi')]$$

$$+ D_1 D_2[2(2\tau\tau'\xi' + 2\tau'^2\xi + \tau'^2 + \tau^2) + 2(\tau'\xi\xi' + \tau\xi'^2 - \tau\xi - \tau) + (\xi + 1)(2\xi + 1)]$$

$$+ D_2^2[2\tau(\tau'\xi' - \tau\xi) + (2\tau\xi^2 + 2\tau\xi'^2 + 3\tau\xi - \tau'\xi') - (\xi^2 + \xi'^2 + \xi)]\} \tag{A5}$$

In the above equations

$$L_1 = -\frac{q^2}{2k \cdot p} = -\frac{1}{\xi + \xi'},$$

$$L_2 = -\frac{q^2}{-2k \cdot p'} = -\frac{1}{\xi - \xi'},$$

$$D_1 = \frac{q^2}{2k \cdot (q - p_q)} = \frac{1}{2\xi - 2\tau + 1}, \qquad \text{(A6)}$$

$$D_2 = \frac{q^2}{2k \cdot p_q} = \frac{1}{2\tau - 1},$$

$$P_{ij} = L_i D_j.$$

These expressions have been obtained with the help of the REDUCE program of Hearn.[14] They can be checked against similar expressions obtained in Ref. 4 by crossing. The rules that should be used are

1. $x \to 1$,
2. Change the overall sign,
3. $\alpha \to \tau + \tau'$, $\alpha' \to \tau' - \tau$, $\beta \to -(\xi + \xi')$, $\beta' \to \xi - \xi'$, $Q^2 \to -q^2$, where α, α', β, β', and Q^2 are defined in Ref. 4.

The hard-photon case of $e^- + e^+ \to \mu^+ + \mu^- + \gamma$ can be obtained from our formulas by replacing the structure function $D_q^h(x)$ by $D_\mu^\mu(x) = \delta(1 - x)$. The formulas we obtain this way agree with formulas given by Berends, Gaemers, and Gastmans.[15] In this limit, our $|T|^2$'s are related to their F_{ij} by

$$|T_{\text{BH}}|^2 = -4m^2\mu^2 \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{F_{ij}}{D_i D_j},$$

$$|T_{\text{int}}|^2 = -4m^2\mu^2 \sum_{i=1}^{2} \sum_{j=3}^{4} \frac{F_{ij}}{D_i D_j},$$

$$|T_C|^2 = -4m^2\mu^2 \sum_{i=3}^{4} \sum_{j=3}^{4} \frac{F_{ij}}{D_i D_j},$$

where the symbols on the right-hand sides are defined in their paper.[15]

APPENDIX B: COMPARISON OF FRACTIONAL- AND INTEGRAL-CHARGED QUARK MODELS

The ratio $R_H^{(3)}(x)/R_H^{(2)}(x)$ and the sum rule, Eq. (15), are sensitive discriminants of fractional- and integral-charged quark-parton models. In the Han-Nambu model with SU(3) color[16] the electromagnetic current has the form

$$J_{\text{em}} \equiv \bar{u}_R u_R + \bar{u}_B u_B - \bar{d}_Y d_Y - \bar{s}_Y s_Y$$

$$= J_{8,1_C} - J_{1,8_C},$$

where

$$J_{8,1_C} = \sum_{C=R,Y,B} \tfrac{2}{3}\bar{u}_C u_C - \tfrac{1}{3}\bar{d}_C d_C - \tfrac{1}{3}\bar{s}_C s_C$$

and

$$J_{1,8_C} = \sum_{q=u,d,s} \tfrac{2}{3}\bar{q}_Y q_Y - \tfrac{1}{3}\bar{q}_B q_B - \tfrac{1}{3}\bar{q}_R q_R.$$

For matrix elements between color singlets only $J_{8,1_C}$ is effective and the results are indistinguishable from the usual fractional-charge quark model with three colors. Additional quark flavors can be readily included; e.g., the charm quark enters in the same way as the u quark, adding $\bar{c}_R c_R + \bar{c}_B c_B$ to J_{em}.

In the $e^+e^- \to \gamma HX$ asymmetry we require the Compton amplitude involving a single quark line. In the scaling region the short-distant real part of the amplitude is required, and virtual intermediate color states will contribute in the Han-Nambu model. We note that

$$\langle 1_C | J_{\text{em}}^2 | 1_C \rangle = \langle 1_C | J_{1,8_C}^2 + J_{8,1_C}^2 | 1_C \rangle$$

$$= \sum_C \tfrac{2}{3}\bar{u}_C u_C + \tfrac{1}{3}\bar{d}_C d_C + \tfrac{1}{3}\bar{s}_C s_C.$$

The sum rule for $R_H^{(3)}(x)$ requires the matrix element

$$\langle 0 | J_{\text{em}}^2 | n \rangle \langle n | J_{\text{em}} | 0 \rangle.$$

We then have

$$S_3 = \sum_H \lambda_H \int_0^1 dx \, R_H^{(3)}(x)$$

$$= \begin{cases} 3(\tfrac{8}{27}\lambda_u - \tfrac{1}{27}\lambda_d - \tfrac{1}{27}\lambda_s) \ \text{(fractional charge)}, \\ 3(\tfrac{4}{9}\lambda_u - \tfrac{1}{9}\lambda_d - \tfrac{1}{9}\lambda_s) \ \text{(Han-Nambu below color} \\ \qquad\qquad\qquad\qquad\qquad \text{production)}, \\ 2\lambda_u - \lambda_d - \lambda_s \ \text{(Han-Nambu above color} \\ \qquad\qquad\qquad\qquad\qquad \text{production)}, \end{cases}$$

which for $\lambda_H = Q_H$ gives $S_3 = \tfrac{2}{3}$, 2, and 4, respectively. Including the charm quark this gives $S_3 = \tfrac{34}{27}$, $3\tfrac{1}{3}$, and 6, respectively. For comparison

$$R = \sigma_{e^+e^- \to \text{had}} / \sigma_{e^+e^- \to \mu^+\mu^-} = 2, \ 2, \ \text{and} \ 4,$$

respectively, below charm threshold, and $3\tfrac{1}{3}$, $3\tfrac{1}{3}$, and 6, respectively, above charm threshold. In addition, the various gauge theories proposed to unify strong, weak, and electromagnetic interactions[17,18] each make a prediction for the value of this sum rule, which then becomes one test of that theory. In particular, a model studied by Pati and Salam[17] would give the same result as the standard fractional charge model in one variant, and the same result as the Han-Nambu model in another variant (at least until we reach the super-high energies needed to test their so-called 'prodigal model").

We also have

$$R_H^{(3)}(x) = \begin{cases} 3[\frac{8}{27}(D_{H/u} - D_{H/\bar{u}}) - \frac{1}{27}(D_{H/d} - D_{H/\bar{d}}) - \frac{1}{27}(D_{H/s} - D_{H/\bar{s}})](\text{fractional charge}) \\ 3[\frac{4}{9}(D_{H/u} - D_{H/\bar{u}}) - \frac{1}{9}(D_{H/d} - D_{H/\bar{d}}) - \frac{1}{9}(D_{H/s} - D_{H/\bar{s}})](\text{Han-Nambu below color production}) \\ 2(D_{H/u} - D_{H/\bar{u}}) - (D_{H/d} - D_{H/\bar{d}}) - (D_{H/s} - D_{H/\bar{s}}) \ (\text{Han-Nambu above color production}) \end{cases}$$

to be compared with

$$R_H^{(2)}(x) = \begin{cases} 3[\frac{4}{9}(D_{H/u} + D_{H/\bar{u}}) + \frac{1}{9}(D_{H/d} + D_{H/\bar{d}}) + \frac{1}{9}(D_{H/s} + D_{H/\bar{s}})] \ (\text{fractional charge or Han-Nambu below} \\ \hspace{9cm} \text{color production}) \\ 2(D_{H/u} + D_{H/\bar{u}}) + (D_{H/d} + D_{H/\bar{d}}) + (D_{H/s} + D_{H/\bar{s}}) \ (\text{Han-Nambu above color production}) \end{cases}$$

The ratio $R_H^{(3)}$ to $R_H^{(2)}$ in the valence region, which is fractional in the standard quark model, is integral for $H = \pi, K$ in the Han-Nambu model whether above or below color-production threshold.

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