Comments and Addenda

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Analytic contribution to the g factor of the electron in sixth order

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We evaluate analytically three more graphs contributing to the g factor of the electron in sixth order. Updated estimates for the value of $(g-2)$ are given.

To discuss the g factor of the electron in sixth order, let us define a_6 by

$$(g-2)/2=\alpha/(2\pi)-0.328\,478\cdots(\alpha/\pi)^2+a_6(\alpha/\pi)^3+O((\alpha/\pi)^4). \qquad (1)$$

There are 40 distinct Feynman graphs contributing to a_6, of which 28 have no fermion loops, 12 involve vacuum polarization, and 2 involve light-by-light scattering. The graphs with vacuum polarization are known analytically, and without including the results of this paper, 14 of the 28 graphs without fermion loops were known analytically.

Continuing the approach developed in previous publications,[1] we have computed the analytic values of 3 more of the 28 graphs without fermion loops contributing to a_6. The graphs are shown in Fig. 1, and the analytic expressions are shown in Table I. (The calculations were done in the Feynman gauge, and infrared-divergent terms proportional to $\ln\lambda$ of $\ln^2\lambda$ are omitted. These can be found in Ref. 2.) Table II presents a comparison of the exact result with previous numerical calculations. As can be seen, Carroll's result[3] is significantly more negative than the correct value. This accounts for part of the previous dis-

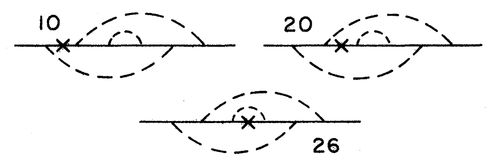

FIG. 1. The three graphs whose contribution to a_6 is evaluated in this paper. The numbering is from Ref. 2.

TABLE I. Analytic expressions for the contribution to a_6 of the graphs in Fig. 1. The infrared pieces proportional to $\ln\lambda$ and $\ln^2\lambda$ are omitted. The values for graphs 10 and 20 have been doubled to include their mirror images.

Graph	
10	$\frac{155}{144}-\frac{493}{432}\pi^2+\frac{71}{432}\pi^4-\frac{1}{18}(\ln 2)^4-\frac{79}{12}\zeta(3)-\frac{4}{3}A_4+\frac{5}{6}\pi^2\ln 2-\frac{4}{9}\pi^2(\ln 2)^2$
20	$-\frac{7}{144}+\frac{221}{144}\pi^2-\frac{191}{1080}\pi^4+\frac{1}{9}(\ln 2)^4+\frac{26}{3}\zeta(3)+\frac{8}{3}A_4-\frac{37}{18}\pi^2\ln 2+\frac{8}{9}\pi^2(\ln 2)^2$
26	$\frac{53}{36}+\frac{313}{432}\pi^2-\frac{11}{4320}\pi^4+\frac{1}{36}(\ln 2)^4-\frac{13}{3}\zeta(3)+\frac{2}{3}A_4-\frac{17}{18}\pi^2\ln 2+\frac{2}{9}\pi^2(\ln 2)^2$
Total	$\frac{5}{2}+\frac{161}{144}\pi^2-\frac{13}{864}\pi^4+\frac{1}{12}(\ln 2)^4-\frac{9}{4}\zeta(3)+2A_4-\frac{13}{6}\pi^2\ln 2+\frac{2}{3}\pi^2(\ln 2)^2$
	$\left[\zeta(3)=\sum 1/(n^3)=1.202\,0569\ldots,\quad A_4=\sum 1/(2^n n^4)=0.517\,479\ldots\right]$

TABLE II. Comparison of analytic and numerical evaluations of the graphs of Fig. 1. Again the values for graphs 10 and 20 represent the sum for the graph and its mirror image.

Graph	Analytic value	Levine and Wright (Ref. 2)	Cvitanovic and Kinoshita (Ref. 4)	Carroll (Ref. 3)
10	0.799 591⋯	0.795(6)	0.7962(46)	
20	−0.152 300⋯	−0.153(6)	−0.1465(72)	
26	−1.889 71 ⋯	−1.888(6)	−1.8846(39)	
Total	−1.242 42 ⋯	−1.246(10)	−1.2349(94)	−1.283(10)

crepancy between his number and that of either Levine and Wright[2] or Cvitanovic and Kinoshita.[4]

Combining these analytic values with those known previously,[5] and with numerical evaluations for graphs whose values are not yet known analytically, we find the following theoretical predictions for a_6:

$a_6 = 1.211 \pm 0.048$ Levine and Wright,[2]

$a_6 = 1.181 \pm 0.015$ Cvitanovic and Kinoshita,[4]

$a_6 = 1.110 \pm 0.037$ Carroll.[3]

The best experimental value is[5]

$$(a_6)_{\text{expt}} = 1.53 \pm 0.33\,.$$

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[1]M. J. Levine and R. Roskies, Phys. Rev. D 9, 421 (1974); M. J. Levine, R. C. Perisho, and R. Roskies, *ibid.* 13, 997 (1976).

[2]M. J. Levine and J. Wright, Phys. Rev. D 8, 3171 (1973).

[3]R. Carroll, Phys. Rev. D 12, 2344 (1975).

[4]P. Cvitanovic and T. Kinoshita, Phys. Rev. D 10, 4007 (1974).

[5]See e.g. the second paper in Ref. 1.