Nonleptonic hyperon decays and octet enhancement in the bag model

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S-wave nonleptonic hyperon decays are discussed in the current-current picture making use of the MIT bag model. It is shown that upon inclusion of an enhancement factor of 3 due to asymptotic freedom the right order of magnitude for these amplitudes is obtained as well as $A(\Sigma_+^+)=0$ and $D/F=-1$.

Considerable interest has recently been devoted to the question of possible octet enhancement in nonleptonic hadron decays.[1-6] In particular, in Refs. 1 and 2 it has been shown that the octet part of the weak Hamiltonian is enhanced relative to the 27-plet by about a factor of 5. However, relative enhancement of only this magnitude is not by itself sufficient to account for the observed ratio of the $\Delta I=\frac{1}{2}$ to the $\Delta I=\frac{3}{2}$ amplitude in nonleptonic decays.

However, note that the statements of Refs. 1 and 2 apply only to the operator part of the weak Hamiltonian. In this paper we wish to investigate the matrix elements themselves by making use of the MIT bag model.[6] One of the main virtues of this model lies in providing a consistent relativistic framework in which the quark wave functions may be explicitly calculated at least in the cavity approximation. In particular, the model has been successfully applied to the calculation of the low-lying hadron mass spectrum[6] and static parameters of light hadrons.[6,7] In view of the successes of the model, it is thus also certainly of interest to study its consequences for nonleptonic decays to see if further light could be shed on the question of octet enhancement.

We start by writing down the matrix elements for the S-wave nonleptonic hyperon decays upon making use of partial conservation of axial-vector current (PCAC) and SU(2) invariance.[8] We then obtain

$$\begin{aligned}\langle B_\alpha \pi_\beta|\mathcal{H}_\omega^{pv}(0)|B_\gamma\rangle &= -\frac{\sqrt{2}}{f_\pi} i\langle B_\alpha|[Q_5^\beta,\mathcal{H}_\omega^{pv}(0)]|B_\gamma\rangle \\ &= -\frac{\sqrt{2}}{f_\pi} i\langle B_\alpha|[Q^\beta,\mathcal{H}_\omega^{pc}(0)]|B_\beta\rangle \\ &\equiv iA(B_\gamma\to B_\alpha\pi_\beta)\,, \quad (1)\end{aligned}$$

where $\mathcal{H}_\omega^{pv}(0)$ and $\mathcal{H}_\omega^{pc}(0)$ denote the parity-violating and parity-conserving parts of the nonleptonic current-current weak Hamiltonian. In order to explicitly calculate the matrix elements in Eq. (1) we then make use of the bag model. In this model the quark wave functions are given by[6]

$$q(\vec{x},t)=\frac{N(x)}{\sqrt{4\pi}}\begin{pmatrix}\left(\dfrac{\omega+m}{\omega}\right)^{1/2} ij_0\left(\dfrac{xr}{R}\right)U\\ -\left(\dfrac{\omega-m}{\omega}\right)^{1/2} j_1\left(\dfrac{xr}{R}\right)\vec{\sigma}\cdot\hat{r}U\end{pmatrix}e^{-i\omega t/R}\,, \quad (2)$$

$$N^{-2}(x)=R^3 j_0^{\,2}(x)\frac{[2\omega(\omega-1/R)+m/R]}{\omega(\omega-m)}\,. \quad (3)$$

In the above R is the radius of the spherical cavity, m is the quark mass, and $\omega(m,R)$ is defined by

$$\omega(m,R)=\frac{1}{R}[x^2+(mR)^2]^{1/2}\,, \quad (4)$$

where $x=x(m,R)$ satisfies the equation

$$\tan x=\frac{x}{1-mR-[x^2+(mR)^2]^{1/2}}\,. \quad (5)$$

The meson and baryon state wave functions may then be expressed in terms of the quark states in Eq. (2) in the standard fashion.[6]

For our consideration we express the parity-conserving part of the weak Hamiltonian in terms of quarks by means of the equation

$$\mathcal{H}_\omega^{pc}(0)=\frac{G}{\sqrt{2}}\sin\theta\cos\theta(\bar{\mathcal{P}}\gamma_\mu\lambda\mathfrak{N}\gamma^\mu\mathcal{P}+\bar{\mathcal{P}}\gamma_\mu\gamma_5\lambda\mathfrak{N}\gamma^\mu\gamma_5\mathcal{P}) +\text{H.c.} \quad (6)$$

The matrix elements in Eq. (1) may then be calculated explicitly, and we obtain

$$A(\Lambda_-^0)=2(\tfrac{3}{2})^{1/2}c\,, \quad (7)$$

$$A(\Xi_-^-)=4(\tfrac{3}{2})^{1/2}c\,, \quad (8)$$

$$A(\Sigma_0^+)=\frac{6}{\sqrt{2}}c\,, \quad (9)$$

and

$$A(\Sigma_+^+)=0\,, \quad (10)$$

where

$$c=\frac{G}{\sqrt{2}}\sin\theta\cos\theta\frac{1}{f_\pi}I\,, \quad (11)$$

with

TABLE I. S-wave amplitudes for $m_1=0$, $m_2=0.279$ GeV, and $R=4.96$ GeV^{-1}.

Amplitudes	$10^7\times$theoretical values	$10^7\times$theoretical values (with enhancement)	$10^7\times$experimental values
$A(\Lambda^0_-)$	0.74	2.22	3.31 ± 0.02
$A(\Xi^-_-)$	1.49	4.47	4.50 ± 0.04
$A(\Sigma^+_0)$	1.29	3.87	3.13 ± 0.20
$A(\Sigma^+_+)$	0	0	0.13 ± 0.04

$$I=\frac{N_1^3N_2}{4\pi}\int_0^R\left\{\left[\left(\frac{\omega_1+m_1}{\omega_1}\right)j_0^2\left(\frac{x_1r}{R}\right)+\left(\frac{\omega_1-m_1}{\omega_1}\right)j_1^2\left(\frac{x_1r}{R}\right)\right]\right.$$
$$\left.\times\left[\left(\frac{\omega_1+m_1}{\omega_1}\right)^{1/2}\left(\frac{\omega_2+m_2}{\omega_2}\right)^{1/2}j_0\left(\frac{x_1r}{R}\right)j_0\left(\frac{x_2r}{R}\right)+\left(\frac{\omega_1-m_1}{\omega_1}\right)^{1/2}\left(\frac{\omega_2-m_2}{\omega_2}\right)^{1/2}j_1\left(\frac{x_1r}{R}\right)j_1\left(\frac{x_2r}{R}\right)\right]\right\}r^2dr \tag{12}$$

and $f_\pi=0.94m_\pi$. In the above m_1 denotes the nonstrange-quark mass and m_2 the strange-quark mass.

Upon comparing the calculation leading to Eqs. (7) to (12) with the classical nonrelativistic SU(6) calculation we note that in the latter only the terms in Eq. (6) involving V_0V_0 and A_kA_k give a contribution. On the other hand, in our case there is also a contribution from V_kV_k since upper and lower components are present in the quark wave functions. Furthermore, we also note that after decomposing in our calculation V_0V_0, V_kV_k, and A_kA_k in terms of Pauli spinors, their space-dependent coefficients are all different. However, the space-dependent parts of V_kV_k and A_kA_k add up to a space-dependent term which is identical to that of V_0V_0, so that the overall change in comparison to nonrelativistic SU(6) amounts merely to a multiplicative factor.

The result in Eq. (10), i.e., $A(\Sigma^+_+)=0$, shows that also in the bag model the 27-plet is absent and hence the Lee-Sugawara relation holds, as might have already been expected from the work in Ref. 5.

Since the results in Eqs. (7) to (11) differ from the nonrelativistic SU(6) results by only a multiplicative factor, it hence also follows that the ratio here obtained is identical to that obtained in the latter, i.e., $D/F=-1$.

We have calculated numerically the integral in Eq. (12) making use of the parameters given in Ref. 6. These are $m_1=0$, $m_2=0.279$ GeV, and $R=4.96$ GeV^{-1} in Table I and $m_1=0.108$ GeV, $m_2=0.353$ GeV, and $R=5.5$ GeV^{-1} in Table II. (Note that we have used the same radius R for initial and final hadrons).

Before presenting our results we once again recall the main conclusion of Refs. (1) and (2). As was shown there, arguments based on asymptotic freedom predict an enhancement of that part of the octet operator term in Eq. (6) which gives a contribution to the matrix elements in Eqs. (7) to (10). We shall take for this enhancement factor the value 3, which is reasonable in view of the arguments presented in those references. We tabulate our results with and without the inclusion of this enhancement factor in Tables I and II.

We thus see that within the bag-model approximations the amplitudes for the S-wave nonleptonic decays in the current-current model are of the right order of magnitude upon inclusion of the enhancement factor due to asymptotic freedom. In particular, the agreement is better in Table I, where smaller values for the quark masses and

TABLE II. S-wave amplitudes for $m_1=0.108$ GeV, $m_2=0.353$ GeV, and $R=5.5$ GeV^{-1}.

Amplitudes	Theoretical values $\times 10^7$	Theoretical values $\times 10^7$ (with enhancement)	Experimental values $\times 10^7$
$A(\Lambda^0_-)$	0.60	1.80	3.31 ± 0.02
$A(\Xi^-_-)$	1.21	3.63	4.50 ± 0.04
$A(\Sigma^+_0)$	1.04	3.12	3.13 ± 0.20
$A(\Sigma^+_+)$	0	0	0.13 ± 0.04

hadronic radius are used.

As we have already stressed, the value for the D/F ratio here obtained, i.e., $D/F = -1$, is identical to that obtained in nonrelativistic SU(6). This value is not in disagreement with the value $D/F \approx -0.9$ obtained in Ref. (9) on the basis of fitting the S- and P-wave amplitudes.

We have neglected throughout possible one-gluon-exchange corrections. Their inclusion could, of course, slightly modify the predictions of the model. However, it is encouraging that the simple model presented here is already in reasonable agreement with some of the main features of the S-wave nonleptonic decays.[10]

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Note added. More recently, J. F. Donoghue and E. Golowich [Phys. Rev. D 14, 1386 (1976)] have calculated nonleptonic decays using the same parameters as are used in our calculation, and have obtained the same results.