Hurley's field equation for arbitrary spin

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A complete set of covariant transforming matrices operating in the spinor space of Hurley's field equation for arbitrary spin are constructed. Similarly, matrices for transitions between spin 1/2 and 3/2 are also constructed.

I. INTRODUCTION

Recently Hurley[1,2,3] has presented a relativistic field equation which transforms according to the $(s,0)\oplus(s-\frac{1}{2},\frac{1}{2})$ [or $(0,s)\oplus(\frac{1}{2},s-\frac{1}{2})$] representation of $SL(2,C)$ as a generalization of the Dirac equation, viz.,

$$(\gamma_\mu p_\mu - im)\psi(x)=0\ , \tag{1}$$

where the field $\psi(x)$ has $6s+1$ components and the generalized γ_μ matrices transform covariantly, as for the Dirac case.[4] The equation needs no subsidiary conditions, and the field has $2s-1$ dependent components which vanish in the rest frame in momentum space. If parity symmetry is to be included in the usual way, the representation must (except for $s=\frac{1}{2}$) be doubled to yield the direct sum of $(s,0)\oplus(s-\frac{1}{2},\frac{1}{2})$ and $(0,s)\oplus(\frac{1}{2},s-\frac{1}{2})$. This, however, leads to difficulties in the field-theoretic treatment of the field satisfying (1).[2,3,5] Hurley shows[3] that covariance of the equation can be realized in the $(6s+1)$-dimensional space by replacing the effect of the parity matrix η by a differential operator which reverses the three-momenta of the components of $\psi(x)$; i.e., the parity transformation is nonlocal (in contrast with the Dirac and Joos-Weinberg cases[6,7,8]).

II. COMPLETE SET OF MATRICES FOR ARBITRARY SPIN

The number and types of such matrices are given by[9,10] the direct product

$$[(s,0)\oplus(s-\tfrac{1}{2},\tfrac{1}{2})]\oplus[(s,0)\oplus(s-\tfrac{1}{2},\tfrac{1}{2})]$$
$$=\left[\sum_{l=0}^{2s}(l,0)\right]\oplus\left[2\sum_{j=1/2}^{2s-1/2}(j,\tfrac{1}{2})\right]\oplus\left[\sum_{l=0}^{2s-1}(l,0)\right]\oplus\left[\sum_{l=0}^{2s-1}(l,1)\right] \tag{2}$$

(l is integer and j half-integer). Let $M_{\mu\nu}=-M_{\nu\mu}$ be the six generators of the homogeneous Lorentz group in the $(s,0)\oplus(s-\frac{1}{2},\frac{1}{2})$ representation. For all spins $s\geq 1$ we define

$$\gamma_5\equiv\frac{-1}{8s(s+1)}\epsilon_{\mu\nu\alpha\beta}M_{\mu\nu}M_{\alpha\beta}\ , \tag{3}$$

$$\gamma_{\mu\nu}\equiv\tfrac{1}{2}[\gamma_\mu,\gamma_\nu]_+ +\Delta^1_{\mu\nu}\ , \tag{4}$$

$$\gamma_{\mu\nu\sigma}\equiv iM_{\mu\nu}\gamma_\sigma+\Delta^2_{\mu\nu\sigma}\ , \tag{5}$$

$$M^{(2)}_{\mu\nu,\alpha\beta}\equiv\tfrac{1}{4}[M_{\mu\nu},M_{\alpha\beta}]_+ +\tfrac{1}{4}[M^D_{\mu\nu},M^D_{\alpha\beta}]_+ +\Delta^3_{\mu\nu,\alpha\beta}\ . \tag{6}$$

$M^D_{\mu\nu}$ is the dual of $M_{\mu\nu}$, i.e., $M^D_{\mu\nu}\equiv\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}M_{\alpha\beta}$. γ_5 is of course not a pseudoscalar matrix for $s\geq 1$ because we have no parity transformation in the spinor space. For $s\geq\frac{3}{2}$ we define

$$\gamma_{\mu\nu\sigma\rho}\equiv\tfrac{1}{2}[\gamma_{\mu\nu\sigma},\gamma_\rho]_+ +\Delta^4_{\mu\nu\sigma\rho}\ , \tag{7}$$

$$\gamma_{\mu\nu\alpha\beta\sigma}\equiv iM^{(2)}_{\mu\nu,\alpha\beta}\gamma_\sigma+\Delta^5_{\mu\nu\alpha\beta\sigma}\ , \tag{8}$$

$$M^{(3)}_{\mu\nu,\alpha\beta,\sigma\rho}\equiv\frac{1}{3!}(M_{\mu\nu}M_{\alpha\beta}M_{\sigma\rho}+\cdots)+\Delta^6_{\mu\nu,\alpha\beta,\sigma\rho}\ . \tag{9}$$

The $\Delta^i_{(\mu)}$'s in (4)–(9) are chosen in such a way that the matrices defined become traceless, i.e., $\gamma_{\mu\mu}=0$, $\gamma_{\mu\nu\nu}=0$, $M^{(2)}_{\mu\alpha,\nu\alpha}=0$, and so on. Moreover, the $\Delta^i_{(\mu)}$'s involve combinations (symmetrized in the right way) of Kronecker δ's, γ_5, and some of the matrices defined in an equation before the actual one.[11] For example,

$$\Delta^1_{\mu\nu}=-\tfrac{1}{4}[(5-2s)+(1-2s)\gamma_5]\delta_{\mu\nu}\ ,$$
$$\Delta^2_{\mu\nu\sigma}=-\tfrac{1}{3}[(2s^2-2s-1)+s(2s-1)\gamma_5]\times(\delta_{\mu\sigma}\gamma_\nu-\delta_{\nu\sigma}\gamma_\mu)\ , \tag{10}$$
$$\Delta^3_{\mu\nu,\alpha\beta}=\tfrac{1}{3}s(s+1)\gamma_5\epsilon_{\mu\nu\alpha\beta}+\tfrac{1}{6}s[-3+(2s-1)\gamma_5](\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha})\ .$$

In Table I the linearly independent matrices for $s=\frac{3}{2}$ are given. [For $s=1$, only the representations up to $(2,0)$ are present, and $(2,0)$ occurs only once for $s=1$ just as $(3,0)$ occurs only once for $s=\frac{3}{2}$.] For a given representation (a,b) the number of corresponding matrices, which are linearly independent of the total set, is $n=(2a+1)(2b+1)$.

The generators $M_{\mu\nu}$ are given by[1,2]

$$M_{kl}=\epsilon_{klm}J_m,\quad M_{k4}=-M_{4k}=-iN_k\ ; \tag{11}$$

TABLE I. Linearly independent matrices for $s=\frac{3}{2}$.

Representation	Matrices	Number of matrices, n
$(0,0)\oplus(0,0)$	$1,\ \gamma_5$	2
$[(1,0)\oplus(0,1)]\oplus(1,0)$	$M_{\mu\nu},\ \gamma_5 M_{\mu\nu}$	9
$(\frac{1}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{1}{2})$	$\gamma_\mu,\ \gamma_5\gamma_\mu$	8
$(1,1)$	$\gamma_{\mu\nu}$	9
$(\frac{3}{2},\frac{1}{2})\oplus(\frac{3}{2},\frac{1}{2})$	$\gamma_{\mu\nu\sigma},\ \gamma_5\gamma_{\mu\nu\sigma}$	16
$(2,0)\oplus(2,0)$	$M^{(2)}_{\mu\nu,\alpha\beta},\ \gamma_5 M^{(2)}_{\mu\nu,\alpha\beta}$	10
$(2,1)$	$\gamma_{\mu\nu\sigma\rho}$	15
$(\frac{5}{2},\frac{1}{2})\oplus(\frac{5}{2},\frac{1}{2})$	$\gamma_{\mu\nu\alpha\beta\sigma},\ \gamma_5\gamma_{\mu\nu\alpha\beta\sigma}$	24
$(3,0)$	$M^{(3)}_{\mu\nu,\alpha\beta,\sigma\rho}$	7
		$\sum n=100=(6\times\frac{3}{2}+1)^2$

$$\vec{J}=\begin{bmatrix}\vec{S} & 0 & 0\\ 0 & & \\ 0 & & \vec{R}\end{bmatrix},\quad -i\vec{N}=\begin{bmatrix}\vec{S} & 0 & 0\\ 0 & & \\ 0 & & \vec{T}\end{bmatrix}, \tag{12}$$

$$\vec{R}=\begin{bmatrix}\vec{S} & 0\\ 0 & \vec{\Sigma}\end{bmatrix},\quad \vec{T}=\frac{1}{s}\begin{bmatrix}(s-1)\vec{S} & \vec{t}^{\dagger}\\ \vec{t} & (s+1)\vec{\Sigma}\end{bmatrix}, \tag{13}$$

where $\vec{S}$ and $\vec{\Sigma}$ are the spin matrices for spins s and $s-1$, respectively, and $\vec{t}$ are $(2s+1)\times(2s-1)$-dimensional vector matrices (which are proportional to the matrices $\vec{\sigma}^{[s-1,s]}$ discussed in a previous paper[10]). For $s=\frac{1}{2}$, $\vec{R}=-\vec{T}=\frac{1}{2}\vec{\sigma}$ and $M_{\mu\nu}$ and $\gamma_5 M_{\mu\nu}$ contain only six linearly independent matrices $\vec{J}$ and $\gamma_5\vec{J}=i\vec{N}$. But when $s\geq 1$, $\vec{R}$ and $\vec{T}$ are linearly independent, which means that $M_{\mu\nu}$ and $\gamma_5 M_{\mu\nu}$ contain nine linearly independent matrices, which can be taken to be $\vec{J}$, $\gamma_5\vec{J}$, and $-i\vec{N}$, say.

In the Kramer-Weyl representation the γ_μ matrices are given by[1,2]

$$\gamma_\mu=\begin{bmatrix}0 & -r_\mu^\dagger\\ -r_\mu & 0\end{bmatrix};\quad r_k=\frac{i}{s}\begin{bmatrix}S_k\\ -t_k\end{bmatrix},\quad r_4=\begin{pmatrix}-1\\ 0\end{pmatrix}. \tag{14}$$

The matrices $\vec{S}$, $\vec{\Sigma}$, $\vec{t}$ satisfy the algebra[1,2]

$$\begin{aligned}
&t_k S_l-\Sigma_l t_k=i\epsilon_{kln}t_n\ ,\\
&t_k S_l-\Sigma_k t_l=is\epsilon_{kln}t_n\ ,\\
&t_k S_l-t_l S_k=(s+1)i\epsilon_{kln}t_n\ ,\\
&\Sigma_k t_l-\Sigma_l t_k=i(1-s)\epsilon_{kln}t_n\ ,\\
&S_k S_l+t_k^\dagger t_l=is\epsilon_{kln}S_n+s^2\delta_{kl}\ ,\\
&t_k t_l^\dagger+\Sigma_k\Sigma_l=-is\epsilon_{kln}\Sigma_n+s^2\delta_{kl}\ ,\\
&t_k^\dagger t_l-t_l^\dagger t_k=(2s-1)i\epsilon_{kln}S_n\ ,\\
&t_k t_l^\dagger-t_l t_k^\dagger=-(2s+1)i\epsilon_{kln}\Sigma_n\ .
\end{aligned} \tag{15}$$

The operator $\vec{T}$ in (13) can be split into two parts:

$$\vec{T}=\vec{R}+\vec{Q};\quad \vec{R}\equiv\begin{bmatrix}\vec{S} & 0\\ 0 & \vec{\Sigma}\end{bmatrix},\quad \vec{Q}\equiv\frac{1}{s}\begin{bmatrix}-\vec{S} & \vec{t}^{\dagger}\\ \vec{t} & \vec{\Sigma}\end{bmatrix}. \tag{16}$$

By means of the algebra in (15) one can show that

$$[R_n,Q_n]_-=0,\quad Q_n{}^2=1 \tag{17}$$

where $R_n\equiv\vec{R}\cdot\vec{n}$, $Q_n\equiv\vec{Q}\cdot\vec{n}$; $|\vec{n}|=1$. From (17) we deduce that

$$\begin{aligned}
e^{\vec{\Omega}\cdot\vec{T}}&=e^{\vec{\Omega}\cdot\vec{Q}}e^{\vec{\Omega}\cdot\vec{R}}\\
&=(\cosh\Omega+\vec{Q}\cdot\hat{\Omega}\sinh\Omega)e^{\vec{\Omega}\cdot\vec{R}}\ ,
\end{aligned} \tag{18}$$

which is a useful relation to study the transformation properties of the wave function $[\Omega\equiv|\vec{\Omega}|,\ \hat{\Omega}\equiv(1/\Omega)\vec{\Omega}]$. A helicity spinor can then be written:

$$\begin{aligned}
u(p\sigma)&=e^{-i\vec{\theta}\cdot\vec{J}}e^{-i\omega N_3}u(0\sigma)\\
&=\frac{e^{\sigma\omega}}{\sqrt{2}}\begin{pmatrix}\chi_\sigma^{[s]}(\hat{p})\\ [\cosh\omega-(\sigma/s)\sinh\omega]\chi_\sigma^{[s]}(\hat{p})\\ \sinh\omega f(s,\sigma)\chi_\sigma^{[s-1]}(\hat{p})\end{pmatrix},
\end{aligned} \tag{19}$$

where

$$u(0\sigma)=\frac{1}{\sqrt{2}}\begin{pmatrix}\chi_\sigma^{[s]}\\ \chi_\sigma^{[s]}\\ 0\end{pmatrix}$$

is the rest spinor. $\chi_\sigma^{[s]}(\hat{p})\equiv e^{i\vec{\theta}\cdot\vec{S}}\chi_\sigma^{[s]}$ and $\chi_\sigma^{[s-1]}(\hat{p})\equiv e^{i\vec{\theta}\cdot\vec{\Sigma}}\chi_\sigma^{[s-1]}$ are helicity spinors which satisfy $\vec{S}\cdot\hat{p}\chi_\sigma^{[s]}(\hat{p})=\sigma\chi_\sigma^{[s]}(\hat{p})$ and $\vec{\Sigma}\cdot\hat{p}\chi_\sigma^{[s-1]}=\sigma\chi_\sigma^{[s-1]}(\hat{p})$, respectively, and $f(s,\sigma)$ is given by $(1/s)t_3\chi_\sigma^{[s]}=f(s,\sigma)\chi_\sigma^{[s-1]}$ $[f(s,\sigma)=0$ for $\sigma=\pm s]$.

Using the algebra in (12) we find

$$\gamma_5 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & & \\ 0 & \frac{1-s}{s}1 & \end{bmatrix}, \tag{20}$$

$$\gamma_\mu\gamma_\mu = (5-2s)+(1-2s)\gamma_5 ,$$

$$M_{\mu\nu}M_{\mu\nu} = 2s[3+(1-2s)\gamma_5] .$$

The matrices corresponding to the representation (1,1) defined in (4) are given by

$$\gamma_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Gamma_{\mu\nu} & \\ 0 & & \end{bmatrix};$$

$$\Gamma_{44} = \frac{1}{4s}\begin{bmatrix} (2s-1) & 0 \\ 0 & -(2s+1) \end{bmatrix}$$

$$\Gamma_{k4} = \Gamma_{4k} = \frac{i}{2s}\begin{bmatrix} 0 & -t_k^\dagger \\ t_k & 0 \end{bmatrix}, \tag{21}$$

$$\Gamma_{kl} = \frac{1}{s^2}\begin{bmatrix} K_{kl}^{[s]} - \frac{1}{12}s(2s-1)\delta_{kl} & -st_{kl}^{(2)\dagger} \\ -st_{kl}^{(2)} & -K_{kl}^{[s-1]} + \frac{1}{12}s(2s+1)\delta_{kl} \end{bmatrix};$$

$$t_{kl}^{(2)} \equiv \frac{1}{2s}(t_kS_l + t_lS_k) .$$

$K_{kl}^{[s]}$ and $K_{kl}^{[s-1]}$ are the quadrupole matrices for spin s and spin $(s-1)$, respectively, i.e.,

$$K_{kl}^{[s]} \equiv \tfrac{1}{2}(S_kS_l + S_lS_k)^{[s]} - \tfrac{1}{3}s(s+1)\delta_{kl} .$$

The matrices $M^{(2)}_{\mu\nu,\alpha\beta}$ in (6) are given by

$$M^{(2)}_{k4,l4} \equiv L_{kl} = \begin{bmatrix} K_{kl}^{[s]} & 0 & 0 \\ 0 & [(s-1)/s]K_{kl}^{[s]} & t_{kl}^{(2)} \\ 0 & t_{kl}^{(2)} & [(s+1)/s]K_{kl}^{[s-1]} \end{bmatrix},$$

$$M^{(2)}_{kl,m4} = \epsilon_{kln}L_{nm} , \tag{22}$$

$$M^{(2)}_{kl,mn} = \epsilon_{klr}\epsilon_{mnt}L_{rt}$$

and the matrices defined in (5) corresponding to the representation $(\frac{3}{2},\frac{1}{2})$ are given by

$$\gamma_{\mu\nu\sigma} = \begin{bmatrix} 0 & -(i\Gamma_{\mu\nu\sigma})^\dagger \\ i\Gamma_{\mu\nu\sigma} & 0 \end{bmatrix};$$

$$\Gamma_{k44} = \frac{1}{3s}\begin{bmatrix} (2s-1)S_k \\ (s+1)t_k \end{bmatrix},$$

$$\Gamma_{k4l} = \frac{i}{s}\begin{bmatrix} -S_lS_k + \frac{1}{3}s(s+1)\delta_{kl} \\ t_lS_k \end{bmatrix},$$

$$\gamma_{klm} = \epsilon_{kln}\gamma_{nm4} - \tfrac{1}{3}(s+1)[(1-2s)-2s\gamma_5]\gamma_4\epsilon_{kln} - \tfrac{1}{3}[(2s^2-5s-1)+2s(s-2)\gamma_5](\delta_{km}\gamma_l - \delta_{lm}\gamma_k) . \tag{23}$$

III. MATRICES FOR TRANSITIONS BETWEEN SPIN $\frac{1}{2}$ AND SPIN $\frac{3}{2}$

In the description of interactions where higher-spin particles are involved, vertices with transitions between different (integer *or* half-integer) spin values are used, for example $N\gamma\Delta$ and $N\pi\Delta$ vertices.[10,12-16] The types of matrices which can operate between a spin-$\frac{3}{2}$ spinor transforming as $(\frac{3}{2},0)\oplus(1,\frac{1}{2})$ and a spin-$\frac{1}{2}$ spinor transforming as $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ are given by the direct product

$$[(\tfrac{3}{2},0)\oplus(1,\tfrac{1}{2})]\otimes[(\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})] = (2,0)\oplus(1,0)\oplus(1,0)\oplus(1,1) \oplus(\tfrac{3}{2},\tfrac{1}{2})\oplus(\tfrac{3}{2},\tfrac{1}{2})\oplus(\tfrac{1}{2},\tfrac{1}{2}) . \tag{24}$$

Because of Lorentz invariance the nonquadratic generalization $\Sigma_{\mu\nu}^{[3/2,1/2]}$ of $M_{\mu\nu}$ must satisfy

$$M_{\sigma\rho}^{[3/2]}\Sigma_{\mu\nu}^{[3/2,1/2]} - \Sigma_{\mu\nu}^{[3/2,1/2]}M_{\sigma\rho}^{[1/2]} = i(\delta_{\nu\sigma}\Sigma_{\mu\rho} - \delta_{\nu\rho}\Sigma_{\mu\sigma} + \delta_{\mu\sigma}\Sigma_{\rho\nu} - \delta_{\mu\rho}\Sigma_{\sigma\nu})^{[3/2,1/2]} . \tag{25}$$

It can be shown that $\Sigma_{\mu\nu}^{[3/2,1/2]}$ can be given by

$$\Sigma_{kl}^{[3/2,1/2]} = \epsilon_{klm}\Sigma_m^{[3/2,1/2]} ,$$

$$\Sigma_{k4}^{[3/2,1/2]} = -\Sigma_{4k}^{[3/2,1/2]} = \Sigma_k^{[3/2,1/2]} ; \tag{26}$$

$$\vec{\Sigma}^{[3/2,1/2]} = \begin{bmatrix} \vec{\sigma}^{[3/2,1/2]} & 0 \\ 0 & \vec{\sigma}^{[3/2,1/2]} \\ 0 & -(1/\sqrt{3})\vec{\sigma}^{[1/2]} \end{bmatrix},$$

where[1,10] $\vec{\sigma}^{[3/2,1/2]} = (1/\sqrt{3})(\vec{t}^{[3/2]})^\dagger$ and $\sigma_k^{[1/2]}$ are the Pauli matrices. Equation (25) is also satisfied by $\Sigma_{\mu\nu}^{[3/2,1/2]}\gamma_5^{[1/2]}$ [$\Sigma_{\mu\nu}^{[3/2,1/2]}$ and $\Sigma_{\mu\nu}^{[3/2,1/2]}\gamma_5^{[1/2]}$ correspond to $(1,0)\oplus(0,1)$]. The matrices corresponding to $(\frac{1}{2},\frac{1}{2})$ are

$$\gamma_\mu^{[3/2,1/2]} \equiv i\Sigma_{\mu\alpha}^{[3/2,1/2]}\gamma_\alpha^{[1/2]} ;$$

$$\vec{\gamma}^{[3/2,1/2]} = i\begin{bmatrix} 0 & 0 \\ 2\vec{\sigma}^{[3/2,1/2]} & 0 \\ (1/\sqrt{3})\vec{\sigma}^{[1/2]} & 0 \end{bmatrix}, \tag{27}$$

$$\gamma_4^{[3/2,1/2]} = \sqrt{3}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1^{[1/2]} & 0 \end{bmatrix}.$$

We observe that $\gamma_\mu^{[3/2,1/2]}\gamma_5^{[1/2]} = \gamma_\mu^{[3/2,1/2]}$. The matrices corresponding to $(\frac{3}{2},\frac{1}{2})\oplus(\frac{3}{2},\frac{1}{2})$ are $\gamma_{\mu\nu\sigma}^{[3/2,1/2]}$ and $\gamma_{\mu\nu\sigma}^{[3/2,1/2]}\gamma_5^{[1/2]}$, where

$$\gamma^{[3/2,1/2]}_{\mu\nu\sigma} \equiv i\Sigma^{[3/2,1/2]}_{\mu\nu}\gamma^{[1/2]}_{\sigma} - \delta_{\nu\sigma}\gamma^{[3/2,1/2]}_{\mu} + \delta_{\mu\sigma}\gamma^{[3/2,1/2]}_{\nu} ;$$

$$\gamma^{[3/2,1/2]}_{k44} = i\begin{bmatrix} 0 & \sqrt{3}\,\sigma^{[3/2,1/2]}_{k} \\ (1/\sqrt{3})\sigma^{[3/2,1/2]}_{k} & 0 \\ -4\sigma^{[1/2]}_{k} & 0 \end{bmatrix}, \qquad (28)$$

$$\gamma^{[3/2,1/2]}_{k4l} = \begin{bmatrix} 0 & -\sqrt{3}\,\sigma^{[3/2,1/2]}_{k}\sigma_{l}^{[1/2]} \\ \sqrt{3}\,\sigma^{[3/2,1/2]}_{k}\sigma^{[1/2]}_{l} & 0 \\ (\delta_{kl} - \sigma^{[1/2]}_{k}\sigma_{l}^{[1/2]}) & 0 \end{bmatrix}.$$

The matrices corresponding to (1, 1) are

$$\gamma^{[3/2,1/2]}_{\mu\nu} \equiv \tfrac{1}{2}(\gamma^{[3/2,1/2]}_{\mu}\gamma^{[1/2]}_{\nu} + \gamma^{[3/2,1/2]}_{\nu}\gamma^{[1/2]}_{\mu}) ;$$

$$\gamma^{[3/2,1/2]}_{44} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \sqrt{3}\,1^{[1/2]} \end{bmatrix},$$

$$\gamma^{[3/2,1/2]}_{k4} = i\begin{bmatrix} 0 & 0 \\ 0 & \sigma^{[3/2,1/2]}_{k} \\ 0 & (2/\sqrt{3})\sigma^{[1/2]}_{k} \end{bmatrix}, \qquad (29)$$

$$\gamma^{[3/2,1/2]}_{kl} = -\begin{bmatrix} 0 & 0 \\ 0 & 2K^{[3/2,1/2]}_{kl} \\ 0 & (1\sqrt{3})\delta_{kl}1^{[1/2]} \end{bmatrix},$$

where $K^{[3/2,1/2]}_{kl} \equiv \frac{1}{2}(\sigma^{[3/2,1/2]}_{k}\sigma^{[1/2]}_{l} + \sigma^{[3/2,1/2]}_{l}\sigma^{[1/2]}_{k})$; $K^{[3/2,1/2]}_{ll} = 0$. The matrices corresponding to (2.0) are

$$Q^{[3/2,1/2]}_{\mu\nu,\alpha\beta} \equiv \tfrac{1}{2}(\Sigma^{[3/2,1/2]}_{\mu\nu}\Sigma^{[1/2]}_{\alpha\beta} + \Sigma^{[3/2,1/2]}_{\alpha\beta}\Sigma^{[1/2]}_{\mu\nu}) - \tfrac{1}{4}(\delta_{\mu\alpha}F_{\nu\beta} - \delta_{\nu\alpha}F_{\mu\beta} + \delta_{\nu\beta}F_{\mu\alpha} - \delta_{\mu\beta}F_{\nu\alpha})^{[3/2,1/2]}, \qquad (30)$$

where

$$\Sigma^{[1/2]}_{\mu\nu} \equiv \frac{1}{2i}[\gamma^{[1/2]}_{\mu}, \gamma^{[1/2]}_{\nu}]_{-} ,$$

$$F^{[3/2,1/2]}_{\mu\nu} \equiv \Sigma^{[3/2,1/2]}_{\mu\alpha}\Sigma^{[1/2]}_{\nu\alpha} + \Sigma^{[3/2,1/2]}_{\nu\alpha}\Sigma^{[1/2]}_{\mu\alpha} .$$

We obtain

$$Q^{[3/2,1/2]}_{k4,l4} = \begin{bmatrix} K^{[3/2,1/2]}_{kl} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \qquad (31)$$

In contrast with the Joos-Weinberg[6,7] case, it is *not* possible to obtain an expression like

$$(a + b\gamma^{[1/2]}_{5})\Sigma^{[1/2,3/2]}_{\mu\nu}p_{\nu}u^{[3/2]}(p\sigma)$$

which can be identified with the Rarita-Schwinger spinor[17] $u_\mu(p\sigma)$ in momentum space.[16] [We have $\Sigma^{[1/2,3/2]}_{\mu\nu} = (\Sigma^{[3/2,1/2]}_{\mu\nu})^\dagger$.] To achieve this, the representation must be doubled.

IV. SOME REMARKS ON INTERACTIONS

If the Feynman rules in momentum space for $s=\frac{1}{2}$ (i.e., the Dirac case) are generalized directly to arbitrary spin (i.e., the Hurley case), the double representation containing both $(s,0) \oplus (s-\frac{1}{2},\frac{1}{2})$ and $(0,s)\oplus(\frac{1}{2},s-\frac{1}{2})$ has to be used. For example, the current in *momentum space* defined as $j_\mu(p'\sigma';p\sigma) = i\tilde{u}(p'\sigma')^\dagger\gamma_\mu u(p\sigma)$ [where $\tilde{u}(p\sigma) = u(\tilde{p}\sigma)$; $\tilde{p} = (-\vec{p}, ip_0)$] contains both vector and pseudovector parts. This can be seen by transforming $j_\mu(p'\sigma'; p\sigma)$ to the Breit frame, where we easily observe that j_μ contains an *electric* dipole moment. [The current in configuration space $j_\mu(x) = \tilde{\psi}(x)^\dagger\gamma_\mu\psi(x)$ is of course a pure four-vector because of the Fourier coefficients, which are not present in $j_\mu(p'\sigma';p\sigma)$.] If the representation is doubled, the matrices $\gamma_\mu, \gamma_5, M_{\mu\nu}, \ldots$ are replaced by[1]

$$\Gamma_\mu = \begin{bmatrix} \gamma_\mu & 0 \\ 0 & \tilde{\gamma}_\mu \end{bmatrix}, \quad \Gamma_5 = \begin{bmatrix} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{bmatrix}, \quad H_{\mu\nu} = \begin{bmatrix} M_{\mu\nu} & 0 \\ 0 & \tilde{M}_{\mu\nu} \end{bmatrix}, \ \ldots \qquad (32)$$

where a tilde indicates space inversion on each four-vector index. The matrix $\eta = \binom{01}{10}$ is now a parity operator in spinor space:

$$\eta\Gamma_\mu\eta = \tilde{\Gamma}_\mu, \ \eta\Gamma_5\eta = -\Gamma_5, \ \eta H_{\mu\nu}\eta = \tilde{H}_{\mu\nu}, \ldots . \qquad (33)$$

The matrix

$$\Gamma_5 = \frac{-1}{8s(s+1)}\epsilon_{\mu\nu\alpha\beta}H_{\mu\nu}H_{\alpha\beta}$$

is now pseudoscalar.

The covariant propagator for (1) in momentum space is

$$S(p) = \frac{-1}{im}\left[\frac{\gamma\cdot p(\gamma\cdot p + im)}{p^2+m^2} - 1\right], \qquad (34)$$

which shows that (1) does not have the drawback of the Joos-Weinberg equation.[6,7] The Joos-Weinberg equation has false mass solutions, which results in unphysical poles in the propagator.[16,18] It is also clear that the propagator (34) does not have the drawback of the (generalized) Rarita-Schwinger[16,19] propagator, which is *not* a pure spin object. Moreover, Hurley[1] has shown that (1) has causal propagation. This was also demonstrated by Nagpal,[20] who used the techniques of Velo and Zwanziger.[21] An attempt to give a consistent use of the Hurley formalism in interactions should therefore be worth further study.

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[4]W. J. Hurley uses β_μ and ϕ instead of γ_μ and ψ. In this paper γ_μ is chosen to stress the fact that Hurley's equation is a higher-spin generalization of Dirac's equation. Moreover, the matrices constructed here have a certain connection with similar matrices to be used in the Joos-Weinberg formalism. See Refs. 9 and 10. Throughout the paper, the metric $a_\mu b_\mu = \vec{a}\cdot\vec{b} + a_4 b_4 = \vec{a}\cdot\vec{b} - a_0 b_0$ is used.

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