

## Pseudoscalar mixing effects on hadronic and photonic decays of the new mesons\*

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We calculate the mixing of  $\eta$ ,  $\eta'$ , and  $\eta_c$  in a cylinder-dominated model and apply our results to the hadronic decay  $\psi' \rightarrow \psi\eta$  and a number of photonic decays, using vector-meson dominance. The results are in excellent agreement with all experimental data.

### I. INTRODUCTION

In Refs. 1, we have discussed violation of the Okubo-Zweig-Iizuka (OZI) rule in the context of a model in which an intermediate state mediates the forbidden transitions. Unitarity requires this intermediate state; whether it is a cut (e.g.,  $\psi \rightarrow DD^* \rightarrow \rho\pi$  in  $\psi$  decay) or a real particle (a gluon bound state, an empty bag, an "O meson," a closed string), it seems to be rather well parametrized by a pole in the  $J^P = 1^-$  and  $0^+$  channels we addressed in Refs. 1, which included some of the more interesting decays of  $\psi$  and  $\psi'$  as well as the classical  $\phi \rightarrow \rho\pi$  rate.

This model will be extended here to OZI-rule-violating transitions in the  $0^-$  channel, which has been treated by several other authors.<sup>2-6</sup> The strikingly large  $\psi' \rightarrow \psi\eta$  rate, considering the small phase space, proves to be an interesting challenge for the model; furthermore, the model provides an interesting alternative to the treatment of Harari,<sup>2</sup> who finds a huge admixture of charm in  $\eta$  and  $\eta'$ , and encounters some problems with photonic decays. (Our results are summarized in Table II.)

Rooted in dual models and dual diagrams, the model<sup>7</sup> correlates deviations from the ideal-mixing mass formula with deviation from ideal mixing in the states via an  $s$ -dependent interaction in the forbidden transition elements. In terms of dual diagrams the OZI-rule-forbidden process is one in which there is a U turn [Fig. 1(a)]. If one views the quarks as being at the ends of a string, then this can be pictured as a closing of the string into a circle, which then reopens, with equal probability, into any quark-antiquark state, according to SU(4) symmetry of the basic interaction. The closed string, or flux ring, sweeps out a cylinder, whose moving-flux-line boundaries, when cut, form tears in the cylinder bounded by quark lines which propagate in time the now open flux lines [Fig. 1(b)].

Within the framework of the topological expan-

sion,<sup>8</sup> the cylinder diagrams are of second order in a perturbation in higher and higher orders of the topology; the lowest-order diagrams are the conventional planar graphs.

In this framework one associates the cylinder with the Pomeron singularity. Freund and Nambu,<sup>7</sup> in the context of a string picture, point out that both senses of flux circulation allow for both charge conjugations, and associate the  $2^{++}, 1^{--}, 0^{++}$  closed strings with the Pomeron trajectory or its daughters. We have used these objects in Refs. 1 to study a number of OZI-rule suppressions.

In all our previous work we have been careful not to restrict ourselves to a particular dynamical structure for the Pomeron and its associated singularities. It is our feeling that these objects are not simple poles even when we often treated them as such as a convenient approximation. In particular, the cylinder corrections may be Pomeron-Reggeon cuts, suggesting cylinders in all quantum-number states which have Reggeons.

Moreover, if cylinders are established in the  $0^+$  and  $1^-$  channels, one can use a topological duality to infer the existence of a cylinder in the  $0^-$  channel. By topological duality we mean that topologically equivalent diagrams are dual in the usual sense. Consider for example the  $\psi \rightarrow \phi\eta$  diagram. This is doubly suppressed and requires two cylinders [see Figs. 1(c)]; it has the topology of a sphere with three holes with a particle attached at each hole. It is topologically equivalent to Fig. 1(d), which has a  $0^+$  cylinder. The line of argument is analogous to pinching an ordinary planar graph in different ways to infer existence of quark-model states in  $s$  and  $t$  channels.

### II. MASS-DEGENERATE MATRIX

As a result of the cylinder correction in the  $0^-$  channel, the ideally mixed "planar" states will be mixed and the masses will be shifted.

We take the cylinder interaction to be

$$Q = \begin{bmatrix} 0 & 0 & 0 & \sqrt{2}f \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & f \\ \sqrt{2}f & f & f & 0 \end{bmatrix}, \quad (1)$$

where the channels are  $\eta$ ,  $\eta'$ ,  $\eta_c$ , and  $O$  meson, respectively. In second order this generates the interaction

$$O = QPQ = \begin{bmatrix} \frac{2f^2}{s-m_O^2} & \frac{\sqrt{2}f^2}{s-m_O^2} & \frac{\sqrt{2}f^2}{s-m_O^2} & 0 \\ \frac{\sqrt{2}f^2}{s-m_O^2} & \frac{f^2}{s-m_O^2} & \frac{f^2}{s-m_O^2} & 0 \\ \frac{\sqrt{2}f^2}{s-m_O^2} & \frac{f^2}{s-m_O^2} & \frac{f^2}{s-m_O^2} & 0 \\ 0 & 0 & 0 & \frac{2f^2}{s-m_{\eta'}^2} + \frac{f^2}{s-m_{\eta}^2} + \frac{f^2}{s-m_{\eta_c}^2} \end{bmatrix}, \quad (2)$$

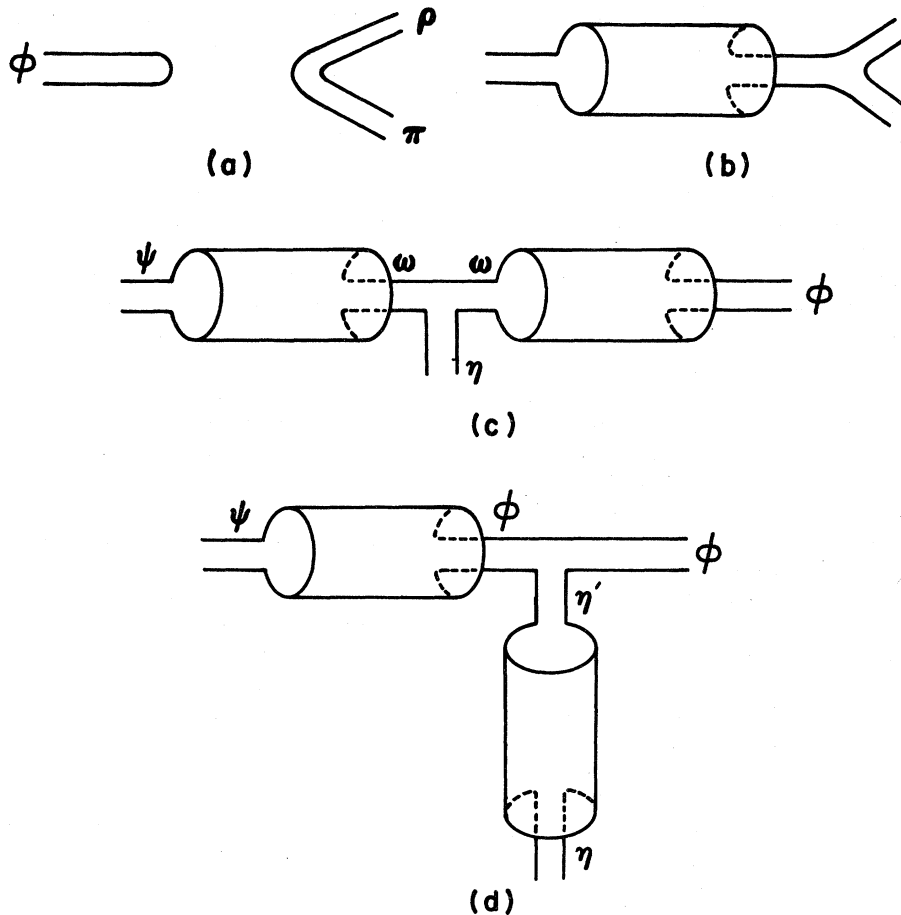


FIG. 1. (a) Dual diagram for  $\phi \rightarrow \rho\pi$ , (b) equivalent cylinder diagram, (c) and (d) equivalent topological diagrams for  $\psi \rightarrow \phi\eta$ .

with

$$P = \begin{bmatrix} (s - m_{\bar{\eta}}^2) & 0 & 0 & 0 \\ 0 & (s - m_{\bar{\eta}'}^2)^{-1} & 0 & 0 \\ 0 & 0 & (s - m_{\bar{\eta}_c}^2)^{-1} & 0 \\ 0 & 0 & 0 & (s - m_{\bar{O}}^2)^{-1} \end{bmatrix}, \quad (3)$$

where the renormalized propagator is

$$\pi^{\alpha\beta} = (P^{-1} - Q)^{-1} = \sum_i \frac{V_i^\alpha V_i^\beta}{s - m_i^2} \equiv \sum_i \frac{\mathcal{R}_{\alpha\beta}^i}{s - m_i^2}, \quad (4)$$

$$m_i = (m_\eta, m_{\eta'}, m_{\eta_c}, m_O) \text{ are solutions of } |\pi| = 0, \quad (5)$$

and

$$V_i = \frac{[\sqrt{2}f(m_i - m_{\bar{\eta}}^2)^{-1}, f(m_i^2 - m_{\bar{\eta}'}^2)^{-1}, f(m_i^2 - m_{\bar{\eta}_c}^2)^{-1}, 1]}{\left(1 + \frac{2f^2}{(m_i^2 - m_{\bar{\eta}}^2)^2} + \frac{f^2}{(m_i - m_{\bar{\eta}'}^2)^2} + \frac{f^2}{(m_i^2 - m_{\bar{\eta}_c}^2)^2}\right)^{1/2}}, \quad (6)$$

The fourth  $O$  channel represents a quarkless state which mediates the OZI-rule violation, and is here approximated by a pole. As implemented, probability leaks into this state and we have  $4 \times 4$  orthogonality and completeness. It is possible to reformulate this problem in a *physically inequivalent* form with a general interaction

$$O = h(s) \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}. \quad (7)$$

Since this is an energy-dependent interaction, orthogonality and completeness hold at a given value of  $s$ . Thus, in evaluating the residues orthogonality is lost but completeness is realized in a  $3 \times 3$  sense. The procedure is technically complicated by subsidiary conditions on  $h(s)$  which guarantee the stability of the physical masses and the  $3 \times 3$  completeness. This will be described in detail elsewhere.<sup>9</sup> The main result of this calculation is that *there exists an alternative to the  $4 \times 4$  system we describe whose numerical output is the  $3 \times 3$  submatrix of the  $4 \times 4$  residue matrix, renormalized so that the diagonal residues sum to unity in the  $3 \times 3$  channel space.*

Continuing with the  $4 \times 4$  theory, with  $m_{\bar{\eta}}^2 = m_{\eta'}^2$  and  $m_{\bar{\eta}}^2 = 2m_K^2 - m_{\eta'}^2$  specified by the ideal-mixing formula, and  $m_\eta^2$ ,  $m_{\eta'}^2$ , and  $m_{\eta_c}^2$  determined by experiment (we take  $m_{\eta_c}^2$  to be recently discovered state at 2.80 GeV), we find that the theory is completely determined, yielding

$$f_{OP}^2 = 0.1916, \quad (8)$$

$$m_{\bar{\eta}_c}^2 = 2.79 \text{ GeV},$$

and the residue matrices of Table I.

With reference to the alternative  $3 \times 3$  theory discussed above, the practical effect is to drop the  $O$  sector, leave the  $\mathcal{R}^\eta$  and  $\mathcal{R}^{\eta_c}$  residues essentially unchanged, and increase all  $\mathcal{R}^{\eta'}$  residues by  $\sim 70\%$ . Effects of this difference on our results will be noted below, where MI refers to the  $4 \times 4$  model and MII to the  $3 \times 3$  model. When the differences are small the particular model will not be identified.

### III. HADRONIC RATES

We have, referring to Fig. 2(a),

$$\Gamma_{\psi' \psi \eta} = \frac{P_\psi^3}{3} \frac{G_{V'VP}^2}{4\pi} \mathcal{R}_{\eta_c \bar{\eta}_c}^\eta. \quad (9)$$

Guided by the experience<sup>1,6</sup> with the vector-vector-scalar vertex which indicates  $G_{V'VS} \approx G_{VVS}$ , we assume that  $G_{V'VP} \approx G_{VVP}$ , and determine  $G_{VVP}$  from (i)  $\omega \rightarrow \pi^0 \gamma$  via vector dominance, (ii)  $\omega \rightarrow 3\pi$  via the Gell-Mann-Sharp-Wagner<sup>10</sup> intermediate- $\rho$ -pole method, (iii) the SU(6) relation  $G_{\omega\rho^0\rho^0} = 4G_{\rho\pi\pi}^2/m_\rho^2$ , and (iv)  $\phi \rightarrow \rho\pi$ , as in Ref. 1. All methods are consistent with  $\frac{1}{2}G_{\omega\rho^0\rho^0}^2/4\pi = G_{VVP}^2/4\pi \approx 9 \pm 2$ . Then

$$\Gamma_{\psi' \psi \eta} \approx (9 \pm 2) \text{ keV} \quad (\text{experiment}^{11}: 9.6 \text{ keV}). \quad (10)$$

A number of other predictions follow easily:

$$\frac{\Gamma_{\psi\eta\omega}}{\Gamma_{\psi\rho\pi}} = \frac{1}{3} \frac{G_{\omega\omega\eta}^2}{G_{\omega\rho\pi}^2} \left| \frac{P_\omega}{P_\rho} \right|^3 \mathcal{R}_{\eta\eta}^\eta \approx 0.03, \quad (11)$$

TABLE I. Residue matrices for  $\eta$ ,  $\eta'$ , and  $\eta_c$  poles in the  $4 \times 4$  model (MI).

$\mathcal{R}^\eta =$	0.39	-0.45	-0.010	0.18
		0.53	0.012	-0.21
$\mathcal{R}^{\eta'} =$			$2.8 \times 10^{-4}$	$4.7 \times 10^{-3}$
				$8.1 \times 10^{-2}$
$\mathcal{R}^{\eta_c} =$	0.19	0.28	-0.018	0.28
		0.40	-0.026	0.40
$\mathcal{R}^{\eta_c} =$			$0.16 \times 10^{-2}$	-0.026
				0.41
$\mathcal{R}^{\eta_c} =$	$2.03 \times 10^{-5}$	$1.5 \times 10^{-5}$	$4.5 \times 10^{-3}$	$2.6 \times 10^{-4}$
		$1.1 \times 10^{-5}$	$3.4 \times 10^{-3}$	$1.9 \times 10^{-4}$
$\mathcal{R}^{\eta_c} =$			0.997	$5.7 \times 10^{-2}$
				$3.2 \times 10^{-3}$

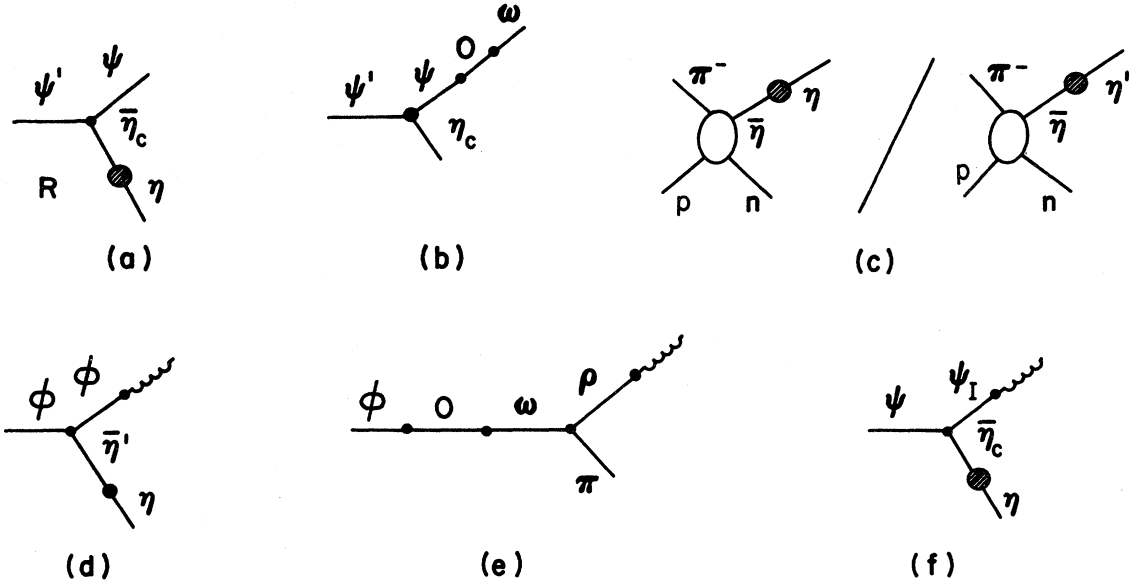


FIG. 2. Diagrams for various OZI-rule-forbidden processes.

$$\frac{\Gamma_{\psi\eta'\phi}}{\Gamma_{\psi\eta\omega}} \approx \left| \frac{P_\phi}{P_\omega} \right|^3 \frac{(m_\psi^2 - m_\omega^2)^2}{(m_\psi^2 - m_\phi^2)^2} \frac{\mathcal{R}_{\eta'\eta}^{\eta'}}{\mathcal{R}_{\eta\eta}^{\eta}} \approx \begin{cases} 0.7 \text{ (MI)} \\ 1.1 \text{ (MII)} \end{cases} \quad (12)$$

$$\frac{\Gamma_{\psi\eta'\omega}}{\Gamma_{\psi\eta\omega}} \approx \left| \frac{P_{\eta'}}{P_\eta} \right|^3 \frac{\mathcal{R}_{\eta'\eta}^{\eta'}}{\mathcal{R}_{\eta\eta}^{\eta}} \approx \begin{cases} 0.38 \text{ (MI)} \\ 0.59 \text{ (MII)} \end{cases} \quad (13)$$

An interesting way of looking for  $\eta_c$  is in the decay  $\psi' \rightarrow \eta_c \omega$ . Rosenzweig<sup>4</sup> predicts that this rate is only slightly ( $\frac{1}{3}$ ) suppressed relative to  $\psi' \rightarrow \psi \eta$ . However, within our framework [see Fig. 2(b)], since the  $0^+$  OZI transitions are more copious than the  $1^-$  OZI transitions (which can be handled perturbatively) we have

$$\frac{\Gamma_{\psi'\omega\eta_c}}{\Gamma_{\psi'\psi\eta}} = \left| \frac{P_{\eta_c}}{P_\eta} \right|^3 \frac{2f_{O\eta}^4}{(m_\omega^2 - m_{O^2})^2 (m_\omega^2 - m_\psi^2)^2} \frac{1}{\mathcal{R}_{\eta_c\eta}^{\eta}} \approx 0.084. \quad (14)$$

This model makes predictions, of course, for OZI-rule-violating production processes as well. Referring to Fig. 2(c), we have

$$\frac{d\sigma(\pi^- p \rightarrow \eta)}{d\sigma(\pi^- p \rightarrow \eta' n)} = \frac{\mathcal{R}_{\eta\eta}^{\eta}}{\mathcal{R}_{\eta'\eta}^{\eta}} \approx \begin{cases} 2.05 \text{ (MI)} \\ 1.3 \text{ (MII)} \end{cases} \quad (15)$$

where both cross sections are evaluated at the same  $s$ ,  $t$ , and  $q^2$ . One hopes that the extrapola-

tion in  $q^2$  from  $m_\eta^2$  to  $m_{\eta'}^2$  is not too serious. However, comparisons should be made at the same  $s$  and  $t$ . We expect the predicted ratio to be much more accurate at  $t=0$ , since production processes at high momentum transfer depend more strongly on the mass of the produced object. Extensive data on this reaction will be available shortly.<sup>12</sup>

It is interesting to note that in our formalism we are able to account for the masses and the OZI suppression with physical  $\eta$  and  $\eta'$  that have extremely small admixture of charm. In particular, referring to Table I, we find 0.028% charm in  $\eta$  and 0.16% charm in  $\eta'$ . This is in sharp contrast with the Harari<sup>2</sup> treatment. The origin of this is clear: In our treatment the OZI-rule violation contributes to both diagonal and off-diagonal terms in the mass matrix. Thus we are freed from the constraint

$$m_{\eta'}^2 + m_{\eta''}^2 + m_{\eta_c}^2 = m_\eta^2 + m_{\eta'}^2 + m_{\eta_c}^2 \quad (16)$$

which forces Harari's large charm admixture.

#### IV. PHOTONIC RATES

Also in contrast with Harari, we find no serious problems with  $\gamma$  decays in the context of the vector-dominance model. The new ingredient here, apart from a different mixing, is the use of  $\psi'$  as another intermediate state in decays involving  $\psi$  as an intermediate state.<sup>6</sup> We choose a judicious relative phase. We do not believe that this is artificial, because there is no reason why (consistent with the assumption  $G_{V'V}^2 \approx G_{VV}^2$ ) we

cannot have  $G_{VVP} \approx -G_{VVP}$ . Moreover, alternating signs considerably enhance the possibility of a convergent generalized-vector-dominance-model sum. This addition of radial excitations, interestingly enough, does not spoil rates such as  $\omega \rightarrow \pi^0 \gamma$  since the  $\rho'$  electronic width is expected to be *considerably* smaller than the  $\rho$  electronic width,<sup>13</sup> in contrast with the  $\psi'$  vs  $\psi$  electronic widths. With these preliminaries, consider the rates [referring to Figs. 2(d) and 2(e)]

$$\Gamma_{\phi \rightarrow \pi \gamma} = \frac{|P\gamma|^3}{3} \frac{G_{VVP}^2}{4\pi} \frac{3\Gamma_{\phi \rightarrow e^+e^-}}{\alpha m_\phi} \mathcal{R}_{\pi\eta}^{\eta_c}, \quad (17)$$

which yields a partial width of  $(44.5 \pm 10)$  keV consistent with recent data<sup>14</sup> indicating an experimental partial width of  $65 \pm 15$  keV. (Our quoted error is determined by the uncertainty in  $G_{VVP}$ .)

Similarly we have

$$\Gamma_{\phi \rightarrow \rho^0 \gamma} = \frac{|P\gamma|^3}{3} \frac{2G_{VVP}^2}{4\pi} \frac{3\Gamma_{\rho e^+e^-}}{\alpha m_\rho} \times \frac{2f_{OV}^4}{(m_\phi^2 - m_0^2)^2 (m_\phi^2 - m_\omega^2)^2}, \quad (18)$$

which yields a partial rate of  $(10.8 \pm 3)$  keV compared with the experimental<sup>14</sup> partial rate of  $5.9 \pm 2.1$  keV.

Referring to Fig. 2(f), with  $\psi_1$  now  $\psi$  and  $\psi'$ , we have for  $\psi \rightarrow \eta \gamma$

$$\begin{aligned} \Gamma_{\psi \rightarrow \eta \gamma} &= \frac{1}{4\pi} \frac{|P\gamma|^3}{3} \mathcal{R}_{\eta c}^{\eta_c} \left| \frac{eF_\psi}{m_\psi} G_{VVP} + \frac{eF_{\psi'}}{m_{\psi'}} G_{VVP} \right|^2, \\ &= \frac{|P\gamma|^3}{3} \frac{G_{VVP}^2}{4\pi} \mathcal{R}_{\eta c}^{\eta_c} \frac{3\Gamma_{\psi e^+e^-}}{\alpha} \frac{1}{m_\psi} B, \end{aligned} \quad (19)$$

where

$$B = \left[ 1 + \frac{G_{VVP}}{G_{VVP}} \left( \frac{\Gamma_{\psi e^+e^-} - m_\psi}{\Gamma_{\psi e^+e^-} - m_{\psi'}} \right)^{1/2} \right]^2. \quad (20)$$

Using  $\Gamma_{\psi \rightarrow \eta \gamma} = 100$  eV (Ref. 15) we find that

$$\frac{G_{VVP}}{G_{VVP}} = -1.24, -1.98; \quad (21)$$

the first root is consistent with our earlier assumption<sup>16</sup>  $G_{VVP}^2 \approx G_{VVP}^2$ . Using this new determination we have

$$\begin{aligned} \Gamma_{\psi \rightarrow \eta \gamma} &= \frac{|P\gamma|^3}{3} \frac{G_{VVP}^2}{4\pi} \mathcal{R}_{\eta c}^{\eta_c} \frac{3\Gamma_{\psi e^+e^-}}{\alpha} \frac{1}{m_\psi} B \\ &= 2.2 \text{ keV}, \end{aligned} \quad (22)$$

which is now a prediction based on  $\Gamma_{\psi \rightarrow \eta \gamma}$ .

Similarly we find

$$\frac{\Gamma_{\psi \rightarrow \eta \gamma}}{\Gamma_{\psi \rightarrow \eta \gamma}} = \begin{cases} 4.6 \text{ (MI)} \\ 7.5 \text{ (MII)} \end{cases} \quad (\text{data}^{14}: 4 \pm 2.5). \quad (23)$$

Treating the  $\psi' \rightarrow \eta_c \gamma$  decay in analogy to the  $\psi \rightarrow \eta_c \gamma$  decay we find

$$\Gamma_{\psi' \rightarrow \eta_c \gamma} = \frac{|P\gamma|^3}{3} \frac{G_{VVP}^2}{4\pi} \mathcal{R}_{\eta_c}^{\eta_c} \frac{3\Gamma_{\psi e^+e^-}}{\alpha} \frac{1}{m_\psi} B', \quad (24)$$

where

$$B' = \left| 1 + \frac{G_{VVP}}{G_{VVP}} \left( \frac{\Gamma_{\psi' e^+e^-} - m_{\psi'}}{\Gamma_{\psi e^+e^-} - m_{\psi'}} \right)^{1/2} \right|^2. \quad (25)$$

If we assume that  $G_{VVP} G_{VVP} \approx G_{VVP}^2$ , we have

$$\Gamma_{\psi' \rightarrow \eta_c \gamma} \approx 50 \text{ keV}. \quad (26)$$

Continuing in this spirit, we have

$$\begin{aligned} \Gamma_{\eta_c \rightarrow \gamma \gamma} &= \frac{G_{VVP}^2}{4\pi} \frac{|P\gamma|^3}{2} \left( \frac{3\Gamma_{\psi e^+e^-}}{\alpha m_\psi} \right)^2 \\ &\times \left[ 1 + \frac{\Gamma_{\psi' m_\psi} G_{VVP}^2}{\Gamma_{\psi m_\psi} G_{VVP}^2} + 2 \frac{G_{VVP}}{G_{VVP}} \left( \frac{\Gamma_{\psi' e^+e^-} - m_{\psi'}}{\Gamma_{\psi e^+e^-} - m_{\psi'}} \right)^{1/2} \right]^2 \\ &\approx 15 \text{ eV}. \end{aligned} \quad (27)$$

(This width should be taken with caution since it depends on the square of the difference of two large numbers; a factor-of-2 variation in the coupling ratio can result in a factor-of-50 increase in the rate.)

Taking  $\Gamma_{\eta_c}$  full width to be 100 keV, which is certainly a lower limit, we find

$$\frac{\Gamma_{\psi' \rightarrow \eta_c \gamma}}{\Gamma_{\psi'}} \frac{\Gamma_{\eta_c \rightarrow \gamma \gamma}}{\Gamma_{\eta_c}} \leq 4 \times 10^{-5}, \quad (28)$$

TABLE II. Table of various rates involving OZI-rule-forbidden transitions. MI is the  $4 \times 4$  model and MII is the  $3 \times 3$  model.

Rates	Theory	Exp.
$\Gamma(\psi' \rightarrow \psi \eta)$	$9 \pm 2$ keV	$9.6 \pm 1.8$ keV
$\Gamma(\psi \rightarrow \eta \omega) / \Gamma(\psi \rightarrow \rho \pi)$	0.03	
$\Gamma(\psi \rightarrow \eta' \phi) / \Gamma(\psi \rightarrow \eta \omega)$	0.7 (MI) 1.1 (MII)	
$\Gamma(\psi' \omega \eta_c) / \Gamma(\psi' \psi \eta)$	0.38 (MI) 0.59 (MII)	
$\Gamma(\psi' \omega \eta_c) / \Gamma(\psi' \psi \eta)$	0.084	
$\frac{d\sigma(\pi^- p \rightarrow \eta \pi)}{d\sigma(\pi^- p \rightarrow \eta' \pi)}$	2.05 (MI) 1.3 (MII)	
$\Gamma(\phi \rightarrow \eta \gamma)$	$44.5 \pm 10$ keV	$65 \pm 15$ keV
$\Gamma(\phi \rightarrow \pi^0 \gamma)$	$10.8 \pm 3$ keV	$5.9 \pm 2.1$ keV
$\Gamma(\psi \eta \gamma)$	normalization	$100 \pm 25$ eV
$\Gamma(\psi \eta_c \gamma)$	2.2 keV	
$\Gamma(\psi \eta' \gamma) / \Gamma(\psi \rightarrow \eta \gamma)$	4.6 (MI) 7.5 (MII)	$4 \pm 2.5$
$\Gamma(\psi' \eta_c \gamma)$	50 keV	
$\Gamma(\eta_c \rightarrow \gamma \gamma)$	15 eV	
$\Gamma(\psi' \rightarrow \rho_0 \rho_0 \gamma)$	$< 50B(\eta_c \rightarrow \rho_0 \rho_0)$ keV	$< 7$ keV

which is well within the experimental<sup>15</sup> bound.

Finally, consider  $\psi' \rightarrow \rho_0 \rho_0 \gamma$ , which Harari points out as a possible problem. We have

$$\Gamma_{\psi' \rightarrow \rho_0 \rho_0 \gamma} \leq \Gamma_{\psi' \rightarrow \gamma \eta_c} B_{\eta_c \rho_0 \rho_0} = 50 \text{ keV}$$

$$(\text{experiment}^{17}: B_{\eta_c \rho_0 \rho_0} \leq 7 \text{ keV}), \quad (29)$$

which is consistent with a plausible branching ratio  $B_{\eta_c \rho_0 \rho_0}$  for  $\eta_c \rightarrow \rho_0 \rho_0$ . Our decay rates are summarized in Table II.

## V. SUMMARY AND CONCLUSIONS

(i) The mixing generated by  $O$ -meson or cylinder correction, controlled by the  $0^-$  masses, yields a correct rate for  $\psi' \rightarrow \psi \eta$  and results in predictions for a number of measurable hadronic rates, different from the predictions of Rosenzweig.<sup>4</sup>

(ii) The admixture of charm in  $\eta$  and  $\eta'$  is much smaller than the model of Harari indicates.

(iii) The photonic rates, calculated using this mixture and the extended vector-dominance-model, yield results consistent with experiment.

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<sup>16</sup>We consider this a determination of the relative sign of  $G_{VVP}$  and  $G_{VVP}$  but only an approximate verification of our assumption  $G_{VVP}/G_{VVP} = 1$ , since the expression  $B$  is extremely sensitive to the vector-dominance-model continuation for  $q^2=0$  to  $m_\psi^2$ .

<sup>17</sup>J. Heintze, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford Univ.* (Ref. 14), p. 97.