PHYSICAL REVIEW D VOLUME 14, NUMBER 9 1 NOVEMBER 1976

μe events in electron-positron annihilation with longitudinally polarized beams

Richard Simard*
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Mahiko Suzuki†
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720
(Received 29 July 1976)

Muon-electron production processes are examined in the case where one of the electron-positron beams is longitudinally polarized. A special emphasis is put on the asymmetry in the angular distribution of muon (or electron) emission to reveal the structure of weak interactions of possible heavy leptons. The angular asymmetry will easily distinguish between $V-A$ and $V+A$ for the weak current of heavy leptons.

I. INTRODUCTION

The μe events discovered at SPEAR[1] indicate the existence of heavy leptons. Details of their decay properties are of special interest from the viewpoint of weak-interaction theory. At the present moment little is known about their identity beyond the fact that they decay into a muon or an electron through three-body decay. The distributions of the collinearity angle and the momenta of μe are insensitive to whether the weak current of the heavy leptons is $V-A$ or $V+A$.[2-5] It is rather difficult to distinguish between $V-A$ and $V+A$ through the collinearity angle and the momentum spectra.

Meanwhile, large transverse polarizations have been obtained at SPEAR in the vicinity of $\sqrt{s}=7.4$ GeV. It is expected that large polarizations will be obtained more easily at PEP and PETRA.[6] Since transverse polarizations are not very useful for the purpose of exploring weak interactions, or parity violation,[7] it is desired to rotate one or both of the beam polarizations into the longitudinal direction.[8] The parallel longitudinal polarizations would be most appropriate for our purpose, but it is almost equally useful if one of the beams is polarized longitudinally and the other beam remains unpolarized or transversely polarized. We examine in this paper the angular distributions of muons and electrons in the μe events when one of the initial beams is polarized longitudinally. With the initial beam polarized longitudinally, properties associated with parity will manifest themselves clearly, for instance, in the asymmetry of the angular distribution. This asymmetry in the emission angles of muons and electrons is large enough to distinguish easily between $V-A$ and $V+A$ for the heavy-lepton currents.

II. ANGULAR DISTRIBUTION OF μe EVENTS

The distributions of the collinearity angle and the momenta have been calculated by many people,[2-5] but they can reveal little of the structure of weak interactions. Being interested in parity-violation effects in heavy-lepton decays, we calculate here the asymmetry in the $\cos\theta$ distribution of muons and electrons in the final states of the μe events (see Fig. 1). The same asymmetry in inclusive muon (electron) production might serve our purpose, but it would be largely contaminated by the muons (electrons) due to charmed hadrons or other heavy hadrons stable against strong and electromagnetic decays. To make sure that final light leptons originate from heavy leptons, one should select out the μe events with no accompanying hadrons.

The interaction for heavy-lepton decay is written in the Fierz-transformed four-fermion form as

$$\begin{aligned}L_{\text{int}} = & (\bar\psi_l\psi_L)[\bar\psi_{\nu L}(C_S+C_S'\gamma_5)\psi_{\nu l}] \\ & +(\bar\psi_l\gamma_\lambda\psi_L)[\bar\psi_{\nu L}\gamma^\lambda(C_V+C_V'\gamma_5)\psi_{\nu l}] \\ & +\tfrac{1}{2}(\bar\psi_l\sigma_{\lambda\kappa}\psi_L)[\bar\psi_{\nu L}\sigma^{\lambda\kappa}(C_T+C_T'\gamma_5)\psi_{\nu l}] \\ & +(\bar\psi_l\gamma_\lambda\gamma_5\psi_L)[\bar\psi_{\nu L}\gamma^\lambda\gamma_5(C_A+C_A'\gamma_5)\psi_{\nu l}] \\ & +(\bar\psi_l\gamma_5\psi_L)[\bar\psi_{\nu L}\gamma_5(C_P+C_P'\gamma_5)\psi_{\nu l}]\,, \end{aligned} \qquad (2.1)$$

where ψ_l and $\psi_{\nu l}$ are Dirac fields of a light lepton (μ or e) and its neutrino, ψ_L and $\psi_{\nu L}$ are Dirac fields of a heavy lepton and its neutrino, and the convention of the γ matrices and the metric tensor is the one that is found in Ref. 9. As is usual,[10]

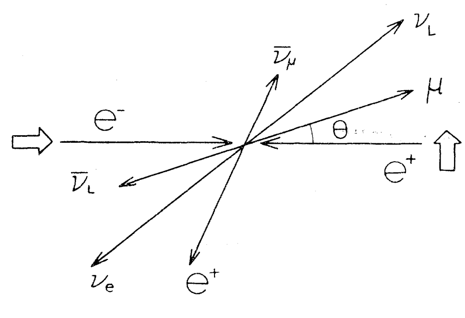

FIG. 1. A μe event with an electron beam polarized longitudinally.

we define

$$
\begin{aligned}
a &= |C_S|^2 + |C_S'|^2 + |C_P|^2 + |C_P'|^2 ,\\
b &= |C_V|^2 + |C_V'|^2 + |C_A|^2 + |C_A'|^2 ,\\
c &= |C_T|^2 + |C_T'|^2 ,\\
\overline{\alpha} &= |C_S|^2 + |C_S'|^2 - |C_P|^2 - |C_P'|^2 ,\\
\overline{\beta} &= |C_V|^2 + |C_V'|^2 - |C_A|^2 - |C_A'|^2 ,\\
a' &= -(C_S C_P'^* + C_S' C_P^* + C_S^* C_P' + C_S'^* C_P) ,\\
b' &= C_V C_A'^* + C_V' C_A^* + C_V^* C_A' + C_V'^* C_A ,\\
c' &= -(C_T C_T'^* + C_T^* C_T') ,\\
\overline{\alpha}' &= -(C_S C_P'^* + C_S' C_P^* - C_S^* C_P' - C_S'^* C_P) ,\\
\overline{\beta}' &= C_V C_A'^* + C_V' C_A^* - C_V^* C_A' - C_V'^* C_A ,
\end{aligned} \tag{2.2}
$$

and also

$$
\begin{aligned}
A_0 &= a + 4b + 6c ,\\
\rho &= (3b + 6c)/A_0 ,\\
\eta &= (\overline{\alpha} - 2\overline{\beta})/A_0 ,\\
\xi &= (-3a' - 4b' + 14c')/A_0 ,\\
\delta &= (3b' - 6c')/A_0\xi .
\end{aligned} \tag{2.3}
$$

If the heavy-lepton current is $V-A$ prior to the Fierz transform, one finds that $\rho = \delta = \frac{3}{4}$ and $\xi = 1$. If it is $V+A$, one finds that $\rho = \delta = 0$ and $\xi = 3$. In either case, $\eta = 0$. In the present paper the mass of the neutrino associated with the heavy lepton is assumed to be zero.

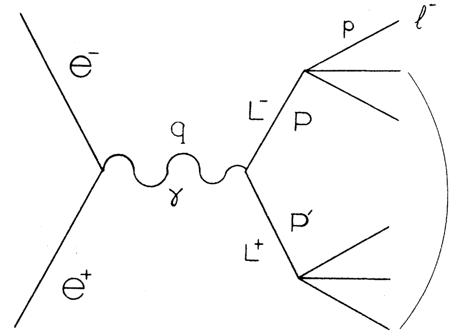

FIG. 2. Kinematics of μe events in the e^+e^- laboratory frame.

Let us first calculate for the process in which the electron beam is polarized longitudinally along its own momentum (as a vector) with the positron beam transversely polarized. We consider the angular distribution of light leptons of negative charge in the final states of such a process (Fig. 2). The other cases are obtained trivially from this case. Only the one-photon annihilation is taken into account. In computing the differential cross section we follow the method of Tsai[2] modified to a manifestly covariant form.

The L^+L^- production cross section with L^+L^- polarized arbitrarily is given in the e^+e^- laboratory frame by

$$\frac{d\sigma}{d\Omega_L} = \frac{\alpha^2\beta}{(q^2)^3} W , \tag{2.4}$$

$$W = W_1 + \mathcal{P} W_2 + O(m_l^{\,2}) , \tag{2.5}$$

$$
\begin{aligned}
W_1 = {} & 2\left[(P\cdot k)(P'\cdot k') + (P\cdot k')(P'\cdot k) + \tfrac{1}{2}q^2M^2\right]\left[1 - (s\cdot s')\right] - q^2(s\cdot P')(s'\cdot P) + \tfrac{1}{2}(q^2)^2(s\cdot s')\\
& - q^2\left[(s\cdot k)(s'\cdot k') + (s\cdot k')(s'\cdot k)\right] + 2(s\cdot P')\left[(P\cdot k)(s'\cdot k') + (P\cdot k')(s'\cdot k)\right]\\
& + 2(s'\cdot P)\left[(P'\cdot k)(s\cdot k') + (P'\cdot k')(s\cdot k)\right] ,
\end{aligned} \tag{2.6}
$$

$$W_2 = -Mq^2\left[(s\cdot k) + (s'\cdot k) - (s\cdot k') - (s'\cdot k')\right] , \tag{2.7}$$

where

$$
\begin{aligned}
&\mathcal{P} = \text{magnitude of } e^+e^- \text{ beam polarizations} ,\\
&s_\mu \ (s_\mu') = \text{four-vector polarization of } L^- \ (L^+) ,\\
&M = \text{mass of } L^\pm\\
&P_\mu \ (P_\mu') = \text{four-momentum of } L^- \ (L^+) ,\\
&k_\mu \ (k_\mu') = \text{four-momentum of initial } e^- \ (e^+) ,\\
&\beta = |\vec{\mathrm{P}}|/P_0, \text{ and } q = k + k' .
\end{aligned} \tag{2.8}
$$

The differential decay rate of $L^\pm$ is given by

$$P_0 \frac{d\Gamma^-}{d^3p} = A^- + (s\cdot p)B^- \tag{2.9}$$

for L^- and by a similar expression for L^+ with A^- and B^- replaced by A^+ and B^+. Here p is the four-momentum of the light lepton. In the approximation of ignoring the square of the light-lepton mass, the covariant decay amplitudes $A^\pm$ and $B^\pm$ are given by

$$
\begin{aligned}
A^\pm = \frac{A_0 M}{96\pi^4 P_0}\, y\biggl[& 3\left(\frac{M}{2} - \frac{y}{M}\right) + 2\rho\left(\frac{4y}{3M} - \frac{M}{2}\right)\\
& + \frac{3mM}{y}\,\eta\left(\frac{M}{2} - \frac{y}{M}\right)\biggr] ,
\end{aligned} \tag{2.10}
$$

$$B^{\pm} = \mp \frac{A_0 M^2}{96\pi^4 P_0}\,\xi\left[\left(\frac{M}{2}-\frac{y}{M}\right)+2\delta\left(\frac{4y}{3M}-\frac{M}{2}\right)\right], \tag{2.11}$$

where $y=(p\cdot P)$.

The final phase volume is fully integrated over L^+, so that the s'-dependent terms in (2.4)–(2.7) do not contribute to the differential production cross section $p_0(d\sigma/d^3p)$. The spin dependence of L^- in $d\sigma/d\Omega_L$ and $p_0(d\Gamma^-/d^3p)$ is correctly included by putting them together as

$$p_0\frac{d\sigma}{d\Omega_L\,d^3p} = \frac{\alpha^2\beta r^2}{(q^2)^3\Gamma}\left[\tilde W_1 A^- - (\tilde p\cdot \tilde W_2)B^-\right], \tag{2.12}$$

where $\tilde W_1$ and $\tilde W_2^\mu$ are defined as

$$\frac{d\sigma}{d\Omega_L} = \frac{\alpha^2\beta}{(q^2)^3}\left[\tilde W_1 + (s_\mu \tilde W_2^\mu) + (s'_\mu \tilde W_3^\mu) + (s_\mu s'_\nu \tilde W_4^{\mu\nu})\right], \tag{2.13}$$

where r is the branching ratio of $L^\pm$ into $l^\pm + \nu_l(\bar\nu_l) + \bar\nu_L(\nu_L)$, Γ is the decay rate in the e^+e^- laboratory frame of $L\to l+\nu_l+\nu_L$ integrated over the entire phase volume, and $\tilde p$ is defined by

$$\tilde p_\mu = p_\mu - P_\mu(p\cdot P)/M^2. \tag{2.14}$$

Equation (2.12) is integrated over $d\Omega_L$ with $\vec p$ kept fixed. Measuring the solid angle of $\vec P$ with respect to $\vec p$, one can integrate over the azimuthal angle Φ. The polar angle Θ should be replaced by a new variable $y=(p\cdot P)=pP_0(1-\beta\cos\Theta)+O(m^2)$. The region of integration over y is restricted by $|\cos\Theta|\le 1$ and $0\le (P-p)^2\le (M-m)^2$ to

$$\mathrm{Max}\left(mM,\ \frac{xM^2}{1+\beta}\right)\le y\le \mathrm{Min}\left(\tfrac{1}{2}M^2,\ \frac{xM^2}{1-\beta}\right) \tag{2.15}$$

up to $O(m^2)$. We thus obtain

$$\frac{d\sigma}{dx\,d\cos\theta} = \frac{16\pi\alpha^2 r^2}{q^2(1+4m\eta/M)}\left[1+\cos^2\theta)G_1(x) + (\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2})G_2(x) - 2\xi\mathcal{P}\cos\theta G_3(x)\right], \tag{2.16}$$

where $x=|\vec p|/k_0$, $\gamma=P_0/M$, θ is the emission angle of the light lepton l^- measured from the direction of the electron beam momentum $\vec k$, and

$$G_1(x) = \left(1+\frac{1}{2\gamma^2}\right)\left\{\left[\tfrac{3}{16}f_2(x)-\tfrac{1}{8}f_3(x)\right]+\rho\left[-\tfrac{1}{8}f_2(x)+\tfrac{1}{9}f_3(x)\right]+\frac{m}{M}\eta\left[\tfrac{3}{4}f_1(x)-\tfrac{3}{8}f_2(x)\right]\right\}, \tag{2.17}$$

$$\begin{aligned} G_2(x) = \frac{1}{x\gamma^2}\Big\{&\left[-\tfrac{1}{8}f_3(x)+\tfrac{3}{32}f_4(x)\right]+\frac{3}{x\gamma^2}\left[\tfrac{1}{128}f_4(x)-\tfrac{1}{160}f_5(x)\right]+\rho\left[\tfrac{1}{12}f_3(x)-\tfrac{1}{12}f_4(x)\right] \\ &+\frac{\rho}{x\gamma^2}\left[-\tfrac{1}{64}f_4(x)+\tfrac{1}{60}f_5(x)\right]+\frac{m\eta}{M}\left[-\tfrac{3}{8}f_2(x)+\tfrac{1}{4}f_3(x)\right]+\frac{m\eta}{Mx\gamma^2}\left[\tfrac{1}{16}f_3(x)-\tfrac{3}{64}f_4(x)\right]\Big\}, \end{aligned} \tag{2.18}$$

$$\begin{aligned} G_3(x) = {}& x\left[\tfrac{1}{4}f_1(x)-\tfrac{1}{8}f_2(x)\right]+\delta x\left[-\tfrac{1}{2}f_1(x)+\tfrac{1}{3}f_2(x)\right]+\left[-\tfrac{1}{16}f_2(x)+\tfrac{1}{24}f_3(x)\right] \\ &+\frac{1}{x\gamma^2}\left[\tfrac{1}{48}f_3(x)-\tfrac{1}{64}f_4(x)\right]+\delta\left[\tfrac{1}{8}f_2(x)-\tfrac{1}{9}f_3(x)\right]+\frac{\delta}{x\gamma^2}\left[-\tfrac{1}{24}f_3(x)+\tfrac{1}{24}f_4(x)\right], \end{aligned} \tag{2.19}$$

$$f_n(x) = \left(\frac{2}{M^2}\right)^n\left[y_{\max}{}^n(x) - y_{\min}{}^n(x)\right]. \tag{2.20}$$

In Eq. (2.20), $y_{\max}(x)$ and $y_{\min}(x)$ stand for the upper and lower limits of integration as given in (2.15).

We write the production cross section of light leptons in the form of

$$\frac{d\sigma}{dx\,d\cos\theta} = r^2\sigma_{\rm tot}(e^+e^-\to\mu^+\mu^-)R(x;\ \cos^2\theta) \times\left[1+\alpha(x;\cos^2\theta)\mathcal{P}\cos\theta\right]. \tag{2.21}$$

The integral of $R(x;\cos^2\theta)$ in $\cos\theta$ and x over the entire physical region tends to unity as $\gamma\to\infty$, thus leading to $\sigma_{\rm tot}(e^+e^-\to L^+L^-)\to\sigma_{\rm tot}(e^+e^-\to\mu^+\mu^-)$.

If one measures the emission angle of l^+ instead of l^- in the process under consideration, one should replace the polarization $\mathcal{P}$ by $-\mathcal{P}$. The same switch of sign should be done when l^- is measured in the annihilation of a transversely polarized electron beam with a positron beam polarized longitudinally along the positron beam momentum (opposite to the electron beam momentum). The formula (2.21) is valid in the case that l^+ is measured in the annihilation of a transversely polarized electron beam and a positron beam polarized along its own momentum.

III. NUMERICAL ESTIMATE

A numerical estimate has been made for the magnitude of the cross section in terms of $R(x;\cos^2\theta)$ and for the asymmetry $\alpha(x;\cos^2\theta)$. Both of these quantities are dependent on the veloc-

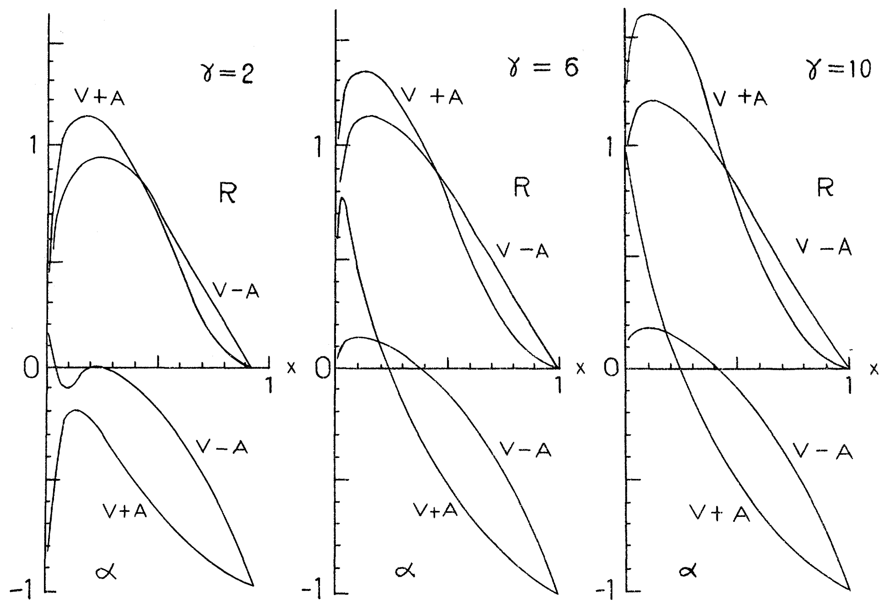

FIG. 3. $R(x;\cos^2\theta)$ and asymmetry parameter $\alpha(x;\cos^2\theta)$ as functions of $x=|\vec{p}|/k_0$ at $\cos\theta=1$. See Equation (2.21) in the text for the definitions of $R(x;\cos^2\theta)$ and $\alpha(x;\cos^2\theta)$.

ity of $L^{\pm}$. We have plotted here the results for the $V-A$ and $V+A$ currents provided that the $(l\nu_l)$ current is always of $V-A$. For $M\gtrsim 2.0$ GeV, $R(x;\cos^2\theta)$ and $\alpha(x;\cos^2\theta)$ are insensitive to the light-lepton mass even near $x=0$, so we have drawn the curves for $m=m_e\simeq 0$. General behaviors of R and α are rather insensitive to values of γ as long as γ is larger than 3 or so. Unlike the collinearity angle or the momentum spectra in the annihilation of unpolarized or transversely polarized beams, the asymmetry α will distinguish $V-A$ and $V+A$ clearly. The magnitude of the asymmetry tends to unity at $\cos\theta=\pm 1$ and $x=1$ as $\gamma\to\infty$, as is expected from helicity conservation. The magnitude of the cross section approaches zero as $x\to 1$ since $L^{\pm}$ decay into three bodies. To obtain the maximum efficiency, one should choose the region

$$0.4\lesssim x\lesssim 0.6 \tag{3.1}$$

or possibly

$$x\lesssim 0.05\,, \tag{3.2}$$

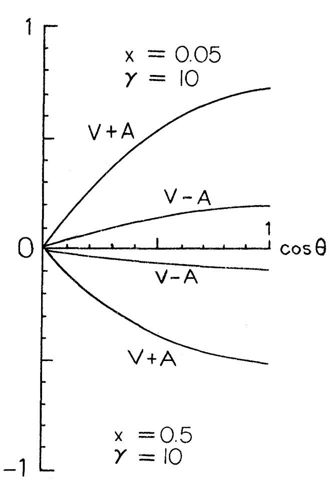

FIG. 4. Asymmetry parameter $\alpha(x;\cos^2\theta)$ as a function of $\cos\theta$ at $x=0.5$ and $x=0.05$. The value of γ is equal to 10 in the both cases.

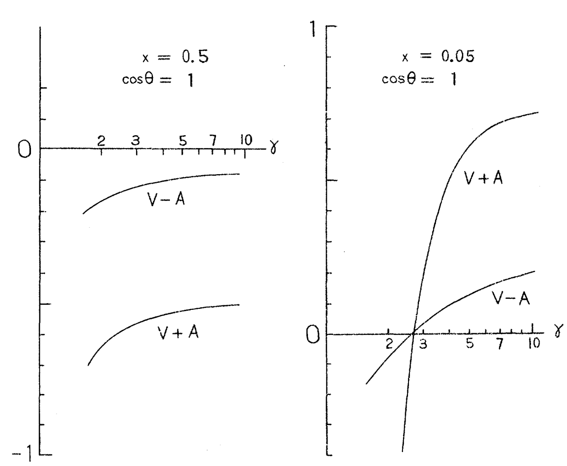

FIG. 5. Dependence on γ of asymmetry parameter $\alpha(x;\cos^2\theta)$ at $x=0.5$ and $x=0.05$. The value of θ is equal to 0° in the both cases.

and stay in the forward-backward cones of, say,

$$|\cos\theta| \lesssim 0.5 . \qquad (3.3)$$

In Fig. 3 are plotted the cross section in terms of quantity $R(x;\cos^2\theta)$ and the asymmetry $\alpha(x;\cos^2\theta)$, both defined in (2.21), as functions of $x=|\vec{p}|/k_0$ at $\gamma=2$, 6, and 10. The $\cos\theta$ dependence of α has been shown in Fig. 4. Finally, fixing values of x at 0.5 and 0.05, we have plotted the γ dependence of α between $\gamma=2$ and $\gamma=10$ in Fig. 5.

IV. COMMENT

Semileptonic decays of charmed hadrons or other heavy hadrons stable against strong and electromagnetic interactions can produce μe events, but the criterion of selecting the μe events with no accompanying hadron, charged or neutral (except for K_L^0), in final states eliminates all but $e^+e^- \to D^+D^-(F^+F^-) \to \mu + e + \nu_\mu + \nu_e$. If stable D^+ and F^+ are of $J^P=0^-$, as the ψ spectroscopy suggests, the branching ratio into $(l\nu_l)$ is negligibly small in the $V\pm A$ interactions. If D^+ and F^+ of $J^P=1^-$ should decay through weak interactions, one might have to separate them out. But strong damping of form factors will suppress two-body hadronic channels so severely that we would hardly worry about this possibility. Because of this we have proposed here to examine only the μe events rather than inclusive μ (or e) production.

---

*Research supported by Energy Research and Development Administration.

†Research supported by National Science Foundation und under Grant No. PHY 74-08174-A02 and in part by Energy Research and Development Administration.

[1]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975).

[2]Y.-S. Tsai, Phys. Rev. D 4, 2821 (1971).

[3]K. Fujikawa and N. Kawamoto, Phys. Rev. Lett. 35, 1560 (1975); Phys. Rev. D 13, 2534 (1976).

[4]S.-Y. Pi and A. I. Sanda, Phys. Rev. Lett. 36, 1 (1976).

[5]A. Pais and S. B. Treiman, Phys. Rev. D 14, 293 (1976); K. J. F. Gaemers and R. Raitio, Phys. Rev. D 14, 1262 (1976).

[6]R. F. Schwitters *et al.*, Phys. Rev. Lett. 35, 1320 (1975).

[7]The only realistic test utilizing transverse polarizations is, to our knowledge, the right-left asymmetry in inclusive hadron production at PEP and PETRA energies. R. Simard and M. Suzuki, Phys. Lett. 63B, 304 (1976).

[8]R. F. Schwitters, Nucl. Instrum. Methods 117, 331 (1974); R. F. Schwitters and B. Richter, SLAC-LBL Note No. SPEAR-175, PEP-87 (unpublished); A. Garren and J. Kadyk, SLAC-LBL Note No. PEP-184 (unpublished). See also A. W. Chao and R. F. Schwiters, SLAC-LBL Note No. SPEAR-194, PEP-217 (unpublished). This points out a serious difficulty in obtaining a large polarization.

[9]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[10]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), p. 185.