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$\Gamma(\psi \to \phi\pi^+\pi^-)/\Gamma(\psi \to \omega\pi^+\pi^-)$ and the Okubo-Zweig-Iizuka rule*

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We report the results of a detailed study of mixing effects in the 0^{++} nonet and their influence on a softening of the Okubo-Zweig-Iizuka (OZI) suppression in the ratio $\Gamma(\psi \to \phi\pi^+\pi^-)/\Gamma(\psi \to \omega\pi^+\pi^-)$. We conclude that the experimental data for these rates are consistent with the scale of OZI violation as determined from $\psi' \to \psi\pi^+\pi^-$.

The branching ratio $\mathcal{R} \equiv \Gamma(\psi \to \phi\pi^+\pi^-)/\Gamma(\psi \to \omega\pi^+\pi^-) = 0.2 \pm 0.1$ (see Ref. 1) has aroused interest and concern because the ϕ decay mode is doubly forbidden by the Okubo-Zweig-Iizuka[2] (OZI) rule whereas the ω decay mode is singly forbidden, suggesting a much smaller $\psi \to \phi\pi^+\pi^-$ rate. It is possible to conclude from this that the SU(4) picture, with the OZI rule for interpreting the anomalous narrow widths of the new resonances, is wrong. In this note we would like to point out that this conclusion is premature in that it rests on the experimentally unfounded assumption that the 0^{++} scalar nonet is close to being ideally mixed. To the contrary, the mass spectrum of this elusive nonet is not described by an ideal-mixing mass formula; hence unitarity corrections to the OZI rule ("cylinder corrections" in the language of Veneziano,[3] "O-meson" effects in the language of Freund and Nambu[4]) are expected to be large, in contrast to the situation met when dealing with respectably ideal nonets such as 1^{--} and 2^{++}.

We shall illustrate this by using a model for OZI-rule violation[4] which we have found to be reliable in a number of OZI-rule-violating decays and production processes.[5,6] The model employs an OZI-rule-violating interaction which is mediated by the "O meson," identified as a unitarity effect associated with daughters of the Pomeron trajectory. Whether the O-meson parameters (mass, width, residue) describe a real particle or merely parametrize a cut in the OZI-violating transitions is a difficult question on which we shall not dwell in this short paper.[7]

Assuming that $\psi(3095)$ is $J^{PC} = 1^{--}$ and isospin zero, it is easy to see that the lowest J^{PC} states for the $\pi\pi$ system are 0^{++} and 2^{++}, and that the isospin is zero. The 0^{++} partial waves will be mediated by the ϵ and ϵ' and the 2^{++} by the f and f', and we will consider only the s-wave ϵ and ϵ' states here since we expect them to give the largest contribution to a softening of the OZI rule in these processes.

The lowest-order diagrams for our model for the decays are contained in Figs. 1(a) and 1(b). The couplings of the "O mesons" to ideally mixed multiplets is given by

$$\mathcal{L}_V = f_{OV} O_\mu(\sqrt{2}\,\omega^\mu + \phi^\mu + \psi^\mu),$$
$$\mathcal{L}_S = f_{OS} O(\sqrt{2}\,\epsilon + \epsilon' + \epsilon_c), \qquad (1)$$

and a simple calculation yields

$$\mathcal{R} = \frac{2 f_{OS}{}^4 (m_\psi{}^2 - m_\omega{}^2)^2}{(m_\psi{}^2 - m_\phi{}^2)^2 (m_\epsilon{}^2 - m_{\epsilon'}{}^2)^2 (m_\epsilon{}^2 - m_0{}^2)^2}, \qquad (2)$$

where we have neglected small relative phase-space differences and the $\phi - \omega$ mass difference, relative to the mass of ψ. f_{OS} has been calculated

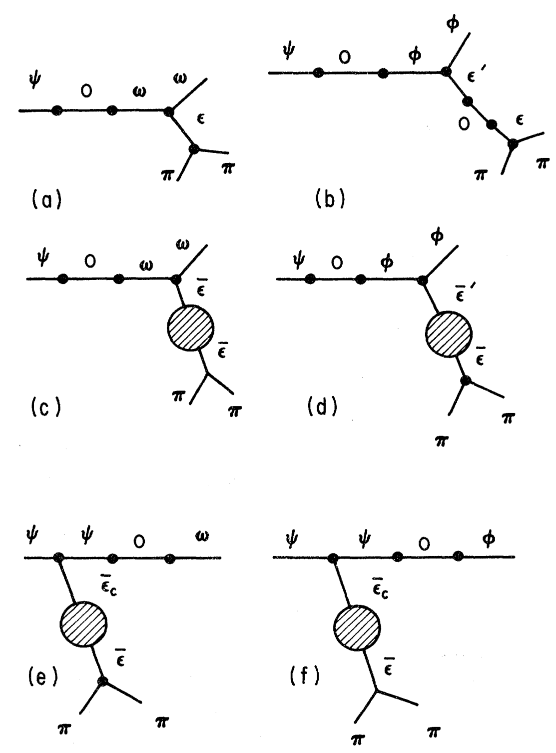

FIG. 1. (a) and (b) The largest lowest-order perturbation diagram contributing to $\mathcal{R}$. (c)–(f) Renormalized diagrams contributing to $\mathcal{R}$.

from data on $\psi' \to \psi\pi\pi$ using this model[4] and has been found to be 0.40 (0.48) GeV^2 for $m_0^2 = 2$ (3) GeV^2.[6] If we take the 0^{++} states to be those appearing in Ref. 8, namely, $\epsilon = \epsilon(700)$ and $\epsilon' = S^*(993)$ and $m_0^2 = 2$, we obtain $\mathcal{R} \approx 0.1$. Thus the suppression is not as large as one might expect.

This calculation should not be taken too seriously, however, since it has two serious problems. First, lowest-order perturbation theory is not justified in the noncharmed sector of SU(4) since the effective coupling is $f_{OS}^2/(m_\epsilon^2 - m_{\epsilon'}^2)^2 \approx 0.65$, and is not small. Secondly, the 0^{++} multiplet may not be ideally mixed; in fact, the states that we are using do not satisfy even approximately the ideal-mixing mass formula.[9] To overcome the first difficulty we will simply sum all the diagrams dictated by the "O-meson" interaction. To overcome the second difficulty we will assume that initially the 0^{++} multiplet is ideally mixed and satisfies the appropriate mass relations, but we will require, to the extent that we can, that the states that emerge after the full "O-meson" interaction is calculated agree with the physical states, that is, the masses are the experimentally measured ones. We will designate the ideally mixed states by overbars. The δ and the κ states are not affected by the "O-meson" mixing, and the ideal-mixing mass formulas (IMMF) give $m_\delta = m_{\bar\epsilon}$ and $m_{\bar\epsilon}^2 + m_\delta^2 = 2m_\kappa^2$. The full propagator Π is given by $\Pi = POP + POPOP + \cdots = P(1-OP)^{-1}$, where

$$P = \begin{pmatrix} (s - m_{\bar\epsilon}^2)^{-1} & 0 \\ 0 & (s - m_{\bar\epsilon'}^2)^{-1} \end{pmatrix}, \quad (3)$$

$$O = \frac{f_{OS}^2}{s - m_{\bar O}^2} \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}. \quad (4)$$

$m_{\bar O}^2$ is the bare mass of the "O meson," and is unknown (a parameter). Summing the series we find

$$\Pi = \frac{\begin{pmatrix} (s - m_{\bar\epsilon'}^2)(s - m_{\bar O}^2) - f_{OS}^2 & \sqrt{2} f_{OS}^2 \\ \sqrt{2} f_{OS}^2 & (s - m_{\bar\epsilon}^2)(s - m_{\bar O}^2) - 2 f_{OS}^2 \end{pmatrix}}{(s - m_{\bar\epsilon}^2)(s - m_{\bar\epsilon'}^2)(s - m_{\bar O}^2) - 2 f_{OS}^2 (s - m_{\bar\epsilon'}^2) - f_{OS}^2 (s - m_{\bar\epsilon}^2)}. \quad (5)$$

This full propagator must now have poles at the physical mass of all possible intermediate particles, i.e., m_ϵ, $m_{\epsilon'}$, and m_O. This implies that

$$(s - m_{\bar\epsilon}^2)(s - m_{\bar\epsilon'}^2)(s - m_{\bar O}^2) - 2 f_{OS}^2 (s - m_{\bar\epsilon'}^2) - f_{OS}^2 (s - m_{\bar\epsilon}^2) = (s - m_\epsilon^2)(s - m_{\epsilon'}^2)(s - m_O^2). \quad (6)$$

Decomposing Π into partial fractions we have

$$\Pi = \sum_{i=1}^{3} \frac{1}{s - m_i^2} \begin{pmatrix} (g_{\bar\epsilon}^i)^2 & g_{\bar\epsilon}^i g_{\bar\epsilon'}^i \\ g_{\bar\epsilon'}^i g_{\bar\epsilon}^i & (g_{\bar\epsilon'}^i)^2 \end{pmatrix}, \quad (7)$$

where $i = 1$, 2, and 3 correspond to ϵ, ϵ', and O, respectively, and where the coupling $g_{\bar i}^j$ is the residue of the renormalized pole j in the ideally mixed channel $\bar i$. Expanding Eq. (6) and equating the coefficient of like powers of s, we find three constraint equations among the bare and physical masses.

If we assume that we know m_δ^2, m_ϵ^2, m_O^2, and m_κ^2, then there are three unknown parameters $m_{\epsilon'}^2$, $m_{\bar O}^2$, and f_{OS}^2 left ($m_{\bar\epsilon}^2$ is known from IMMF) for which we have solved numerically. This determines all masses, the coupling f_{OS}^2 (which can be compared with its independent determination from the charmed sector in Ref. 6), and the residues of Eq. (7).

We then proceed to calculate the renormalized diagrams [Figs. 1(c) and 1(d)]. The result is

$$\Gamma_{\psi\to\omega\pi\pi} = \frac{2 f_{OV}^4 G_{VVS}^2}{2(m_\psi^2 - m_O^2)^2 (m_\psi^2 - m_\omega^2)^2} \times I^\omega((g_{\bar\epsilon}^\epsilon)^2, (g_{\bar\epsilon}^{\epsilon'})^2, (g_{\bar\epsilon}^O)^2) \quad (8)$$

and

$$\Gamma_{\psi\to\phi\pi\pi} = \frac{f_{OV}^4 G_{VVS}^2}{(m_\psi^2 - m_O^2)^2 (m_\psi^2 - m_\phi^2)^2} \times I^\phi(g_{\bar\epsilon}^\epsilon g_{\bar\epsilon'}^\epsilon, g_{\bar\epsilon}^{\epsilon'} g_{\bar\epsilon'}^{\epsilon'}, g_{\bar\epsilon}^O g_{\bar\epsilon'}^O), \quad (9)$$

where

$$I^x(f_1, f_2, f_3) = \int_{4\mu^2}^{(M_\psi - M_x)^2} dy\, \rho(y) \left| \sum_i \frac{f_i}{y - m_i^2 + i m_i \Gamma_i} \right|^2, \quad (10)$$

with $x = \omega$, ϕ, and $\rho(y)$ as defined in Ref. 6, coming from phase space and the chiral coupling for the pseudoscalars. The known width $\Gamma_{\psi'\to\psi\pi\pi} = 600$ MeV determines the scale from which $\Gamma_{\epsilon'}$ and Γ_O can be calculated:

$$\Gamma_i = 3\frac{G^2}{4\pi}\left(\frac{M_i^2}{4} - m_\pi^2\right)^{1/2} \left(\frac{1}{4} - \frac{m_\pi^2}{2M_i^2}\right)^2 (g_{\bar\epsilon}^i)^2 + 2\frac{G^2}{4\pi}\left(\frac{M_i^2}{4} - m_K^2\right)^{1/2} \left(\frac{1}{4} - \frac{m_K^2}{2M_i^2}\right)^2 \left(g_{\bar\epsilon'}^i + \frac{1}{\sqrt{2}} g_{\bar\epsilon}^i\right)^2. \quad (11)$$

Here $G^2/4\pi$ is determined by $\Gamma_{\epsilon\to\pi\pi}$, a θ function is understood where phase space vanishes, and $i = \epsilon'$ or 0. We have used the invariant coupling $p_\mu p'_\mu G/$

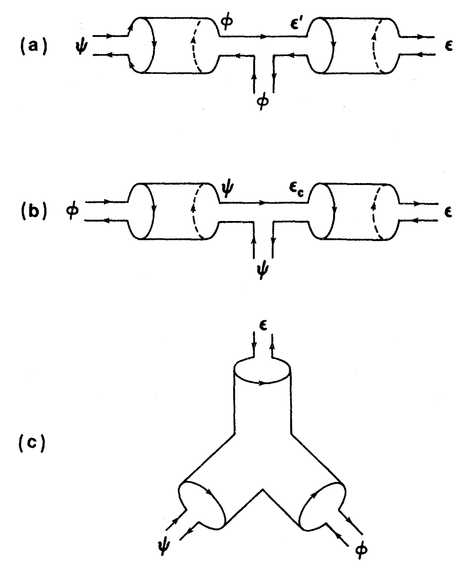

FIG. 2. (a), (b), and (c) "Cylinder" diagram corresponding to 1(d), 1(f), and the triple O coupling, respectively.

$(q^2)^{1/2}$ for $\epsilon(q) \to \pi(p) + \pi(p')$, which preserves the Adler zero observed in the analogous decay $\psi' \to \psi\pi\pi$, results in a dimensionless coupling constant with which the nonet-symmetry scheme for "bare" couplings is implemented, and tends to a constant in the high-q^2 limit. Since $\rho(y)$ is a slowly varying function, it was evaluated at the particle pole. The six terms were then integrated analytically with masses, couplings, and widths previously determined.

The topological-expansion diagram corresponding to the double-OZI-suppressed process is shown in three configurations in Figs. 2(a), 2(b), and 2(c). Since these diagrams are topologically equivalent, they are dual to one another. The usual phenomenological procedure, and the one we will use here to avoid double (in this case, triple) counting, is to select the configuration that can best be saturated by a few terms in the kinematic region of interest and to calculate the leading term or terms. One should then not add terms from other configurations, even if they appear to make contributions in the kinematic region of interest.

We have considered the duality-equivalent sets in Figs. 1(e), 1(f), and 1(c), 1(d). Figures 1(e) and 1(f) are skewed towards very high mass because of the effect of the $\bar{\epsilon}_c$ propagator, and therefore make relatively smaller contributions to the region of experimentally accessible $\pi\pi$ invariant mass, as compared to Figs. 1(c) and 1(d), which are peaked in the experimentally accessible region. This effect persists in the nonperturbative calculation. The argument can be further supported by considering the recurrences of $\bar{\epsilon}_c$ in Figs. 1(e) and 1(f), which show a slower convergence than the $\bar{\epsilon}$ and $\bar{\epsilon}'$ in Figs. 1(c) and 1(d). Thus the indications are, indeed, that Figs. 1(c) and 1(d) are the most strongly convergent diagrams and are more accurately estimated in leading terms.

It is appropriate here to point out that the question of double counting is a subtle and controversial one which has been handled differently by other authors,[10] who generally add coherently their estimates for the different configurations.

To summarize the calculation, an input of $m_\epsilon = 700$ MeV, $\Gamma_{\epsilon\to\pi\pi} = 600$ MeV, $m_O{}^2 = 2$ or 3 GeV^2, and a range of m_δ and m_κ, results in the outputs $f_{OS}{}^2$, $m_{\epsilon'}$, $\Gamma_{\epsilon'}$, Γ_O, $\Gamma_{\delta\to\eta\pi}$ (calculated analogous to $\Gamma_{\epsilon'}$ described above) and $\mathcal{R} = \Gamma_{\psi\to\phi\pi^+\pi^-}/\Gamma_{\psi\to\omega\pi^+\pi^-}$. Two hypotheses were entertained in the input choice of m_κ and m_δ: (1) that $m_\kappa = 1300$[8] (varying m_δ) and (2) that $m_\delta = 976$[8] (varying m_κ). Large amounts of output were then scanned by physicists to determine the behavior and analytic character of the solutions. Because of level crossings, some solutions occur on branches which are not analytically connected to the perturbative origin $f_{OS}{}^2 = 0$, $m_\epsilon = \bar{m}_\epsilon$, $m_{\epsilon'} = \bar{m}_{\epsilon'}$, $m_O = \bar{m}_O$. A cut was then made to require that $f_{OS}{}^2$ was reasonably close to its independent determination in Ref. 6 [$f_{OS}{}^2 = 0.162$ (0.235) for $m_O{}^2 = 2$ (3) GeV^2]. Thus the inputs are $f_{OS}{}^2$, m_O; m_κ or m_δ; and m_ϵ and Γ_ϵ. The results are presented in Table I, *with inputs underlined.* Where solutions were double-valued (C and D) the branch was chosen which had the most reasonable masses and widths. Solution C is not on a perturbative branch. We have the following observations:

(1) Solution A requires a high-mass ϵ' which can-

TABLE I. Solutions for $f_{OS}{}^2$, nonet masses and widths, and $\mathcal{R}$, with input underlined.

Solution	$f_{OS}{}^2$ (GeV^4)	m_O (MeV)	Γ_O (MeV)	$m_{\epsilon'}$ (MeV)	$\Gamma_{\epsilon'}$ (MeV)	m_δ (MeV)	Γ_δ (MeV)	m_κ (MeV)	Γ_κ (MeV)	$\mathcal{R}$
A	<u>0.15</u>	<u>1414</u>	460	1697	83	837	69	<u>1300</u>	190	0.06
B	<u>0.25</u>	<u>1732</u>	995	1442	426	894	72	<u>1300</u>	196	0.07
C	<u>0.16</u>	<u>1414</u>	1800	975	515	<u>976</u>	110	975	346	0.11
D	<u>0.26</u>	<u>1732</u>	2368	917	1270	<u>976</u>	177	922	537	0.23

not be identified with $S^*(993)$ and a δ with rather too low a mass to be identified with $\delta(976)$, although the widths are good. Solution B has $m_{\epsilon'}$ and m_δ closer to $S^*(993)$ and $\delta(976)$, but the ϵ' has broadened unacceptably for an $S^*(993)$ identification. Morgan's[11] analysis, however, indicates that table values for scalar masses and widths may be misleading because of the large background in the phase shifts. Considered in that context A and B may be an acceptable mass and width fit.

(2) Solutions C and D have acceptable $m_{\epsilon'}$ for $S^*(993)$ identification, at the expense of a large ϵ' width, and a low-mass κ meson. It should be noted, however, that κ parameters are not well known; again the remarks in (1) apply.

(3) *All our solutions are consistent with large values of* $\mathfrak{R}$ *reported in Ref. 1.* With $m_\kappa = 1300$ our preferred solutions indicate $\mathfrak{R} = 0.06$ ($m_0 = 1414$) and $\mathfrak{R} = 0.07$ ($m_0 = 1732$). With $m_\delta = 976$ our preferred solutions indicate $\mathfrak{R} = 0.11$ ($m_0 = 1414$) and $\mathfrak{R} = 0.23$ ($m_0 = 1732$).

(4) We do not believe that our results are peculiar to the details of the specific mixing model we have employed. We believe they are typical of any model which realistically confronts the 0^{++} enhancement spectrum. *In particular, we have run our programs with the Morgan assignment,* ϵ*(1100–1400), and still find large values of* $\mathfrak{R}$, *as in Table* I.

In conclusion, we have carried through a detailed calculation of the ratio $\psi \to \phi\pi^+\pi^-$: $\psi \to \omega\pi^+\pi^-$, concentrating on contributions from the s-wave dipion component. We find no evidence to support the necessity of a strongly forbidden $\psi \to \phi\pi^+\pi^-$ rate relative to $\psi \to \omega\pi^+\pi^-$ rate. The propagator-matrix study indicates broad s-wave isoscalar and isovector phase-shift action in the high mass range, with large deviations from ideal mixing in the SU(3) sector, while maintaining the relative purity of the $c\bar{c}$ states. To cast doubt on the general SU(4) scheme, with its appeal to the OZI rule, on the basis of what we know of the 0^{++} spectrum and this rate ratio, would seem to be premature.

Finally, a $\pi\pi$ mass spectrum for these transitions would be enormously useful in sorting out the spectroscopy of the s-wave intermediate states. The integrated spectrum indicates large mixing; the differential spectrum would be a unique window into the detailed physics of the uncertain 0^{++} system.

The authors would like to acknowledge several stimulating discussions with G. Kramer on the importance of mixing in the 0^{++} nonet when interpreting the OZI rule in connection with $\psi \to \pi^+\pi^-\phi$, and discussion with J. Shapiro on the question of double counting.

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