# Proposed experiment to test the nonseparability of quantum mechanics

Alain Aspect
*Institut d'Optique, Expériences Fondamentales en Optique, Université de Paris XI, 91405 Orsay, France*
(Received 2 December 1975; revised manuscript received 17 March 1976)

As a criterion between quantum mechanics and local hidden-variable theories, the so-called Einstein-Podolsky-Rosen paradox is mainly tested in the form of the statistical correlation between polarizations of photons issuing from a cascade transition. It has been stated more than once that an improved form of the test would make use of polarizers, the orientation of which would change randomly in a time comparable with the time of flight of the two photons; the Bell locality assumption could then be replaced by a weaker assumption also considered by Bell: The Einstein principle of separability. However, to our knowledge, no workable experimental scheme has yet been proposed, and we believe the one described in this paper to be a workable one. After explaining the difference between the Bell locality assumption and the Einstein principle of separability, we briefly discuss the theoretical implications of the modified experiment. The overall scheme of the apparatus we are proposing is described, and the generalized Bell inequalities, modified for our case, are derived. As in previous experiments, supplementary assumptions are made in order to derive experimentally testable inequalities. Finally, we describe the device we intend to use to carry out the proposed scheme.

## I. TEST OF THE PRINCIPLE OF SEPARABILITY

The so-called nonlocality paradox of Einstein, Podolsky, and Rosen[1] has been much discussed. Bell[2] has shown the possibility of bringing the question into the experimental domain. Then, several experiments have been proposed and performed.[3-8] All these experiments are able to discriminate between quantum mechanics and "local" hidden-variable theories that fulfill Bell's condition of locality: The setting of a measuring device does not influence the result obtained with another remote measuring device (nor does it influence the way in which particles are emitted by a distant source). Most of the experiments contradict these local hidden-variable theories,[4,5,7] although conflicting results exist.[6,8]

Although such a condition of locality looks highly reasonable, it is not prescribed by any fundamental physical law. Following a suggestion made by Bell,[2] we are proposing an experiment able to discriminate between quantum mechanics and "separable" hidden-variable theories fulfilling Einstein's principle of separability[9,10] that we can formulate in the following way for the experiments under consideration: The setting of a measuring device at a certain time (event $A$) does not influence the result obtained with another measuring device (event $B$) if the event $B$ is not in the forward light cone of event $A$ (nor does it influence the way in which particles are emitted by a source if the emission event is not in the forward light cone of event $A$).

Any theory fulfilling Bell's condition of locality also obeys Einstein's principle of separability. But one can conceive separable theories that do not fulfill Bell's condition of locality; such theories take into account the possibility of interactions between remote measuring devices (i.e., these theories do not fulfill Bell's condition of locality), but these interactions do not propagate with velocity greater than that of light (i.e., these theories obey Einstein's principle of separability). These theories were not within the reach of previous experiments, but they could be tested with a modification of these experiments.

To discuss this point, let us recall the optical transposition of Bohm's "*Gedankenexperiment*"[11] as performed by Freedman and Clauser[4] (Fig. 1). Letting $N(a_i, b_j)$ be the joint detection rate when the polarizers are in orientations $a_i$ and $b_j$ (the value $\infty$ represents the removal of the corresponding polarizer), one considers the quantiy

$$S=[1/N(\infty,\infty)][N(a_1,b_1)-N(a_1,b_2)+N(a_2,b_1)+N(a_2,b_2)-N(a_2,\infty)-N(\infty,b_1)], \tag{1}$$

where $a_1, a_2, b_1, b_2$ denote specific orientations of the polarizers in successive measurements. Local hidden-variable theories fulfilling Bell's condition of locality predict (modulo a supplementary assumption on the detector's efficiency) that $S$ is constrained by the generalized Bell inequalities[3,12-15]

$$-1 \leqslant S \leqslant 0\,. \tag{2}$$

For certain values of the orientation parameters $a_1, a_2, b_1, b_2$ the quantum-mechanical predictions violate the inequalities (2). Hence an experimental test between the conflicting theories is possible.

It has been emphasized that a crucial point in the derivation of the Bell inequalities is the locality assumption.[16] These inequalities could not be

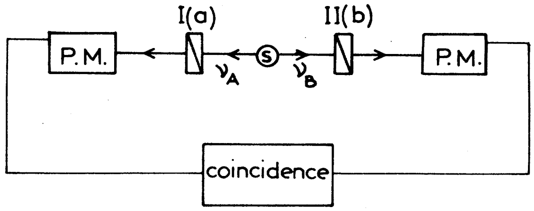


FIG. 1. Optical transposition of Bohm's "*Gedankenexperiment.*" The correlated photons $\nu_A$ and $\nu_B$, issuing from the cascade source $S$, impinge upon the linear polarizers I and II in orientations $a$ and $b$. The rate of joint detections by the photomultipliers is monitored for various couples of orientations $(a,b)$.

proved if the response of one polarizer was depending on the orientation of the other one (or if the emission of the pairs of photons was depending on the orientations of the polarizers). In the previous experiments, such interactions are not precluded by the principle of separability because "the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light".[2] Therefore the separable theories that do not fulfill Bell's condition of locality cannot be tested by the previous experiments.

To test these theories, it has been proposed[2,13,15,17] to change rapidly, repeatedly, and independently the orientations of the polarizers. Then one finds as a consequence of the principle of separability that the response of one polarizer, when analyzing a photon, cannot be influenced by the orientation of the other polarizer at the same time (when analyzing the coupled photon); likewise, the way in which a pair of photons is emitted cannot be influenced by the orientations of the polarizers when later analyzing this pair. Therefore, for such improved experiments, inequalities (2) can be derived from the principle of separability, with no further locality assumption made. Since inequalities (2) still conflict with the quantum-mechanical predictions, such modified experiments would be able to discriminate between quantum mechanics and separable hidden-variable theories.

A result consonant with the quantum-theory predictions would imply the rejection of separable hidden-variable theories. But as a matter of fact, it would imply more.[18] According to a recent analysis[19] it would constitute an experimental confirmation of the reality of the nonseparability introduced formally in the quantum theory. More generally, d'Espagnat[20] pointed out that such a result would entail consequences practically amounting to a disproof of the principle of separability, and he showed that these consequences (violation of some general assumptions) could be derived without reference to any specific theory (in particular without incorporating in the general assumptions just mentioned any *a priori* hypothesis about the existence of hidden variables[21]). Then, some reinterpretations of the quantum theory would be untenable, while others would be upheld.[22]

## II. PROPOSED EXPERIMENTAL SCHEME AND CORRESPONDING GENERALIZED BELL INEQUALITIES

Several authors[2,13,15,17] have already proposed to change rapidly and repeatedly the orientations of the polarizers, but few experimental practical suggestions have been given. One could think of using Kerr or Pockels cells, allowing changes in the polarization orientations in less than one nanosecond. Unfortunately, there are several drawbacks: Only very narrow beams could be transmitted, yielding very low coincidence rates; as these cells heat up, and then become inoperative, long runs would be prohibited. Last, a very sophisticated system would be needed for monitoring the change in time of the orientations; the calibration of the system would thus be exceedingly difficult.

We believe that these difficulties could be overcome by using optical commutators (Fig. 2). During a short time interval, the commutator $C_A$ directs the photons $\nu_A$ towards the polarizer $\mathrm{I}_1$; then its state changes and, during the following period, it directs $\nu_A$ towards the polarizer $\mathrm{I}_2$. The commutator $C_B$ works similarly with the photons $\nu_B$, independently of $C_A$. The time intervals between two commutations are taken to be stochastic, so that two states of the commutator, separated by a time longer than the autocorrelation time, are statistically independent. The autocorrelation time

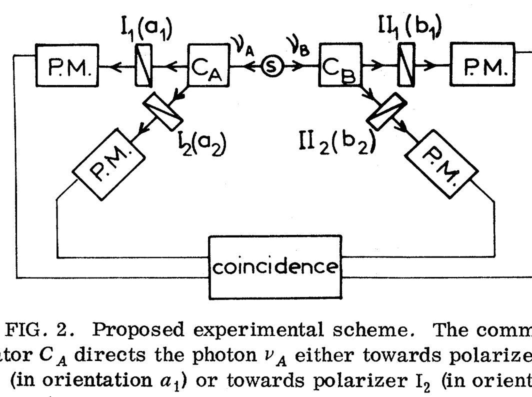


FIG. 2. Proposed experimental scheme. The commutator $C_A$ directs the photon $\nu_A$ either towards polarizer $\mathrm{I}_1$ (in orientation $a_1$) or towards polarizer $\mathrm{I}_2$ (in orientation $a_2$). Similarly $C_B$ directs $\nu_B$ towards $\mathrm{II}_1$ or $\mathrm{II}_2$ (in orientations $b_1$ and $b_2$). The two commutators work independently in a stochastic way. The four joint detection rates are monitored, and the orientations $a_1$, $a_2$, $b_1$, and $b_2$ are not changed for the whole experiment.

of each commutator is taken as short as $L/c$; $L$ denotes the distance between the commutators and $c$ denotes the speed of light.

One guesses that the separability assumption then leads to inequalities analogous to Bell's, with the arguments the same as in Sec. I[23,24]

Let us proceed to the actual derivation of the modified inequalities. We use the formalism of hidden-variable theories, but, as emphasized in Sec. I, the validity of the result will be more general.

Each emitted pair of photons is assumed to be characterized by a hidden variable $\lambda$ ($\lambda$ might stand for any number of parameters). We denote by $t_1$ the time of emission[25] of a pair of photons in the laboratory frame, and by $\hat{u}(\lambda, t_1)$ the probability distribution of $\lambda$. At the time $t_2 = t_1 + L/2c$ the photons $\nu_A$ and $\nu_B$ impinge upon the commutators. We define "commutation functions" $\hat{\alpha}_i(t)$ and $\hat{\beta}_j(t)$ ($i=1$ or 2 and $j=1$ or 2), the values of which are 1 or 0 according as, respectively, the photons are sent along the corresponding channel or not; $\hat{\alpha}_i(t)$ and $\hat{\beta}_j(t)$ are of course $\lambda$ independent. We denote by $t_3 > t_2$ the time at which the photons are anlayzed by the polarizers, and by $\hat{\mathfrak{A}}_i(\lambda, t_3)$ and $\hat{\mathfrak{B}}_j(\lambda, t_3)$ the corresponding response functions, the values of which are 1 or 0 according as the corresponding photon does or does not pass the polarizer. Thus $\hat{\alpha}_i(t_2)\hat{\beta}_j(t_2) \times \hat{\mathfrak{A}}_i(\lambda, t_3)\hat{\mathfrak{B}}_j(\lambda, t_3)$ assumes the value 1 if both photons $\nu_A$ and $\nu_B$ emerge from the polarizers $\mathrm{I}_i$ and $\mathrm{II}_j$ and 0 otherwise. Finally, the probability that a pair of photons $\nu_A$ and $\nu_B$ emerge in coincidence from $\mathrm{I}_i$ and $\mathrm{II}_j$ is

$$P_{ij} = \lim_{T\to\infty} \frac{1}{T}\int_T dt_1 \int d\lambda\, \hat{u}(\lambda, t_1)\hat{\alpha}_i(t_2)\hat{\beta}_j(t_2) \times \hat{\mathfrak{A}}_i(\lambda, t_3)\hat{\mathfrak{B}}_j(\lambda, t_3), \quad (3)$$

where the considered $\lambda$ is the initial value at the time $t_1$ of emission.

Following Bell, we can generalize this to the case where the polarizers themselves contain hidden variables contributing to the result. We denote by $\lambda'_{\mathrm{I}_i}$ and $\lambda'_{\mathrm{II}_j}$ the instrumental parameters of each polarizer; then the response of each polarizer is $\hat{\mathfrak{A}}_i(\lambda, \lambda'_{\mathrm{I}_i}, t_3)$ and $\hat{\mathfrak{B}}_j(\lambda, \lambda'_{\mathrm{II}_j}, t_3)$. This formalism is also appropriate for a stochastic theory,[16] since $\lambda'_{\mathrm{I}_i}$ and $\lambda'_{\mathrm{II}_j}$ can be taken as random variables without any specific interpretation. Then the probability $P_{ij}$ assumes the form

$$P_{ij} = \lim_{T\to\infty} \frac{1}{T}\int_T dt_1 \int d\lambda \int \cdots \int d\lambda'_{\mathrm{I}_1} \cdots d\lambda'_{\mathrm{II}_2}\hat{u}(\lambda, \lambda'_{\mathrm{I}_1}, \ldots, \lambda'_{\mathrm{II}_2}, t_1, t_3) \times \hat{\alpha}_i(t_2)\hat{\beta}_j(t_2)\hat{\mathfrak{A}}_i(\lambda, \lambda'_{\mathrm{I}_i}, t_3)\hat{\mathfrak{B}}_j(\lambda, \lambda'_{\mathrm{II}_j}, t_3), \quad (4)$$

where $\hat{u}(\cdots)$ is the probability distribution of $\lambda$ at time $t_1$ and of the instrumental parameters at time $t_3$.

Correlations may exist between the instrumental parameters $\lambda'_{\mathrm{I}_i}$ and $\lambda'_{\mathrm{II}_j}$. However, in accordance with the principle of separability, correlations at time $t_3$ can be produced only by common causes at times $t \le t_1$. Introducing these common causes in formula (4), one remarks that these common causes are always coupled with $\lambda$; then for simplicity these common causes are included as a part of the fully general $\lambda$.

In accordance with the principle of separability, the given $\lambda$ at times $t \le t_1$ describe in a complete manner all the correlations between the instrumental responses at time $t_3$; therefore the conditional probability distribution of the instrumental parameters, given a particular value of $\lambda$, is factorized.[26] Hence, with $\hat{u}(\lambda, \lambda'_{\mathrm{I}_1}, \ldots, \lambda'_{\mathrm{II}_2}, t_1, t_3)$ expressed as a function of the probability distribution $\hat{\rho}(\lambda, t_1)$ of $\lambda$ at time $t_1$ and of the conditional probability distributions of each instrumental parameter, $P_{ij}$ (formula 4) assumes the form

$$P_{ij} = \lim_{T\to\infty} \frac{1}{T}\int_T dt_1 \int d\lambda\, \hat{\rho}(\lambda, t_1)\hat{\alpha}_i(t_2)\hat{\beta}_j(t_2) \times \hat{A}_i(\lambda, t_3)\hat{B}_j(\lambda, t_3), \quad (5)$$

where $\hat{A}_i(\lambda, t_3)$ and $\hat{B}_j(\lambda, t_3)$ are the average values—over the respective instrumental parameters—of the instrumental response functions at time $t_3$.

When integrating over time, we must consider the possibility of interactions between the commutators and the other devices. The principle of separability precludes instantaneous interactions but not retarded ones. Hence the response of polarizer $\mathrm{I}_i$ (for instance) at time $t_3$ might depend upon the state of commutator $C_A$ at times $t \le t_2$ and also upon the state of commutator $C_B$ at times $t \le t_2 - L/c$. Similarly, the emission at time $t_1$ might depend upon the states of both commutators at times $t \le t_1 - L/2c$. Taking into account these possible interactions, it can be shown (see Appendix)—by using the assumed properties of the commutators (stochastic independent workings, autocorrelation times shorter than $L/c$)—that the probability of coincident emergence factorizes as

$$P_{ij}=\alpha_i\beta_j\int d\lambda\,\rho(\lambda)A_i(\lambda)B_j(\lambda)\,, \qquad (6)$$

where $\alpha_i$ and $\beta_j$ denote the averages over time of the commutation functions $\hat{\alpha}_i(t)$ and $\hat{\beta}_j(t)$, $\rho(\lambda)$ is a function with the properties of a probability distribution, and $A_i(\lambda)$ and $B_j(\lambda)$ are functions related to the responses of polarizers $\mathrm{I}_i$ and $\mathrm{II}_j$, constrained by the inequalities

$$\begin{aligned}&0\le A_i(\lambda)\le 1\,,\\&0\le B_j(\lambda)\le 1\,.\end{aligned} \qquad (7)$$

With the same notations, the probabilities that one photon emerges from $\mathrm{I}_i$ (or $\mathrm{II}_j$) are (respectively)

$$\begin{aligned}&P_{i0}=\alpha_i\int d\lambda\,\rho(\lambda)A_i(\lambda)\,,\\&P_{0j}=\beta_j\int d\lambda\,\rho(\lambda)B_j(\lambda)\,.\end{aligned} \qquad (8)$$

Thus, using the principle of separability, we have derived a factorized form similar to the definition of objective local theories.[15] Nevertheless, there is a large difference: The response of one polarizer, which (given $\lambda$) depends upon its orientation, might also depend upon the orientations of the other polarizers (since we do not make the Bell locality assumption). Similarly, the way in which the pairs of photons are emitted [and therefore $\rho(\lambda)$] might depend upon the polarizers' orientations. But, in our experiment, all the orientations remain unchanged during the whole course of the experiment; hence, in formula (6), $\rho(\lambda)$ does not depend upon the indices $i$ or $j$; likewise, $A_i(\lambda)$ does not depend upon the index $j$ [nor does $A_j(\lambda)$ depend upon $i$].

Therefore, the derivation of Clauser and Horne[15] holds. Inequalities (7) entail

$$-1\le U\le 0\,, \qquad (9)$$

with ($\lambda$ is dropped for simplicity)

$$U\equiv A_1B_1-A_1B_2+A_2B_1+A_2B_2-A_2-B_1\,.$$

After multiplication by $\rho(\lambda)$ and integration, one obtains

$$-1\le S\le 0\,, \qquad (10)$$

with

$$S\equiv\frac{P_{11}}{\alpha_1\beta_1}-\frac{P_{12}}{\alpha_1\beta_2}+\frac{P_{21}}{\alpha_2\beta_1}+\frac{P_{22}}{\alpha_2\beta_2}-\frac{P_{20}}{\alpha_2}-\frac{P_{01}}{\beta_1}\,. \qquad (11)$$

Inequalities (10) are isomorphic to the generalized Bell inequalities.[15] On the other hand, the quantum-mechanical predictions, which are

$$\begin{aligned}&P_{ij}=\tfrac{1}{2}\alpha_i\beta_j\cos^2(a_i,b_j)\,,\\&P_{i0}=\tfrac{1}{2}\alpha_i\,,\\&\text{and}\\&P_{0j}=\tfrac{1}{2}\beta_j\,,\end{aligned} \qquad (12)$$

lead to a violation of the inequalities (10), the maximum of which occurs respectively for

$$(a_1,b_1)=(b_1,a_2)=(a_2,b_2)=\begin{cases}22.5^\circ\\ \text{or}\\ 67.5^\circ\end{cases}$$

and (13)

$$(a_1,b_2)=\begin{cases}67.5^\circ\\ \text{or}\\ 202.5^\circ\,.\end{cases}$$

For these specific orientations, we indeed have

$$S=\begin{cases}0.207\\ \text{or}\\ 1.207\,.\end{cases}$$

All the quantities involved in formula (11) are probabilities of coincident—or single—emergence and could, in principle, be measured. We thus have a test for separable hidden-variable theories. This test will be operational if we find means for measuring these probabilities and designing commutators obeying the assumptions we have made.

## III. TESTABLE INEQUALITIES

Although theoretically measurable, the probabilities in formula (11) cannot be measured directly for two reasons.[3,15] First, optical photomultipliers have a low quantum efficiency; hence the rate of joint detection will be lower than the true rate of coincident emergence from the corresponding polarizers. Second, in atomic cascades only a fraction of pairs fly in opposite directions (since we have a three-body decay); hence a photon $\nu_A$ (for instance) may impinge upon commutator $C_A$ and be analyzed while the corresponding $\nu_B$ is lost; therefore the rate of single detection is not a faithful measurement of the probability that a single photon is transmitted by the corresponding polarizer.

Denoting by $N$ the average rate of emission of processed pairs (i.e., with both photons impinging upon the commutators) and by $\epsilon_{ij}$ a numerical factor accounting for the quantum efficiency of the corresponding photomultipliers, the rate of joint detection may be expressed as

$$N_{ij}(a_i,b_j)=\epsilon_{ij}P_{ij}N\,. \qquad (14)$$

As in previous work,[3,4,8,12] we assume that $\epsilon_{ij}$ does not depend on whether or not the photons have passed through a polarizer.[27] We also assume that the average rate of pair emissions $N$ is not changed when a polarizer is removed [although, as stated in Sec. II, we accept that $\hat{u}(\lambda, t)$ might be changed]. Therefore the rate of joint detection in channels $I_i$ and $II_j$ when the corresponding polarizers are removed will be

$$N_{ij}(\infty, \infty) = \epsilon_{ij}\, \alpha_i \beta_j N, \tag{15}$$

where $\alpha_i \beta_j$ is the probability of a coincident emergence from the commutators $C_A$ and $C_B$ along the corresponding channels.

We thus obtain

$$\frac{P_{ij}}{\alpha_i \beta_j} = \frac{N_{ij}(a_i, b_j)}{N_{ij}(\infty, \infty)} \tag{16}$$

and then the four first terms in formula (11) are put as functions of measurable quantities.

One more assumption is needed to render the two last terms of $S$ measurable: that the probability that a photon emerges out of a polarizer does not depend upon whether or not another polarizer has been removed[28] (although, as stated above, we accept that the elementary response might be changed).

Therefore, if the polarizer $II_j$ is removed, the probability of joint emergence of $\nu_A$ from polarizer $I_i$ and of $\nu_B$ along channel $II_j$ is $\beta_j P_{i0}$ (see Appendix), and the rate of joint detections, with polarizer $II_j$ removed, is

$$N_{ij}(a_i, \infty) = \epsilon_{ij}\, \beta_j P_{i0} N. \tag{17}$$

We finally obtain

$$\frac{P_{i0}}{\alpha_i} = \frac{N_{ij}(a_i, \infty)}{N_{ij}(\infty, \infty)}, \qquad \frac{P_{0j}}{\beta_j} = \frac{N_{ij}(\infty, b_j)}{N_{ij}(\infty, \infty)}, \tag{18}$$

and the two last terms in formula (11) can be measured.

On the whole, $S$ in formula (11) is expressed as

$$S = \frac{N_{11}(a_1, b_1)}{N_{11}(\infty, \infty)} - \frac{N_{12}(a_1, b_2)}{N_{12}(\infty, \infty)} + \frac{N_{21}(a_2, b_1)}{N_{21}(\infty, \infty)} + \frac{N_{22}(a_2, b_2)}{N_{22}(\infty, \infty)} - \frac{N_{2j}(a_2, \infty)}{N_{2j}(\infty, \infty)} - \frac{N_{i1}(\infty, b_1)}{N_{i1}(\infty, \infty)}. \tag{19}$$

The four quantities $N_{ij}(a_i, b_j)$ are measured during one single run of the experiment, and this is a very significant difference between our proposal and the previous schemes. The other quantities are measured in auxiliary calibrations.

Concluding this section, we have been able to define a practical scheme for measuring the probabilities, modulo some reasonable assumptions which, however, restrict somewhat the generality of the derivation of Sec. II.

## IV. OVERALL EXPERIMENTAL SETUP

Clauser *et al.*[3] have already discussed the case of nonideal polarizers and extended beams. These features of a realistic experiment decrease somewhat the quantum-mechanical violation of the inequalities (10). Significant experiments have nevertheless been carried out to test the locality condition. Our experiment could be built with the same sort of source, polarizers, and detecting devices.

The specificity of our experiment is the presence of two optical commutators. These can consist of acoustic standing waves working as adjustable gratings (Fig. 3). The deviation of a light beam by strong interactions with an acoustic wave has been studied[29] both theoretically and experimentally. We are planning to use commutators with a surface of 4 cm$^2$ and an angular aperture of $1° \times 30°$. By appropriately adjusting the various parameters, we should be able to obtain one single beam diffracted, i.e., two channels.

The transmitted and diffracted beams will be modulated in opposition at twice the frequency of the sound wave. We expect the modulation rate of the transmitted beam to be over 90%, and polarization independent.

We finally must discuss to what extent our commutators obey the assumptions stated in Sec. II. Since the modulation rate is not exactly 100% and the commutation is not instantaneous, the values of the commutation functions $\hat{\alpha}_i(t)$ and $\hat{\beta}_j(t)$ are not restricted to 0 and 1. Nevertheless, the reasoning in Sec. II still holds if, with $\hat{\alpha}_i(t)$ and $\hat{\beta}_j(t)$ denoting the probability that a photon is directed towards channel $I_i$ or $II_j$, these commutation func-

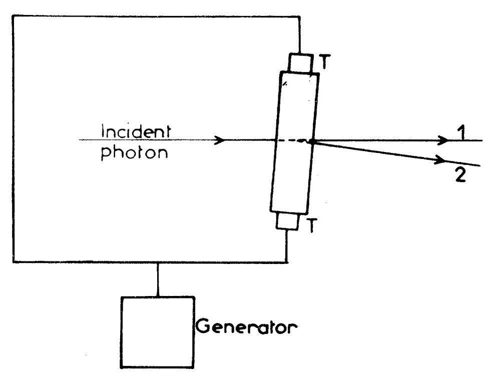


FIG. 3. Optical commutator. The generator supplies two identical transducers $T$, producing an acoustic standing wave in a crystal. The diffracted beam (channel 2) is modulated at twice the frequency of the standing wave. The transmitted beam (channel 1) can be modulated at a rate greater than 90%; hence the commutation is nearly complete.

tions do not depend on the hidden variable $\lambda$. This makes one more assumption. If it is unsatisfied, a fraction of the photons could be directed one way or the other, depending on the value of $\lambda$, and a "conspiracy" of the commutators and polarizers could decrease the difference between the quantum-mechanics predictions and the separable hidden-variable theories predictions. However, if the quantum-mechanics predictions were vindicated for various modulation rates, the occurrence of such a conspiracy might appear as *a priori* unlikely. Nevertheless, such a "conspiracy" could be avoided by inhibiting the detectors when the commutation functions assume values significantly different from 0 or 1 (during the rise time).

Another difficulty is that the commutators are operated periodically and not in a truly stochastic way. However, the significant requirement is that we have two independent commutation functions, each with autocorrelation time shorter than $L/c$—say, shorter than 20 nsec if $L$ is 6 m. It seems that these conditions will be fulfilled if the commutators are separately driven by macroscopic generators whose frequencies deviate independently. We can drive a pseudorandom deviation of the frequency of each commutator, and a direct action of the source upon the driving mechanisms (and possibly upon the operator's descision; see footnote 13 of Ref. 15) seems very unlikely. For instance, the standing-wave frequency can vary between 100 and 125 MHz, and the commutation frequency will vary between 200 and 250 MHz. Then the autocorrelation time, which is of the order of the inverse of the line width, is about 20 nsec. If a sweeping of the frequency over a broad line turned out to be too difficult, a supplementary assumption should be exhibited: The polarizers have no "memory," i.e., they can be influenced by signals received at a certain time from the commutators (with a certain delay) but they cannot store all this information for a long time and extrapolate in the future even if there is some regularity in the working of the commutators. With this very natural supplementary assumption, the experiment would be significant even with periodic commutations.

## V. BRIEF CONCLUSION AND ACKNOWLEDGMENTS

We believe that the experimental scheme we are proposing, although it is not an ideal one, is interesting in that it embodies a device for changing the orientations of the analyzers in a time comparable to the time of flight of the photons. Such a feature has been considered a crucial one by quite a few workers in the field, and therefore such experiments are worth making, even if they are not ideal.

The author gratefully acknowledges Professor C. Imbert and Dr. O. Costa de Beauregard for having suggested this study and for many fruitful discussions. He especially thanks Dr. J. S. Bell for his encouragement, and Professor B. d'Espagnat for his thorough consideration and discussion of the theoretical aspects of our scheme.

## APPENDIX

The probability $P_{ij}$ in Eq. (11) is the time average of the quantity [see Eq. (5)]

$$\hat{F}_{ij}(\lambda, t_1) = \int d\lambda\, \hat{\rho}(\lambda, t_1)\hat{\alpha}_i(t_2)\hat{\beta}_j(t_2) \times \hat{A}_i(\lambda, t_3)\hat{B}_j(\lambda, t_3),$$

where $t_2 = t_1 + L/2c$ and $t_3 - t_2$ is the time of flight of photons between a commutator and a polarizer.

Since the commutators work in a stationary stochastic way, we may replace the time average by ensemble averages. Denoting by $X(t)$ and $Y(t)$ random variables specifying the states at the time $t$ of commutators $C_A$ and $C_B$, respectively, we replace the time functions ^ by functions ~ of these random variables. Taking care of the various possible interactions (as explained in Sec. II) we obtain

$$\hat{\alpha}_i(t_2) \to \tilde{\alpha}_i[X(t_2)],$$

$$\hat{\beta}_j(t_2) \to \tilde{\beta}_j[Y(t_2)],$$

$$\hat{\rho}(\lambda, t_1) \to \tilde{\rho}[\lambda, X(t_0), Y(t_0)],$$

$$\hat{A}_i(\lambda, t_3) \to \tilde{A}_i[\lambda, X(t_0), X(t_2), X(t_4), Y(t_0)],$$

$$\hat{B}_j(\lambda, t_3) \to \tilde{B}_j[\lambda, Y(t_0), Y(t_2), Y(t_4), X(t_0)],$$

and in general

$$\hat{F}_{ij}(\lambda, t_1) \to \tilde{F}_{ij}[\lambda, X(t_0) \cdots Y(t_4)],$$

where

$$t_0 \leq t_1 - L/2c$$

and

$$t_1 - L/2c < t_4 < t_2.$$

As the two commutators are working independently, $X(t)$ and $Y(t)$ are independent random variables. As the autocorrelation time of each commutator is shorter than $L/c$, $X(t_0)$ and $X(t_2)$ are independent random variables, and so are $Y(t_0)$ and $Y(t_2)$.

We average $\tilde{F}_{ij}$, i.e., we integrate after multiplying by the probability distribution

$$\tilde{g}[X(t_0), X(t_2), X(t_4)]\tilde{h}[Y(t_0), Y(t_2), Y(t_4)]$$

in a factorized form since $X(t)$ and $Y(t)$ are independent. Integrating over $X(t_2)$ and $X(t_4)$, we then define

$$A_i[\lambda, X(t_0), Y(t_0)] = \{\alpha_i \tilde{g}'[X(t_0)]\}^{-1} \times \iint dX(t_2)\, dX(t_4)\, \tilde{g}[X(t_0), X(t_2), X(t_4)]\, \tilde{\alpha}_i[X(t_2)]\, \tilde{A}_i[\lambda, X(t_0), X(t_2), X(t_4)],$$

where

$$\tilde{g}'[X(t_0)] = \iint dX(t_2)\, dX(t_4)\, \tilde{g}[X(t_0), X(t_2), X(t_4)]$$

is the probability distribution of $X(t_0)$ and $\alpha_i$ is the average of $\tilde{\alpha}_i[X(t_2)]$. Remembering that $\tilde{A}_i$ assumes values 0 or 1 and that all the factors are positive, we obtain

$$0 \leq A_i[\lambda, X(t_0), Y(t_0)] \leq \{\alpha_i \tilde{g}'[X(t_0)]\}^{-1} \iint dX(t_2)\, dX(t_4)\, \tilde{g}[X(t_0), X(t_2), X(t_4)]\, \tilde{\alpha}_i[X(t_2)],$$

hence, remarking that $\int dX(t_4)\, \tilde{g}[X(t_0), X(t_2), X(t_4)]$ factorizes, because $X(t_0)$ and $X(t_2)$ are independent random variables, we obtain

$$0 \leq A_i[\lambda, X(t_0), Y(t_0)] \leq 1 .$$

By a completely similar fashion, with $\tilde{h}'[Y(t_0)]$ replacing $\tilde{g}'[X(t_0)]$, we obtain

$$0 \leq B_j[\lambda, X(t_0), Y(t_0)] \leq 1 .$$

The average of $\tilde{F}_{ij}$ is then

$$P_{ij} = \alpha_i \beta_j \iiint d\lambda\, dX(t_0)\, dY(t_0)\, \tilde{g}'[X(t_0)]\, \tilde{h}'[Y(t_0)]\, \tilde{\rho}[\lambda, X(t_0), Y(t_0)]\, A_i[\lambda, X(t_0), Y(t_0)]\, B_j[\lambda, X(t_0), Y(t_0)] .$$

For simplicity we can include $X(t_0)$ and $Y(t_0)$ into the parameter $\lambda$, thus obtaining formulas (6) and (7).

Similarly, the single probability that a photon emerges from polarizer $\mathrm{I}_i$ is the time average of

$$\int d\lambda\, \hat{\rho}(\lambda, t_1)\hat{\alpha}_i(t_2)\hat{A}_i(\lambda, t_3),$$

and, through the same procedure as above, we obtain formula (8).

Finally, the joint probability of photon $\nu_A$ emerging from polarizer $\mathrm{I}_i$ and photon $\nu_B$ emerging from commutator $C_B$ into the channel $\mathrm{II}_j$ is the time average of

$$\int d\lambda\, \hat{\rho}(\lambda, t_1)\hat{\alpha}_i(t_2)\hat{\beta}_j(t_2)\hat{A}_i(\lambda, t_3) .$$

Remembering that $X(t_2)$ and $Y(t_2)$ are independent, we obtain $\beta_j P_{i0}$ as the expression of this probability; this expression is used in Sec. III [Eq. (17)].

---

[1]A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).

[2]J. S. Bell, Physics 1, 195 (1965).

[3]J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).

[4]S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. 28, 938 (1972).

[5]L. Kasday, in *Foundations of Quantum Mechanics*, Proceedings of the International School of Physics "Enrico Fermi," Course 49, edited by B. d'Espagnat (Academic, New York, 1972).

[6]G. Faraci, D. Gutkowski, S. Notarrigo, and A. R. Pennisi, Lett. Nuovo Cimento 9, 607 (1974).

[7]M. Lamehi-Rachti and W. Mittig, Phys. Rev. D (to be published).

[8]R. A. Holt, Ph.D. thesis, Harvard University, 1973 (unpublished).

[9]The words "locality" and "separability" (and likewise "local" and "separable") are sometimes taken as synonymous. However, the specific definitions given here are natural and, in the context of the present paper, useful.

[10]The expression "principle of separability of Einstein" is used by B. d'Espagnat in *Conceptual Foundations of Quantum Mechanics* (Benjamin, Reading, Mass., 1971). This principle is applied to "the real, factual situation of the system" [A. Einstein, in *Albert Einstein: Philosopher-Scientist*, edited by P. Schilpp (Open Court, La Salle, Ill., 1949)]; it is more stringent than the principle of causality; for instance, in the Einstein-Podolsky-Rosen situation, the principle of separability may be violated, although one cannot send orders

faster than light.

[11]D. J. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, N. J., 1951).

[12]M. A. Horne, thesis, Boston Univ., 1970 (unpublished).

[13]A. Shimony, in *Foundations of Quantum Mechanics* (Ref. 5).

[14]E. P. Wigner, Am. J. Phys. 38, 1005 (1970).

[15]J. F. Clauser and M. A. Horne, Phys. Rev. D 10, 526 (1974).

[16]J. S. Bell (in Ref. 5) and Clauser and Horne (Ref. 15) have proved that the hypothesis of determinism is not necessary to derive inequalities conflicting with some quantum-mechanical predictions. The latter have shown that such inequalities hold for objective local theories which are "essentially...stochastic hidden variable theories with a certain local character considered by Bell" (see Ref. 15).

[17]Y. Aharonov and D. Bohm, Phys. Rev. 108, 1970 (1957).

[18]The hypothesis of Furry (that the wave function of a many-body system tends to factorize into a product of localized states when the two parts are separated) would be rejected without the locality assumption (see Ref. 17).

[19]H. P. Stapp, LBL Report No. 3837, 1975 (unpublished).

[20]B. d'Espagnat, Phys. Rev. D 11, 1424 (1975).

[21]The idea that the hypothesis of the existence of a hidden variable would not be necessary to derive Bell's inequalities is in several papers [see for instance L. E. Ballentine, Phys. Today 27 (No. 10), 53 (1974); A. Baracca, C. J. Bohm, and B. J. Hiley, report (unpublished); see also Ref. 15 and a recent paper of J. S. Bell: CERN Report No. TH. 2053, 1975 (unpublished)].

[22]Among the authors who have considered a possible rejection of the principle of separability, Costa de Beauregard considers "the paradoxical scheme accepting an indirect spacelike connection via the two timelike vectors connecting the cascade event to the commutation events (and to the two subsequent counting events)." He goes on to say the following: "The corresponding space-time channels are indeed carrying waves and, moreover, the calculation of the quantal correlation probabilities does imply the phase binding between these waves. In this sense the (indirect) spacelike correlation, or nonseparability, predicted by the quantum theory, can be said to be physical *and* mathematical *and* logical.

"This correlation is nevertheless paradoxical, inasmuch as it asserts that the two (apparently) independent reductions of the wave function occurring in the distant pieces of apparatus are indeed in contact through their common past (and, thus, not really independent).

"This implied in some sense that Einstein's prohibition to telegraph into the past does not hold at the level of the individual quantal stochastic event (transition, or $\Psi$ collapse). This is in line with the intrinsic time-symmetry also postulated in classical statistical mechanics for the individual stochastic event" (private communication).

See also O. Costa de Beauregard, unpublished work and work reported in *Proceedings of the International Conference on Thermodynamics*, edited by P. T. Landsberg (Butterworths, London, 1970), p. 540.

[23]A. Aspect, Phys. Lett. 54A, 117 (1975).

[24]A modification of our scheme has been suggested by J. F. Clauser (private communication). The two detectors $I_1$ and $I_2$ (Fig. 2) could be "or-wired", giving the equivalent of a single-variable polarizer with an orientation jumping from $a_1$ to $a_2$ and reversely. The time of change of the orientation should be monitored in order that the experiment be significant.

[25]The time of emission is not actually the same for the two photons, but our derivation remains valid when the lifetime of the medium state is shorter than the time of flight of the photons.

[26]A similar argument is used by Bell for a very general derivation of inequalities [J. S. Bell, CERN Report No. TH.2053, 1975 (unpublished)].

[27]Clauser and Horne (Ref. 15) found a weaker assumption for previous experiments. However, the widening of their reasoning to our case is not straightforward.

[28]This hypothesis may appear as strong as the Bell condition of locality. Actually it is quite a different hypothesis, concerning the probability and not the elementary response. It could then be experimentally tested with an ideally efficient photomultiplier. We can practically make several tests; for instance, we can verify that

$$\frac{N_{i1}(a_i,\infty)}{N_{i1}(\infty,\infty)}=\frac{N_{i2}(a_i,\infty)}{N_{i2}(\infty,\infty)}\,.$$

Such verifications would give good confidence in our hypothesis.

[29]J. M. Bauza, C. Carles, and R. Torguet, Acustica 30, 137 (1974); V. N. Mahajan and Jack D. Gaskill, Optica Acta 21, 893 (1974).