Chiral-SU(3)-symmetry breaking for the pseudoscalar mesons*

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In the framework of the quark–vector-gluon model, with chiral symmetry broken by a quark mass term, careful use of the algebraic properties of the Hamiltonian and of the divergences of axial-vector currents leads to a consistent picture of symmetry breaking in the pseudoscalar nonet. The $\eta'(960)$ emerges with a large gluon component.

Although quark–vector-gluon models have been remarkably successful in accounting for many properties of hadrons, the η meson has remained a puzzle in this context for several reasons. Such models always involve an (approximately) conserved axial-vector current, from which the usual current-algebra arguments lead to the existence of a unitary-singlet pseudoscalar Goldstone boson with a mass of the order of a pion mass.[1] Furthermore, in the conventional picture of η-η' mixing, there appears to be a conflict between the mixing angle determined from the quadratic mass formula for pseudoscalars and the η radiative-decay width and that determined from the ratio of η-to-η' production cross sections via $\pi^- p \to \eta(\eta')n$. We will show here that a consistent picture of the pseudoscalar mixing and spectrum is obtained if one includes a mixing of quark and gluon states as the quark–vector-gluon model naturally suggests. The admixture of a gluon component for vector mesons and light pseudoscalars is known to be small, while the situation for the η' is not so clear. In the present analysis, it is found that the η' should have a large (in fact dominant) contribution of the gluon state in its constituent-quark-basis decomposition. We will first set out our arguments supporting these assertions, and at the end we will discuss some of the consequences of the results.

There have been two approaches taken to the problem of the pseudoscalar masses: attempting to diagonalize a mass (or mass-squared) matrix[2-4] and analyzing the equations which arise from computing the axial-vector-current divergences.[5-7] We will first use the mass-squared matrix-diagonalization technique and then show how the current-divergence equations are consistent with the results obtained.

In the quark–vector-gluon model, the nonstrange piece of the mass-squared matrix has the form

$$M^2 = \begin{pmatrix} \hat{m}^2 & 0 & 0 & 0 \\ 0 & \hat{m}^2 & 0 & 0 \\ 0 & 0 & m_s^2 & 0 \\ 0 & 0 & 0 & m_G^2 \end{pmatrix} + \kappa \begin{pmatrix} 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \\ 1&1&1&0 \end{pmatrix} + \lambda \begin{pmatrix} 1&1&1&0 \\ 1&1&1&0 \\ 1&1&1&0 \\ 0&0&0&0 \end{pmatrix}, \tag{1}$$

where the matrix rows and columns correspond to quark states $\bar{u}u$, $\bar{d}d$, $\bar{s}s$, and the pseudoscalar two-gluon state[2] G. The term proportional to the parameter κ arises from the annihilation of a quark pair into the gluon state G, while the term proportional to λ arises from quark-pair annihilation and reconstitution via G as an intermediate state. The parameter m_G^2 is presumably large compared to all pseudoscalar masses; it arises at least in part from transitions of G to a quark pair and back to G. We will take the parameters κ, λ, $\hat{m}^2$, m_s^2, and m_G^2 to be free and look for solutions that fit the known masses of the pseudoscalar nonet (π, K, η, η'). There is a tenth state in this scheme which will be fixed in terms of the fitted parameters. Note that if one ignores the state G as an "external" state, then one finds there is no solution for fixed λ. This is the approach taken by De Rújula *et al.*[3] They conclude that λ is mass dependent; in fact, they require it to vary rapidly with mass.

It is easy to show that

$$\hat{m}^2 = m_\pi^2, \quad m_s^2 = 2m_K^2 - m_\pi^2, \tag{2}$$

leaving only the three isoscalar states to be determined. The matrix for the isoscalar states is

$$\begin{pmatrix} 2\lambda + \hat{m}^2 & \sqrt{2}\lambda & \sqrt{2}\kappa \\ \sqrt{2}\lambda & \lambda + m_s^2 & \kappa \\ \sqrt{2}\kappa & \kappa & m_G^2 \end{pmatrix}. \tag{3}$$

Clearly, if the coupling of quark pairs to the gluon state is ignored, the eigenstates are the "ideal" nonet states $\bar{u}u+\bar{d}d$ and $\bar{s}s$, with masses $\hat{m}$ and m_s, and a pure gluon state[8] G with mass m_G. This is far from the observed state of affairs, so we must solve the matrix-diagonalization problem in detail. Define the eigenvectors

$$\eta=N\begin{bmatrix}\cos\alpha\\ \sin\alpha\\ \epsilon\end{bmatrix},\quad \eta'=N'\begin{bmatrix}-\sin\alpha'\\ \cos\alpha'\\ \epsilon'\end{bmatrix},\quad \eta''=N''\begin{bmatrix}-\sin\alpha''\\ \cos\alpha''\\ \epsilon''\end{bmatrix}, \tag{4}$$

where N, N', and N'' are normalization constants. Then, demanding that η and η' be the eigenvectors corresponding to the physical squared masses of the observed particles η and η', one finds

$$\alpha=-50.6°,\quad \alpha'=-36.5°, \tag{5}$$

independent of the parameters κ, λ, and m_G^2. In terms of the singlet-octet mixing angles θ, θ' defined by

$$\eta=N[\cos\theta|\eta_8\rangle+\sin\theta|\eta_1\rangle+\epsilon|G\rangle],$$
$$\eta'=N'[-\sin\theta'|\eta_8\rangle+\cos\theta'|\eta_1\rangle+\epsilon'|G\rangle]. \tag{6}$$

This means

$$\theta=4.1°,\quad \theta'=18.2°. \tag{7}$$

Contrary to the usual treatment of η-η' mixing, the η and η' mesons are not related to each other by a common mixing angle. A similar conclusion has been reached recently by Inami *et al.*[9] for independent reasons. Since we have three parameters in the matrix and only two known masses, there will be one parameter undetermined, which we choose to be m_G^2. In Table I we display some solutions for various allowed values of m_G^2. There are no solutions for $m_G^2<m_{\eta'}^2$.

If one arbitrarily sets $\kappa=0$ in the matrix of Eq. (3), one recovers the conventional eigenvalue problem. The physical masses of the η and the η' are not eigenvalues of this matrix. Since the approach is in terms of a mass-squared matrix, one should compare predicted and observed squared masses. Then m_η^2 is predicted to be 10% too low and $m_{\eta'}^2$ is predicted to be 15% too high. What is one to conclude? Isgur[4] takes the attitude that the disagreement is not large, and so he is content in this result. De Rûjula *et al.*[3] feel that the disagreement is significant, and that therefore λ (their β) must vary rapidly with mass. The viewpoint taken in this paper is that the disagreement is real, but that λ should not vary much and

TABLE I. Solutions of the matrix diagonalization for some values of the parameter m_G^2.

m_G^2 (GeV^2)	$-\epsilon$	$-\epsilon'$	$m_{\eta''}^2$ (GeV^2)	ϵ''
0.917	0	∞	3.275	0
0.95	0.0323	8.36	3.438	0.116
1	0.0494	5.47	3.685	0.178
2	0.114	2.36	8.63	0.406
3	0.126	2.15	13.6	0.445
4	0.131	2.07	18.5	0.462
10	0.138	1.96	48.2	0.488
100	0.143	1.89	493	0.502
1000	0.143	1.89	4943	0.503

so another resolution of the problem should be sought.

The gluon interaction leads to an anomalous term in the divergence of the ninth axial-vector current.[10] Therefore we write[11]

$$\partial^\mu \hat{A}_\mu=\hat{m}\hat{v}+\mathcal{G}. \tag{8}$$

Unfortunately, since matrix elements involving the two-gluon state are not *a priori* related to those involving pure quark-antiquark states we cannot use this last equation to constrain the parameters describing the pseudoscalar mesons. It is worthwhile making this explicit. Define

$$\langle 0|\partial^\mu\hat{A}_\mu|G\rangle\equiv g, \tag{9}$$

$$\langle 0|\hat{A}_\mu|G\rangle\equiv Fp_\mu. \tag{10}$$

Then

$$\langle 0|\partial^\mu\hat{A}_\mu|\eta\rangle=N[\epsilon F+f\cos\alpha]m_\eta^2$$
$$=N[\epsilon g+fm_\pi^2\cos\alpha], \tag{11}$$

$$\langle 0|\partial^\mu\hat{A}_\mu|\eta'\rangle=N'[\epsilon' F-f\sin\alpha']m_{\eta'}^2$$
$$=N'[\epsilon' g-fm_\pi^2\sin\alpha'], \tag{12}$$

so

$$\frac{m_\eta^2-m_\pi^2}{m_{\eta'}^2-m_\pi^2}\,\frac{\cos\alpha}{-\sin\alpha'}=\frac{\epsilon}{\epsilon'}\,\frac{g-Fm_\eta^2}{g-Fm_{\eta'}^2}. \tag{13}$$

However, since g and F are unknown we can draw no conclusions.

Note that we have not assumed that v_0 and v_8 belong to the same nonet as v_3—only that the linear combination $\hat{v}$ does.[11] This means that we have fewer constraints than do Caser and Testa,[7] so we can fit the pseudoscalar masses and mixing angles consistently, while they do not.

It is worthwhile examining the behavior of the equations if one allows the quark masses $\hat{m}$ and m_s to vanish. We will do this as follows: Set $\hat{m}=0$, then let $m_s\to 0$. In this limit,

$$\begin{aligned}&\epsilon=0,\\&\alpha=\alpha'=\alpha''=-\tan^{-1}\sqrt{2}\cong-54.7°,\\&\theta=\theta'=0,\\&\epsilon\epsilon'=0,\\&\epsilon'\epsilon''=-1,\\&m_\eta{}^2\to\tfrac{2}{3}m_s{}^2,\\&g\to-\lim\frac{fm_\eta{}^2}{\sqrt{3}\epsilon},\end{aligned}\tag{14}$$

which implies an octet of pseudoscalar Goldstone bosons and two massive pseudoscalar unitary-singlet states. If κ, λ, and $m_G{}^2$ are kept fixed during this limiting process, then the η', η'' masses and mixing angles do not appreciably change.

The conventional quark model for η and η' mesons gives predictions for their radiative decays.[12] In the present scheme, results for the η decays are essentially unchanged from the "small"-mixing-angle results (i.e., the ones obtained using quadratic mass mixing). However, the η' will have its decay amplitudes suppressed because of its reduced quark content. The success of the Zweig rule suggests that the decay of a pure gluon configuration proceeds either into other gluon states or into excited hadrons[2]; however, decays which require coupling of gluons to a $\bar{q}q$ pair are suppressed. Since at present one only has an upper bound established for the η' width, there is no disagreement with experiment here.

By comparing the reactions $\pi N\to\eta(\eta')N,\Delta$ at high energies, the quark-model relation

$$\frac{\sigma(\eta)}{\sigma(\eta')}=\frac{\cos^2\alpha}{\sin^2\alpha'}\left(\frac{N}{N'}\right)^2\tag{15}$$

may be tested. A difficulty with this approach is that the energy must be high enough so that kinematic effects due to the η-η' mass difference are negligible.

At present, some data above 10 GeV exist[13,14]; however, they do not appear to be sufficient to make a definitive test of the relation. The experimental uncertainty arises both from the measurement of the production cross section for $\pi^-p\to\eta' n$, which is roughly 30%, and from the branching ratio $\eta'\to2\gamma/\eta'\to$ all, which is known to be about 20%. There is room for our gluon mixing scheme to be valid.[15] Furthermore, there may be some contribution to η' production via gluon emission, which would increase $\sigma(\eta')$ beyond that given by the above equation. Finally, both the conventional model and the present scheme would expect the energy variations of $\sigma(\eta)$ and $\sigma(\eta')$ to be the same; experiment suggests a somewhat different behavior.

One can fix the mixing by an analysis of the reactions $\pi^-p\to\pi^0n,\eta n,\eta' n$; $K^+\eta\to K^0p$; and $K^-p\to\bar{K}^0n$. Denote the differential cross sections for these processes by $\sigma(\pi)$, $\sigma(\eta)$, $\sigma(\eta')$, $\sigma(K)$, and $\sigma(\bar{K})$, respectively. These reactions should be dominated by ρ and A_2 Regge exchange at high energies, so that

$$\begin{aligned}&\sigma(\pi)+3\sigma(\eta_8)=\sigma(K)+\sigma(\bar{K}),\\&\sigma(\eta_1)+\sigma(\eta_8)=\sigma(\eta)+\sigma(\eta').\end{aligned}\tag{16}$$

If we use S_T to denote the ratio of the singlet to octet couplings of the η to πA_2, then

$$\begin{aligned}&\sigma(\eta)=\sigma(\eta_8)(\cos\theta+S_T\sin\theta)^2N^2,\\&\sigma(\eta')=\sigma(\eta_8)(-\sin\theta'+S_T\cos\theta')^2(N')^2.\end{aligned}\tag{17}$$

It has been traditional to assume that S_T is independent of t (and $\theta=\theta'$, $N'=N=1$).

The ratio

$$\frac{\sigma(\eta)}{\sigma(\eta_8)}=\frac{3\sigma(\eta)}{\sigma(K)+\sigma(\bar{K})-\sigma(\pi)}\tag{18}$$

is consistent with unity, as expected, but S_T has been found[16-18] to be roughly half the quark-model value of $\sqrt{2}$.

There are preliminary results in which a measurement of the shape of differential cross sections for η and η' production at 8.4 GeV/c has been made.[19] It was found that $\sigma(\eta')$ and $\sigma(\eta)$ do not have the same shape. One is then faced with the problems of deciding how to use the data to extract the η-η' mixing angle. The principle we adopt is that Eqs. (15) and (16) apply only to the pure Regge-pole part of the amplitude and not to cuts or other types of background. There is general agreement among the various phenomenological analyses of $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta^0n$ that the spin-flip amplitudes are well described by a simple Regge formula, while spin-nonflip amplitudes are complicated by cuts, absorption, etc. Therefore, we will interpret Eqs. (15) and (16) to refer to spin-flip cross sections.

At low momentum transfers, we can use the parameterization

$$\sigma=\sigma(t=0)(1-at)e^{bt},\tag{19}$$

where we have assumed for simplicity that the flip and nonflip amplitudes have the same exponential falloff. This form works well[20] for $\pi^-p\to\pi^0\eta$ at 6 GeV/c. For η production at 8.4 GeV/c we find that

$$\sigma(\eta)=30(1-48t)e^{7t}\ \mu\text{b/GeV}^2\tag{20}$$

is a good fit to the data if $-t<0.9$ GeV^2. Clearly, there is a large spin-flip component. If we require that $\sigma(\eta')$ satisfy Eq. (19) with $b=7$, then we find that

$$\sigma(\eta') = 30(1 - 13t)e^{7t} \quad (21)$$

is a good fit to the data if $-t < 0.6\ \text{GeV}^2$. The ratio of the coefficients of t is just the ratio of the spin-flip cross sections:

$$\frac{\sigma(\eta',\ \text{flip})}{\sigma(\eta,\ \text{flip})} = 0.27. \quad (22)$$

With the values of α and α' we have deduced, the ratio Eq. (15) is just $(N/N')^2$, so that the experimental result Eq. (22) together with the expectation that $N \approx 1$ leads to

$$N' \approx 3.7,$$

$$|\epsilon'| \approx 1.6,$$

which is almost consistent with our scheme (see Table I).

A somewhat speculative proposal for studying the question of the gluon content of the η' is the following. As has been shown by Einhorn and Quigg,[21] the nonleptonic weak decays of pseudoscalar charmed mesons lead predominantly into states in which an η' occurs. For example, the relative ratios for D^0 decay into two pseudoscalars are $\bar{K}^0\eta' : K^-\pi^+ : \bar{K}^0\pi^0 : \bar{K}^0\eta = 8:6:3:1$. Similar results obtain for other decay modes and for other charmed pseudoscalars. A reduction in the quark content of the η' should suppress decay modes involving η' relative to those involving η.

Again, on a speculative note, we would point out that if the asymptotic power of hadronic form factors at large spacelike momenta is indeed related to the number of "valence" constituents,[22] then one would find η' production at large transverse momentum to be qualitatively different from the production of other mesons such as the η. This may show up in e^+e^- annihilation, if hard-gluon bremsstrahlung is the dominant source of hadrons with large momenta transverse to the main jet axes.[23]

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