

## Asymmetry in $e + \bar{e} \rightarrow K^+ + K^-$ and $\eta \rightarrow \pi^0 + \gamma + \gamma$

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The final-state interaction for the off-shell-photon process  $\gamma + \gamma \rightarrow K^+ + K^-$  is discussed in terms of an "intermediate representation" where one of the photons is on shell. This allows the evaluation of the absorptive part and the soft-photon contribution for  $e + \bar{e} \rightarrow K^+ + K^-$  from the two-photon intermediate state. The asymmetry due to the interference between the one-photon and the two-photon amplitudes for this process, near the  $f'$ -resonance position, is found to be quite large, but goes through zero. Generalization of the analysis of the final-state interaction in the  $s$ -wave state to SU(3) allows us to predict a decay width  $\Gamma(\eta \rightarrow \pi^0 + \gamma + \gamma) \approx 0.05$  keV, which is in good agreement with the experimental number.

### I. INTRODUCTION

The analysis of  $e\bar{e}$  annihilation is important for testing the validity of quantum electrodynamics as well as for the observation of small but significant effects due to weak neutral currents<sup>1-4</sup> and higher-order radiative corrections.<sup>2-7</sup> In particular, the asymmetry<sup>3-7</sup> resulting from the interference between the charge-conjugation  $C = -1$  hadron states produced by the one-photon intermediate state and the  $C = +1$  hadron states produced by the two-photon intermediate state is of considerable interest because of its enhancement due to

- (i) the infrared effects and
- (ii) the final-state interaction near the  $C = +1$  positions.

These two effects may produce a large asymmetry, of the order of 20%, near the  $C = +1$  resonance positions, in contrast to the generally expected asymmetry of the order of the fine-structure constant  $\alpha$ , i.e., about 1%.

In a recent note,<sup>7</sup> which we will denote as (I), we discussed the results of an analysis of the asymmetry due to the interference between one-photon and two-photon amplitudes for

$$e + \bar{e} \rightarrow \pi^+ + \pi^-, \quad (1)$$

$$e + \bar{e} \rightarrow \pi^+ + \pi^- + \pi^0 \quad (2)$$

in the region of the  $C = +1$  resonances with  $J = 0, 1, 2$ . There we found that the asymmetry near the  $f$ -meson resonance position for process (1), and near the  $A_1, A_2$  meson resonance positions for process (2), is about 15-20%, but goes through zero. These are, indeed, large effects and should be easily observed experimentally.

In the first part of our present note, we analyze the asymmetry due to the interference between one-photon and two-photon amplitudes for the process

$$e(p_1) + \bar{e}(p_2) \rightarrow K^+(k_1) + K^-(k_2) \quad (3)$$

in the region of the  $f'$ -meson resonance position. Though the techniques we have used for the evaluation of the two-photon amplitude here are similar to those used in (I) for processes (1) and (2), there are some major differences and improvements. These are as follows:

- (i) The  $K$ -meson mass is not negligible, unlike the  $\pi$ -meson mass which is quite small and could be neglected at the energies under consideration.
- (ii) In (I), the final-state interaction for the soft-photon amplitude was introduced for processes (1) and (2) in analogy with the treatment for the on-shell photon amplitude. While this is plausible, it does not follow logically, and requires a detailed treatment in terms of the off-shell invariant amplitudes for a rigorous justification. Here we have analyzed the final-state interaction in terms of an intermediate representation for invariant amplitudes, with only one photon off-shell. Use of such a representation is valid for the discussion of both the absorptive part of the amplitude as well as the soft-photon contribution to the amplitude.

(iii) We have calculated the radiative corrections leading to the  $K^+K^-$  state with charge conjugation  $C = +1$  as well as  $C = -1$ . However, the radiative corrections leading to the hadron state with  $C = -1$ , which were neglected in (I), are calculated only in the soft-photon limit. Any improvement over this would require a very sophisticated treatment of the strong interaction.

The main result of the analysis is that the asymmetry in the forward and backward events for process (3), near the  $f'$ -meson resonance position, is of the order of 35%, but goes to zero within about half a width from the resonance.

In the second part of our note, we consider the decay rate for the process

$$\eta \rightarrow \pi^0 + \gamma + \gamma. \quad (4)$$

In some of the earlier attempts<sup>8</sup> to evaluate this decay, one used either the vector-dominance mod-

el or an effective Hamiltonian. These models lead to a rather small rate for this decay, though the latter model, which is cutoff-dependent, can give a larger rate. Here we assume that the two photons are in an  $s$ -wave state. We relate the amplitude for process (4) to that for  $\gamma + \gamma \rightarrow K^+ + K^-$  in the  $I = 1$  state by crossing and  $SU(3)$  symmetry, which is then evaluated by saturating it with the  $\delta$  meson. We predict a partial width

$$\Gamma(\eta \rightarrow \pi^0 + \gamma + \gamma) = 4.8 \times 10^{-2} \text{ keV}, \quad (5)$$

which is in good agreement with the average experimental value<sup>9</sup> of  $8.15 \times 10^{-2}$  keV, though our prediction tends to support some of the larger<sup>10</sup> experimental values.

## II. ASYMMETRY IN $e(p_1) + \bar{e}(p_2) \rightarrow K^+(k_1) + K^-(k_2)$

With the assumption of vector-meson dominance and  $\omega$ - $\phi$  mixing, the one-photon amplitude for process (3) is

$$T_1 = e^2 \frac{g(s)}{s} \bar{v}(p_2)(\not{k}_1 - \not{k}_2)u(p_1), \quad (6)$$

with

$$g(s) = \frac{1}{2} \left( \frac{m_\rho^2}{m_\rho^2 - s - i\Gamma_\rho m_\rho} + \frac{m_\phi^2 \cos^2 \theta_m}{m_\phi^2 - s - i\Gamma_\phi m_\phi} + \frac{m_\omega^2 \sin^2 \theta_m}{m_\omega^2 - s - i\Gamma_\omega m_\omega} \right), \quad (7)$$

where  $s = p^2$ ,  $p = (p_1 + p_2)$ ,  $m_\rho$ ,  $m_\phi$ , and  $m_\omega$  are the masses of the  $\rho$ ,  $\phi$ , and  $\omega$  mesons, respectively,  $\Gamma_\rho$ ,  $\Gamma_\phi$ , and  $\Gamma_\omega$  are the corresponding widths, and  $\theta_m$  is the  $\omega$ - $\phi$  mixing angle. The region of major interest for the analysis of the asymmetry in this process is around the position of the  $f'$  resonance with  $I = 0$  and  $J = 2$ , which is produced through the two-photon intermediate state. Therefore, for the

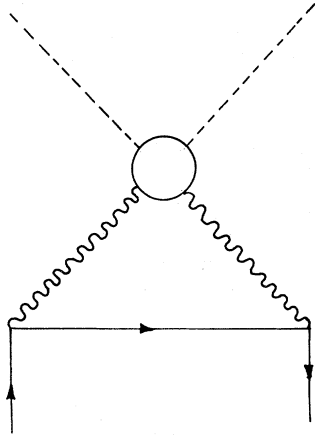


FIG. 1. Diagram for the two-photon amplitude for the process  $e\bar{e} \rightarrow K^+K^-$ .

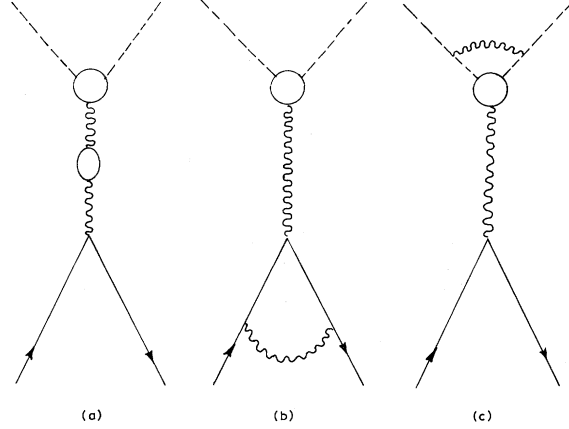


FIG. 2. The diagrams for (a) vacuum polarization, and (b), (c) vertex corrections for the process  $e\bar{e} \rightarrow K^+K^-$ .

evaluation of this asymmetry, we will need to calculate

(i) the two-photon amplitude corresponding to Fig. 1, including the  $I = 0$ ,  $J = 2$  term for the  $K^+K^-$  state with the final-state interaction, and the background amplitude,

(ii) vacuum polarization and vertex corrections due to the electromagnetic interaction corresponding to Fig. 2, and

(iii) the bremsstrahlung contributions, corresponding to process (3), accompanied by the emission of a soft photon, as shown in Fig. 3. In the soft-photon approximation, diagrams with the  $\pi\pi\gamma\gamma$  point interaction do not contribute to (ii) or (iii).

We begin with a calculation of the two-photon amplitude. In the following calculations, the electron mass is taken to be small compared to  $s^{1/2}$ .

*Two-photon amplitude.* The two-photon amplitude corresponding to Fig. 1 is

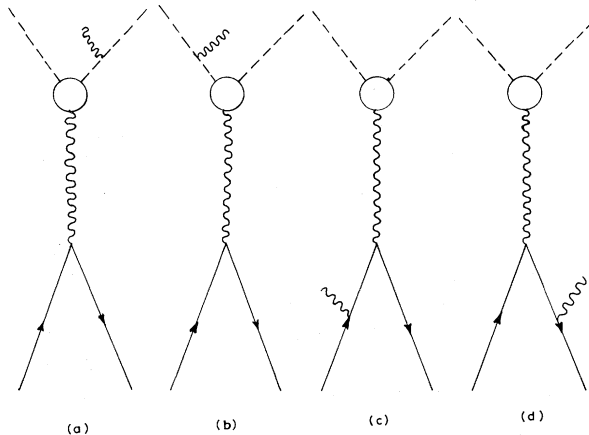


FIG. 3. The diagrams for the bremsstrahlung processes associated with  $e\bar{e} \rightarrow K^+K^-$ .

$$T_2 = -\frac{ie^2}{(2\pi)^4} \int \frac{d^4 q_1}{q_1} \frac{d^4 q_2}{q_2} \delta^4(q_1 + q_2 - p) \\ \times \bar{v}(p_2) \gamma_\nu \frac{1}{\not{p}_1 - \not{q}_1 - m} \gamma_\mu u(p_1) B_{\mu\nu}, \quad (8)$$

where  $m$  is the electron mass and  $B_{\mu\nu}$  describes the off-shell photon process

$$\gamma_\mu(q_1) + \gamma_\nu(q_2) \rightarrow K^+(k_1) + K^-(k_2). \quad (9)$$

The general structure of  $B_{\mu\nu}$  is quite formidable with five invariant amplitudes.<sup>4</sup> However, there are two parts of  $T_2$  which are important for the discussion of asymmetry in  $K^+K^-$  production for process (3). One is the absorptive part of  $T_2$ , resulting from on-shell photons, which gives the dominant contribution to asymmetry at the  $C=+1$  resonance positions. The other is the soft-photon contribution, which is important because of the enhancement due to infrared effects. Both of these contributions can be evaluated in terms of what we call an "intermediate representation" of  $B_{\mu\nu}$ , where one of the two photons is on its mass shell, say  $q_1^2=0$ . This representation has the form

$$B_{\mu\nu} = \sum_i (t_i)_{\mu\nu} B_i, \\ (t_1)_{\mu\nu} = q_1 \cdot q_2 g_{\mu\nu} - q_{2\mu} q_{1\nu}, \\ (t_2)_{\mu\nu} = (q_2 \cdot Q)^2 g_{\mu\nu} - q_1 \cdot q_2 Q_\mu Q_\nu \\ + q_2 \cdot Q Q_\mu q_{1\nu} - q_2 \cdot Q q_{2\mu} Q_\nu, \\ (t_3)_{\mu\nu} = q_1 \cdot q_2 Q_\mu q_{2\nu} - q_1 \cdot Q q_{2\mu} q_{2\nu} \\ + q_2^2 q_1 \cdot Q g_{\mu\nu} - q_2^2 Q_\mu q_{1\nu}, \quad (10)$$

$$B_{\mu\nu}^B = e^2 g(q_2^2) \left[ \frac{(2k_1 - q_1)_\mu (-2k_2 + q_2)_\nu}{-2k_1 \cdot q_1} + \frac{(2k_2 - q_1)_\mu (-2k_1 + q_2)_\nu}{-2k_2 \cdot q_1} - 2g_{\mu\nu} \right], \quad (13)$$

and the corresponding helicity amplitudes are

$$T_{1,1}^B = -e^2 g(q_2^2) \left[ |\vec{k}_1|^2 (\sin^2 \theta') \left( \frac{1}{q_1 \cdot k_1} + \frac{1}{q_1 \cdot k_2} \right) - 2 \right], \\ T_{1,-1}^B = e^2 g(q_2^2) \left[ |\vec{k}_1|^2 (\sin^2 \theta') \left( \frac{1}{q_1 \cdot k_1} + \frac{1}{q_1 \cdot k_2} \right) \right], \quad (14) \\ T_{1,0}^B = e^2 g(q_2^2) \left[ \left( \frac{q_2^2}{2} \right)^{1/2} |\vec{k}_1| (\sin \theta') \left( \frac{1}{q_1 \cdot k_1} - \frac{1}{q_1 \cdot k_2} \right) \right].$$

For incorporating the effect of the  $f'$  meson, one first obtains the  $J=2$  projections of these amplitudes:

where  $Q = \frac{1}{2}(k_1 - k_2)$  and  $B_1$ ,  $B_2$ , and  $B_3$  are the invariant amplitudes. The helicity amplitudes<sup>11</sup> are related to these invariant amplitudes by

$$T_{1,1} = -q_1 \cdot q_2 B_1 - \frac{1}{2} |\vec{k}_1|^2 (q_1 \cdot q_2 \sin^2 \theta' + 2 |\vec{q}_1|^2 \cos^2 \theta') B_2 \\ + q_2^2 |\vec{q}_1| |\vec{k}_1| (\cos \theta') B_3,$$

$$T_{1,-1} = \frac{1}{2} q_1 \cdot q_2 |\vec{k}_1|^2 (\sin^2 \theta') B_2, \quad (11)$$

$$T_{1,0} = \left( \frac{q_2^2}{2} \right)^{1/2} [ |\vec{q}_1|^2 |\vec{k}_1|^2 (\sin \theta') (\cos \theta') B_2 \\ + s^{1/2} |\vec{q}_1| |\vec{k}_1| (\sin \theta') B_3 ],$$

where the subscripts of  $T$  describe the helicities of  $\gamma_\mu(q_1)$  and  $\gamma_\nu(q_2)$ ,  $|\vec{k}_1| = \frac{1}{2}(s - 4\mu^2)^{1/2}$ ,  $q_{20} = (s + q_2^2)/2s^{1/2}$ ,  $|\vec{q}_1| = |\vec{q}_2| = (s - q_2^2)/2s^{1/2}$ ,  $\mu$  is the kaon mass, and  $\theta'$  is the scattering angle in the center-of-mass (c.m.) frame. Here we have used the representations

$$\epsilon_{+1}(q_1) \equiv \left( 0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0 \right), \\ \epsilon_{-1}(q_1) \equiv \left( 0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0 \right), \\ \epsilon_{\pm 1}(q_2) = -\epsilon_{\mp 1}(q_1) \quad (12) \\ \epsilon_0(q_2) = (q_2^2)^{-1/2} (-|\vec{q}_2|, 0, 0, q_{20})$$

for the photon polarization vectors, where the subscripts denote the helicities.

The Born approximation for  $B_{\mu\nu}$ , due to the kaon exchange diagrams and the sea-gull diagram, which we designate by  $B_{\mu\nu}^B$ , is

$$\begin{aligned}
T_{1,1}^B(J=2) &= \frac{5}{8}(3 \cos^2 \theta' - 1) \int_{-1}^1 d \cos \theta (3 \cos^2 \theta - 1) T_{1,1}^B, \\
T_{1,-1}^B(J=2) &= \frac{15}{16} \sin^2 \theta' \int_{-1}^1 d \cos \theta (\sin^2 \theta) T_{1,-1}^B, \\
T_{1,0}^B(J=2) &= \frac{15}{4} \sin \theta' \cos \theta' \int_{-1}^1 d \cos \theta (\sin \theta \cos \theta) T_{1,0}^B.
\end{aligned} \tag{15}$$

These integrals are carried out, and then, using relations (11), one obtains the corresponding invariant amplitudes  $B_1$ ,  $B_2$ , and  $B_3$ . We define

$$\begin{aligned}
B_1(J=2) &\equiv a_0(s) + a_1(s) \cos^2 \theta', \\
B_2(J=2) &\equiv a_2(s), \\
B_3(J=2) &\equiv a_3(s) \cos \theta',
\end{aligned} \tag{16}$$

for which the Born approximations for  $a_0, a_1, a_2, a_3$  are

$$\begin{aligned}
a_0^B(s) &= -\frac{15e^2 g(q_2^2) s}{4(s - q_2^2)^2} \left[ \frac{10}{3} + \frac{(4\mu^2 - s/2)}{|\mathbf{k}_1|^2} - \frac{2\mu^2(\mu^2 + 4|\mathbf{k}_1|^2/3)}{s^{1/2}|\mathbf{k}_1|^3} \ln \frac{s^{1/2} + 2|\mathbf{k}_1|}{s^{1/2} - 2|\mathbf{k}_1|} \right], \\
a_1^B(s) &= \frac{30e^2 g(q_2^2)}{(s - q_2^2)^2} \left[ -\frac{7}{4} q_2^2 + \frac{s(\frac{9}{8}q_2^2 + 3\mu^2)}{2|\mathbf{k}_1|^2} - \frac{(\frac{1}{2}\mu^2 q_2^2 + sq_2^2 + 2s|\mathbf{k}_1|^2 + 3s\mu^2)}{2s^{1/2}|\mathbf{k}_1|^3} \mu^2 \ln \frac{s^{1/2} + 2|\mathbf{k}_1|}{s^{1/2} - 2|\mathbf{k}_1|} \right], \\
a_2^B(s) &= \frac{30e^2 g(q_2^2) s}{(s - q_2^2)^2 (s - 4\mu^2)} \left[ \frac{10}{3} - \frac{s}{2|\mathbf{k}_1|^2} + \frac{2\mu^4}{s^{1/2}|\mathbf{k}_1|^3} \ln \frac{s^{1/2} + 2|\mathbf{k}_1|}{s^{1/2} - 2|\mathbf{k}_1|} \right], \\
a_3^B(s) &= \frac{30e^2 g(q_2^2) s^{1/2}}{(s - q_2^2)^2 |\mathbf{k}_1|} \left[ -\frac{13}{8} + \frac{5s}{8|\mathbf{k}_1|^2} - \frac{\mu^2(s + \mu^2)}{2s^{1/2}|\mathbf{k}_1|^3} \ln \frac{s^{1/2} + 2|\mathbf{k}_1|}{s^{1/2} - 2|\mathbf{k}_1|} \right].
\end{aligned} \tag{17}$$

The analyticity properties of these Born terms indicate that the amplitudes which are free from kinematic singularities, and which are suitable for writing partial-wave dispersion relations, are

$$\begin{aligned}
\bar{a}_0(s) &= \frac{(s - q_2^2)^2}{s|\mathbf{k}_1|^2} a_0(s), \\
\bar{a}_1(s) &= \frac{(s - q_2^2)^2}{|\mathbf{k}_1|^2} a_1(s), \\
\bar{a}_2(s) &= \frac{(s - q_2^2)^2}{s} a_2(s), \\
\bar{a}_3(s) &= \frac{(s - q_2^2)^2}{|\mathbf{k}_1|s^{1/2}} a_3(s).
\end{aligned} \tag{18}$$

The Born approximations for these amplitudes go as  $1/s$  for  $s \rightarrow \infty$  and do not contain undesirable threshold factors.

The introduction of the final-state interaction in the  $I=0, J=2$  state is considered through two approaches. In the approach which is discussed here, one writes dispersion relations for the  $S$ -matrix elements and uses the  $N/D$  representation for a suitable partial-wave scattering amplitude for  $K\bar{K}$  scattering. In the other approach, which is discussed in the Appendix, we analyze the solutions

for the amplitude for  $K\bar{K} \rightarrow \gamma\gamma$  scattering with the  $K\bar{K}$  intermediate state, in the Logunov-Tavkhelidze equation.<sup>12</sup> The effects of the final-state interaction in both these approaches are rather similar though there are some relatively minor differences between them.

In the  $S$ -matrix approach, one writes<sup>13</sup> dispersion relations for  $[\bar{a}_i(s)]^{I=0} D_{K\bar{K}}(s)$  where  $[\bar{a}_i(s)]^{I=0}$  are the  $I=0$  projections of  $\bar{a}_i(s)$  and  $D_{K\bar{K}}(s)$  is the  $D$  function for the  $I=0, J=2$  partial-wave  $KK$  scattering amplitude. Approximating the left-hand discontinuity by its Born approximation, we get

$$[\bar{a}_i(s)]^{I=0} D_{K\bar{K}}(s) = \frac{1}{\pi} \int_{-\infty}^0 \frac{D_{K\bar{K}}(s') \text{Im}[\bar{a}_i^B(s')]^{I=0}}{s' - s} ds'. \tag{19}$$

Now since the  $\bar{a}_i^B(s')$  are quite steep near  $s' \approx 0$ , we may set  $D_{K\bar{K}}(s') \approx D_{K\bar{K}}(0)$  and obtain

$$[\bar{a}_i(s)]^{I=0} \approx \frac{D_{K\bar{K}}(0)}{D_{K\bar{K}}(s)} [\bar{a}_i^B(s)]^{I=0}. \tag{20}$$

On saturating the  $I=0, J=2$  partial-wave  $K\bar{K}$  scattering amplitude by the  $f'$ -meson resonance, these expressions lead to relations between  $[B_i(J=2)]^{I=0}$  and their Born approximations  $[B_i^B(J=2)]^{I=0}$ ,

$$[B_i(J=2)]^{I=0} = \left( \frac{m_{f'}^2}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} \right) [B_i^B(J=2)]^{I=0}, \quad (21)$$

where  $m_{f'}$  is the mass of the  $f'$ -meson resonance and  $\Gamma_{f'}$  is its width. We assume that the final-state interaction in other states, i.e.,  $J=2$ ,  $I=1$ , and  $J \neq 2$ ,  $I=0$  or  $1$ , is negligible, so that the amplitudes in those states may be represented by their Born approximations. We then have for process (9)

$$B_{\mu\nu} = B_{\mu\nu}^B + \frac{1}{2} \left( \frac{m_{f'}^2}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} - 1 \right) \times \sum_i (t_i)_{\mu\nu} B_i^B(J=2), \quad (22)$$

where the  $(t_i)_{\mu\nu}$  are given in (10),  $B_{\mu\nu}^B$  is given by (13), and  $B_i^B(J=2)$  are obtained from (16) using the Born approximations (17). This  $B_{\mu\nu}$  may be substituted in (8) and the integrals carried out. In the integrations, we will be primarily interested in the contributions to the integrals from two regions which are important for the discussion of asymmetry in process (3). These are the absorptive part and the soft-photon contribution.

*Absorptive part.* The absorptive part of the two-photon amplitude  $T_2$  is generally smaller than the soft-photon contribution to (8), but gives the dominant interference term at the resonance position  $s \approx m_{f'}^2$ , since the resonant soft-photon term is out of phase with the one-photon term (6) at  $s \approx m_{f'}^2$ . The resonant absorptive term is obtained by using Cutkosky rules<sup>14</sup>

$$\text{abs } T_2 = \frac{e^2}{8\pi^2} \int d^4 q_1 d^4 q_2 \delta^4(p - q_1 - q_2) \delta(q_1^2) \theta(q_{10}) \delta(q_2^2) \theta(q_{20}) \times \bar{v}(p_2) \gamma_\nu \frac{1}{\not{p}_1 - \not{q}_1 - m} \gamma_\mu u(p_1) \frac{m_{f'}^2}{2(m_{f'}^2 - s - im_{f'}\Gamma_{f'})} [(t_1)_{\mu\nu} B_1^B(J=2) + (t_2)_{\mu\nu} B_2^B(J=2)], \quad (23)$$

where  $B_i^B(J=1)$  are obtained from (16) using the Born approximations (17). Finally, carrying out the integrals, we get

$$\text{abs } T_2 \approx e^2 \bar{v}(p_2) (\not{k}_1 - \not{k}_2) u(p_1) \frac{1}{s} \left( -i \frac{0.6 \alpha m_{f'}^2 \cos \theta}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} \right) \quad (24)$$

for  $s \approx m_{f'}^2$ , where  $\alpha$  is the fine-structure constant, and  $\theta$  is the scattering angle in the c.m. frame for process (3).

*Soft-photon contribution.* The leading contribution in the soft-photon limit is given by

$$(T_2)_{\text{sp}} \approx \frac{2ie^2}{(2\pi)^4 s} \int \frac{d^4 q_1}{q_1^2 (2\hat{p}_1 \cdot q_1)} \bar{v}(p_2) \gamma_\nu \not{p}_1 \gamma_\mu u(p_1) \left[ B_{\mu\nu}^B + \frac{1}{2} \left( \frac{m_{f'}^2}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} - 1 \right) \sum_i (t_i)_{\mu\nu} B_i^B(J=2) \right], \quad (25)$$

where sp stands for soft-photon contribution. The evaluation of the integrals gives the result

$$(T_2)_{\text{sp}} \approx e^2 \frac{g(s)}{s} \bar{v}(p_2) (\not{k}_1 - \not{k}_2) u(p_1) \left( \frac{2\alpha}{\pi} \ln \frac{s^{1/2}}{m\gamma} \right) \times \left[ \ln \frac{s^{1/2} + 2|\vec{k}_1| \cos \theta}{s^{1/2} - 2|\vec{k}_1| \cos \theta} + 0.9 \cos \theta \left( \frac{m_{f'}^2}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} - 1 \right) \right], \quad (26)$$

where  $m\gamma$  is a small mass assigned to the photon.

*Vacuum polarization and vertex corrections.* For vacuum polarization, we include only the electron-loop diagram [Fig. 2(a)] which gives<sup>2</sup>

$$T_p = \frac{\alpha}{\pi} \left( -\frac{5}{9} + \frac{2}{3} \ln \frac{s^{1/2}}{m} \right) T_1. \quad (27)$$

The electron vertex correction [Fig. 2(b)] has been calculated before and is<sup>2</sup>

$$T_V(ee) = \frac{\alpha}{\pi} \left[ -1 + \frac{\pi^2}{3} + \frac{2}{3} \ln \frac{s^{1/2}}{m} - \left( \ln \frac{s^{1/2}}{m} \right)^2 + \left( 1 - 2 \ln \frac{s^{1/2}}{m} \right) \ln \left( \frac{m}{m\gamma} \right) \right] T_1. \quad (28)$$

The kaon vertex correction is complicated by the strong interaction. However, in the soft-photon limit, the  $K\bar{K}$  intermediate state dominates. The amplitude corresponding to Fig. 2(c) is

$$T_V(KK) = \frac{\alpha}{2\pi} \left[ \left( \ln \frac{2\mu}{m\gamma} - 1 \right) I_a + I_b \right] T_1, \quad (29)$$

where

$$I_a = 2 \left\{ 1 - \frac{2\mu^2 - s}{[s(s - 4\mu^2)]^{1/2}} \ln \frac{s^{1/2} - (s - 4\mu^2)^{1/2}}{s^{1/2} + (s - 4\mu^2)^{1/2}} \right\},$$

$$I_b = \int_0^1 \frac{dx(1+x^2)\ln[x^2/(1-x^2)]}{x^2 - [(s - 4\mu^2)/s]}.$$

In the soft-photon limit, this gives

$$T_V(KK) \xrightarrow{m\gamma \rightarrow 0} \frac{\alpha}{\pi} \left( \ln \frac{2\mu}{m\gamma} \right) \left\{ 1 + \frac{s - 2\mu^2}{[s(s - 4\mu^2)]^{1/2}} \ln \frac{s^{1/2} - (s - 4\mu^2)^{1/2}}{s^{1/2} + (s - 4\mu^2)^{1/2}} \right\} T_1. \quad (30)$$

*The asymmetry.* The differential cross section to process (3), along with the bremsstrahlung corrections (Fig. 3), is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma_0}{d\cos\theta} \left[ \frac{1}{|T_1|^2} |T_1 + \text{abs}T_2 + (T_2)_{\text{sp}} + T_p + T_V(ee) + T_V(KK)|^2 + \delta^B(ee) + \delta^B(KK) + \delta^B(eK) \right], \quad (31)$$

where

$$\frac{d\sigma_0}{d\cos\theta} = \frac{\pi\alpha^2 \sin^2\theta}{4s} |g(s)|^2 \left( \frac{s - 4\mu^2}{s} \right)^{3/2}, \quad (32)$$

and where  $\delta^B$  are the corrections from the bremsstrahlung processes. In the soft-photon approximation, these terms are

$$\delta^B(ee) = \frac{2\alpha}{\pi} \left[ \left( \ln \frac{s^{1/2}}{m} \right)^2 + \left( 1 - 2 \ln \frac{s^{1/2}}{m} \right) \ln \left( \frac{s^{1/2} m \gamma}{4m \Delta E} \right) \right], \quad (33)$$

as given in Ref. 1, where  $\Delta E$  is the maximum energy of the soft photon emitted, and

$$\delta^B(KK) = \frac{2\alpha}{\pi} \left( \ln \frac{m\gamma}{\Delta E} \right) \left\{ 1 + \frac{s - 2\mu^2}{[s(s - 4\mu^2)]^{1/2}} \ln \frac{s^{1/2} - (s - 4\mu^2)^{1/2}}{s^{1/2} + (s - 4\mu^2)^{1/2}} \right\}, \quad (34)$$

$$\delta^B(eK) = \frac{4\alpha}{\pi} \left( \ln \frac{m\gamma}{\Delta E} \right) \text{Re} \left[ \ln \frac{s^{1/2} + 2|\vec{k}_1| \cos\theta}{s^{1/2} - 2|\vec{k}_1| \cos\theta} + 0.9 \cos\theta \left( \frac{m_{f'}^2}{m_{f'}^2 - s - i m_{f'} \Gamma_{f'}} - 1 \right) \right]. \quad (35)$$

The asymmetry in the  $K^+K^-$  distribution is due to the presence of the "C = +1" terms,  $\text{abs}T_2$ ,  $(T_2)_{\text{sp}}$ , and  $\delta^B(eK)$ . It is conveniently represented<sup>7</sup> in terms of an asymmetry parameter

$$A = \frac{F - B}{F + B}, \quad (36)$$

where  $F$  and  $B$  are the forward and backward events, respectively. The expected behavior of the asymmetry parameter near the  $f'$ -resonance position is shown in Fig. 4. The plot is for  $m_{f'} \approx 1516$  MeV and  $\Gamma_{f'} \approx 40$  MeV.

*Discussion.* The asymmetry in the  $K^+K^-$  distribution for the process  $e\bar{e} \rightarrow K^+K^-$  is predicted to be quite large, about 35% near the  $f'$ -resonance position, but goes through zero a little below the resonance, around  $s^{1/2} \approx m_{f'} - \Gamma_{f'}/4$ . The asymmetry in this case is larger than in the case of  $\pi^+\pi^-$  production,<sup>7</sup> but is sharper in distribution. This is to be expected, since the  $f'$  meson has a smaller width than the  $f$  meson. At the resonance, i.e.,  $s^{1/2} = m_{f'}$ , the asymmetry is  $A = 0.28$ . Our numbers for the asymmetry at the resonances,

both for  $\pi^+\pi^-$  and  $K^+K^-$  production, are in good agreement with the estimates<sup>6</sup> based on the quark model. They agree with the prediction<sup>5</sup> for  $\pi^+\pi^-$  production based on the universal coupling of the  $f$  meson but differ from the prediction for  $K^+K^-$  production based on the universal coupling of the  $f'$  meson. We regard our results as a support for

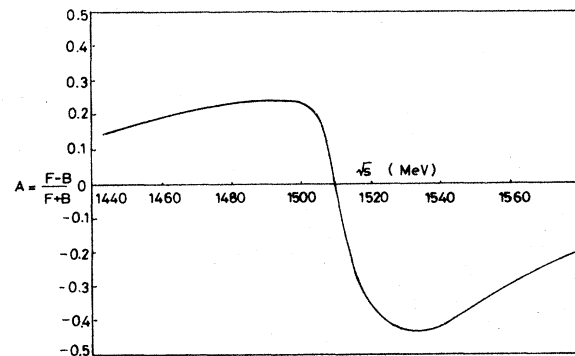


FIG. 4. The asymmetry  $A$  in  $e\bar{e} \rightarrow K^+K^-$  plotted as a function of  $s^{1/2}$ .

the universal coupling<sup>15</sup> of the  $f$  meson but a non-universal coupling for the  $f'$  meson, which would be plausible if  $f$  were primarily a singlet.

A detailed observation of the asymmetry is important for understanding the properties of  $f$  and  $f'$  and the radiative corrections to processes  $e\bar{e} \rightarrow K^+K^-, \pi^+\pi^-$ .

### III. DECAY RATE FOR $\eta \rightarrow \pi^0 + \gamma + \gamma$

The decay process  $\eta \rightarrow \pi^0 + \gamma + \gamma$  is related to

$$\gamma_\mu(q_1) + \gamma_\nu(q_2) \rightarrow \pi^0(k_1) + \eta(k_2) \quad (37)$$

by crossing. Furthermore, since the phase space for the decay is quite small, it is possible to assume that the two photons are in the  $J=0$ ,  $s$ -wave state.

The representation (10) is valid for the process (37) also, except that  $B_2$  and  $B_3$  do not contribute to the  $J=0$  amplitude. We decompose  $B_1$  into the various SU(3) ( $\underline{8} \times \underline{8}$ ) states and write for the  $J=0$  amplitude

$$B_{\mu\nu} = (q_1 \cdot q_2 g_{\mu\nu} - q_{2\mu} q_{1\nu}) B_1, \quad (38)$$

$$B_1 = \tau_8 + \tau_{27},$$

where  $\tau_8$  and  $\tau_{27}$  are contributions to (37) from the symmetric  $\underline{8}$  and the  $\underline{27}$  states of SU(3), respectively. The  $B_1$  is related to the  $J=0$  projection of the helicity amplitude  $T_{1,1}$  by

$$B_1 = -\frac{1}{s} \int_{-1}^1 T_{1,1} d\cos\theta. \quad (39)$$

Now the Born approximation for process (37) is zero, i.e.,  $B_1=0$ , though the  $\tau_i$  themselves are not zero. Therefore, it is the final-state interaction which will give a nonzero amplitude for (37).

One expects a dominant final-state interaction for the process (37) in the symmetric  $\underline{8}$  state, since there is the  $\delta$  resonance<sup>9</sup> which presumably has the quantum numbers  $J^P=0^+$ , and which decays mainly into  $\pi\eta$ . For introducing the final-state interaction, we first obtain the Born approximation for  $\tau_8$ , which may be obtained from, say  $\gamma + \gamma \rightarrow K^+K^-$ :

$$\tau_8^B = \left(\frac{\sqrt{3}}{5}\right) \frac{e^2}{s} \int_{-1}^1 \left[ |\vec{k}_1|^2 \sin^2\theta \left( \frac{1}{q_1 \cdot k_1} + \frac{1}{q_1 \cdot k_2} \right) - 2 \right] \times d\cos\theta \quad (40)$$

$$= \left(\frac{\sqrt{3}}{5}\right) \frac{8e^2\mu^2}{s^{3/2}(s-4\mu^2)^{1/2}} \ln \frac{s^{1/2} - (s-4\mu^2)^{1/2}}{s^{1/2} + (s-4\mu^2)^{1/2}},$$

where the factor  $\sqrt{3}/5$  comes from the SU(3) projections, and  $\mu$  is some average pseudoscalar octet mass, taken to be approximately the kaon mass. The final-state interaction is introduced by writing dispersion relations for  $s\tau_8(s)D_8(s)$ , where

$s\tau_8(s)$  is free from kinematic singularities, and  $D_8(s)$  is the  $D$  function for pseudoscalar-pseudoscalar scattering in the  $J=0$ , symmetric  $\underline{8}$  state. Approximating the left-hand discontinuity of  $\tau_8(s)$  by its Born approximation, we get

$$s\tau_8(s)D_8(s) = \frac{1}{\pi} \int_{-\infty}^0 \frac{s'D_8(s')\text{Im}\tau_8^B(s')}{s'-s} ds'. \quad (41)$$

For the integration over the left cut, i.e.,  $s' < 0$ , we represent the  $D$  function as a pole at  $s' = s_0 > 0$ . This is similar<sup>16</sup> to what is frequently done for the  $N$  function, i.e., it is represented as a pole at  $s' = s_0 < 0$  for integration over  $s' > 0$ . Furthermore, we note that there should be a zero in  $D_8(s)$  at  $s \approx m_\delta^2$ , where  $m_\delta$  is the mass of the  $\delta$  resonance and we require that  $D_8(s) \rightarrow \text{constant}$  for  $s \rightarrow \infty$ . These features together determine the form of  $D_8(s)$  uniquely,

$$D_8(s) = 1 - \frac{s}{m_\delta^2} \left( \frac{m_\delta^2 - s_0}{s - s_0} \right), \quad (42)$$

where we expect  $s_0$  to be of the order of  $2m_\delta^2$ .

We assume that the final-state interaction in the  $\underline{27}$  state is negligible so that  $\tau_{27} \approx \tau_{27}^B$ , so that we have

$$B_1(s) = \frac{1}{\pi s D_8(s)} \int_{-\infty}^{\infty} \frac{s'D_8(s')\text{Im}\tau_8^B(s')}{s'-s} ds' - \tau_8^B(s), \quad (43)$$

where we have used the fact that  $\tau_8^B(s) + \tau_{27}^B(s) = 0$ . The integral here is evaluated numerically and the  $B_{\mu\nu}$  in (38) with the  $B_1$  thus obtained is used as the matrix element for the decay of  $\eta \rightarrow \pi^0 + \gamma + \gamma$ . We obtain a numerical value of

$$\Gamma(\eta \rightarrow \pi^0 + \gamma + \gamma) \approx 4.8 \times 10^{-2} \text{ keV} \quad (44)$$

for  $s_0 = 2m_\delta^2$  and a somewhat larger value for  $s_0 > 2m_\delta^2$ . This number should be compared with the experimental value<sup>9</sup> of  $8.1 \times 10^{-2}$  keV. Actually, the experimental value quoted here is the average value. Some experiments,<sup>10</sup> however, give a considerably larger value.

We note that in one of the earlier attempts, Oppo and Oneda<sup>8</sup> assumed that the  $\pi^0$  and one of the photons in the decay  $\eta \rightarrow \pi^0 + \gamma + \gamma$  are in  $1^-$  state saturated by the vector mesons, thereby introducing a considerable angular-momentum barrier, which gave a very small value for the decay rate, viz.,  $\Gamma_{\eta \rightarrow \pi^0 \gamma} \approx 0.44$  eV. On the other hand, in the effective-Hamiltonian model of Okubo and Sakita<sup>8</sup> one got  $\Gamma_{\eta \rightarrow \pi^0 \gamma} \approx 8$  eV, though this number is cut-off-dependent. Therefore, with reference to the average experimental value for the decay width, our result (44) should be regarded as quite satis-

factory and our model for the decay  $\eta \rightarrow \pi^0 + \gamma + \gamma$  should be regarded as essentially correct.

#### APPENDIX

The final-state interaction for the process may be introduced through an integral equation of the Bethe-Salpeter type. Here we will consider the Logunov-Tavkhelidze equation,<sup>12</sup> which is an approximation to the Bethe-Salpeter equation. This equation for the partial-wave amplitude for the process  $\gamma + \gamma \rightarrow K + \bar{K}$  has the form

$$T_i(s, \nu') = T_i^B(s, \nu') + \frac{1}{\pi} \int_0^\infty \frac{d\nu''}{\nu'' - \nu} \left( \frac{\nu''}{\nu'' + \mu^2} \right)^{1/2} \times T_i(s, \nu'') v_i(\nu'', \nu', s), \quad (\text{A1})$$

where  $T_i(s, \nu')$  is the  $I=0$ ,  $J=2$  partial-wave projection of a suitable helicity amplitude for  $\gamma + \gamma \rightarrow K + \bar{K}$ ,  $T_i^B(s, \nu')$  is the Born approximation for  $T_i(s, \nu')$ ,  $v_i(\nu'', \nu', s)$  is the "potential" or the Born approximation for the  $I=0$ ,  $J=2$  partial-wave amplitude for  $K\bar{K}$  scattering, i.e., the Born approximation for  $[(\nu + \mu^2)/\nu]^{1/2} e^{i\delta_2} \sin\delta_2$ , and  $\nu = s/4 - \mu^2$ . We take  $\nu' = \nu$  and assume  $T_i(s, \nu'') \approx T_i(s, \nu)$  to obtain

$$T_i(s, \nu) = T_i^B(s, \nu)/D(s), \quad (\text{A2})$$

$$D(s) = 1 - \frac{1}{\pi} \int_0^\infty \frac{d\nu''}{\nu'' - \nu} \left( \frac{\nu''}{\nu'' + \mu^2} \right)^{1/2} v_i(\nu'', \nu, s).$$

This result is qualitatively similar to the result (20) in that the final-state interaction multiplies the Born term by a "resonant factor." For the analysis of this effect, we assume that the denominator produces a resonance corresponding to the  $f'$  meson with the observed width. Therefore, if we expand the denominator at the point  $s = s_r = m_{f'}^2 - i\Gamma_{f'} m_{f'}$ , we get

$$D(s) = [(s - m_{f'}^2) + i\Gamma_{f'} m_{f'}] \frac{dD(s)}{ds} \Big|_{s=s_r}. \quad (\text{A3})$$

The determination of  $dD(s)/ds|_{s=s_r}$  requires a dynamical model for  $v_i(\nu'', \nu', s)$ .

A simple model for  $v_i(\nu'', \nu', s)$  is to assume that it is generated by the exchanges of particles with  $J=1, 2, 3$ . We evaluate the numerator of the pole terms at the pole positions, so that the potentials are asymptotically well behaved, and obtain the off-shell potential

$$v_i(\nu'', \nu', s) = \sum_n f_n(s) \frac{1}{2(\nu' \nu'')^{1/2}} \times Q_2 \left[ \frac{m_n^2 + \nu' + \nu''}{2(\nu' \nu'')^{1/2}} \right], \quad (\text{A4})$$

where the summation is over particles with spins 1, 2, and 3. The functions  $f_n(s)$  for exchanges of  $J=1, 2, 3$  particles are

$$\begin{aligned} f_1(s) &= 3C_{0I}\gamma_1 \left( \frac{1}{4}m_1^2 - \mu^2 \right) P_1 \left( 1 + \frac{2s}{m_1^2 - 4\mu^2} \right), \\ f_2(s) &= 5C_{0I}\gamma_2 \left( \frac{1}{4}m_2^2 - \mu^2 \right) P_2 \left( 1 + \frac{2s}{m_2^2 - 4\mu^2} \right), \\ f_3(s) &= 7C_{0I}\gamma_3 \left( \frac{1}{4}m_3^2 - \mu^2 \right) P_3 \left( 1 + \frac{2s}{m_3^2 - 4\mu^2} \right), \end{aligned} \quad (\text{A5})$$

respectively, where  $C_{0I}$  are the crossing matrix elements for the exchange of a particle with isospin  $I$ , for  $K\bar{K}$  scattering in the isospin-zero state:

$$C_{I'I} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}. \quad (\text{A6})$$

The values of  $\gamma'_I$  are related to the coupling constants of these particles with the  $K\bar{K}$  system and are related to the experimental widths<sup>9</sup> of these particles. For the  $J=1$  case, we have actually two exchanges, one with  $I=1$  corresponding to the  $\rho$  exchange, and another with  $I=0$  corresponding to the exchange of the unmixed  $\omega_8$  meson. Together they give an effective contribution of  $C_{0I}\gamma_1 \approx 1.4$ . For the  $J=2$  exchange, we have the  $f'$  meson which gives  $C_{0I}\gamma_2 \approx 2.0/m^2$  where  $m=1$  GeV. The widths of the  $J=3$  resonances are not so well established and we adjust it to produce the required zero in  $D(s)$  at  $s \approx 2.3$  (GeV)<sup>2</sup>. This gives us a value of  $C_{0I}\gamma_3 \approx 1.1/m^4$  corresponding to the partial width  $\Gamma(g \rightarrow \pi\pi) \approx 150$  MeV. This determines the  $v_i(\nu'', \nu', s)$  needed for the calculation of  $D(s)$  in (A2). The value of  $dD(s)/ds|_{s=s_r}$  obtained from this  $D(s)$  is approximately  $-1.3/m_{f'}^2$ , so that

$$T_i(s, \nu) \approx \left( \frac{0.75m_{f'}^2}{m_{f'}^2 - s - im_{f'}\Gamma_{f'}} \right) T_i^B(s, \nu), \quad (\text{A7})$$

which is similar in form to the result (21), but is smaller by a factor of 0.75. Even this can be traced to our taking a rather large width for  $\Gamma(g \rightarrow \pi\pi)$  so that one may conclude that there is consistency between the two approaches.

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