SU(2) ⊗ U(1) ⊗ U(1)′ weak-interaction model and $\Delta I = \frac{1}{2}$ enhancement*

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The $\Delta I = \frac{1}{2}$ enhancement in nonleptonic decays is achieved in an SU(2) ⊗ U(1) ⊗ U(1)′ model bv the role of an additional neutral gauge boson, X_μ. The new vector boson couples to the hadronic neutral current that has predominantly isoscalar $\Delta S = 0$ term plus a small $\Delta S \neq 0$ term but it does not interact with leptons. The associated $\Delta S = 2$ processes can be made small under due conditions on parameters. Several manifestations of X_μ as a gauge boson or gluon bound state are speculated.

I. INTRODUCTION

The vector theories of fundamental interactions, in which the interactions are mediated by vector particles such as photon A_μ, heavy vector bosons $W_\mu^\pm$ and Z_μ, and gluons G_μ, have attracted a great deal of attention recently. These vector models have a common feature in that the gauge symmetry is assumed to be an exact symmetry for strong interactions, but is assumed to be a spontaneously broken symmetry for weak and electromagnetic (em) interactions. The gauge-symmetry group of the original Weinberg-Salam (WS) model[1] of weak and electromagnetic interactions is SU(2) ⊗ U(1). This WS model combined with the Glashow-Iliopoulos-Maiani (GIM) construction of currents[2] can account for most of the observed phenomenology of the semileptonic weak interactions. However, it fails[3] to give a persuasive account for the enhancement of $\Delta I = \frac{1}{2}$ nonleptonic decays compared to the accompanying $\Delta I = \frac{3}{2}$ transitions.

In order to explain the observed $\Delta I = \frac{1}{2}$ dominance of nonleptonic decays, the SU(2) ⊗ U(1) WS model with GIM currents can be extended in the context of *vector* models to the following:

(a) SU(2) ⊗ U(1) theories with additional (so far unknown) currents that can enhance the $\Delta I = \frac{1}{2}$ nonleptonic interactions compared to the $\Delta I \geq \frac{3}{2}$ part, or

(b) SU(2) ⊗ U(1) ⊗ ··· theories where the dots denote the yet unknown weak- and electromagnetic-interaction gauge group which is responsible for the observed $\Delta I = \frac{1}{2}$ enhancement in hadronic decays.

In fact, a model belonging to the first category has recently been suggested by De Rújula, Georgi, and Glashow (DGG).[4] They observed that the "$V+A$" charm-changing charged current $\bar{c}\gamma_\mu(1-\gamma_5)d$ can induce a large $\Delta I = \frac{1}{2}$ effective interaction for strange-particle decays. But it turns out that such a charm-changing current not only induces too large a $\Delta S = 2$ effect,[5] but also it is not favored from the observed rate and decay parameters of $K \to 3\pi$ decay.[6] Furthermore, this charm-changing current should accompany yet further charged currents in order to give an anomaly-free theory, and, therefore, many more undiscovered quarks and leptons are needed.[4]

In a previous paper, we proposed a model belonging to the second category.[7] Specifically, we built a weak and electromagnetic interaction model based on the gauge group SU(2) ⊗ U(1) ⊗ U(1)′.[8] We were led to this symmetry group by the following considerations: The WS model picks up a particular weak and em subgroup SU(2) ⊗ U(1) which is then spontaneously broken from the full gauge group SU(2) ⊗ U(1) ⊗ $SU(3)_{color}$ ⊗ ··· . Owing to the absence of a successful explanation of $\Delta I = \frac{1}{2}$ enhancement in the SU(2) ⊗ U(1) scheme with a minimal number of fermions, we simply enlarge the weak and em gauge group to SU(2) ⊗ U(1) ⊗ U(1)′, where U(1)′ may be a subgroup of $SU(3)_{color}$ ⊗ ··· or the phenomenological manifestation of the second- or higher-order effects of color group $SU(3)_{color}$. Also the symmetry of quark flavors (i.e., $m_u = m_d \neq m_s \ll m_c$) suggests a gauge-symmetry group SU(2) ⊗ U(1) ⊗ U(1)′.

In this SU(2) ⊗ U(1) ⊗ U(1)′ gauge model, a triplet gauge field A_μ^α ($\alpha = 1, 2, 3$) and two singlet fields B_μ and C_μ are introduced and coupled to the isospin, hypercharge, and charm groups $\vec{I}$, Y, and C, respectively. The charge assignments of the fermion and scalar fields are as usual dictated by the Gell-Mann–Nishijima formula

$$Q = I_3 + \frac{Y+C}{2} \tag{1}$$

for given multiplets. Since only Q and I_3 can be given in a multiplet, we have the freedom to choose Y or C for a fixed combination of $Y + C = 2(Q - I_3)$. This arbitrariness introduces one more parameter for every multiplet and makes us go beyond the simple nature of the SU(2) ⊗ U(1) models. However, this freedom can be constrained in the leptonic sector by imposing that the additional gauge boson X_μ (which will be explicitly given in Sec. II) does not interact with leptons. We introduce the minimum number of fermions as given in the WS model, i.e., electron, muon and their neutrinos, and four quark flavors u, d, s, and c (with three colors). To break the gauge symmetry spontaneously except in the em group, the scalar fields are introduced. We choose the simplest nontrivial representation for scalars, a doublet ϕ and a singlet ϕ'. The other choices, for example two doublets, will work as well. This simplest choice poses all the problems and solutions in our $\Delta I=\frac{1}{2}$ enhancement scheme, but with the fewest number of parameters to be introduced.

The $\Delta I=\frac{1}{2}$ enhancement is attributed to the role of the neutral boson X_μ which does not interact with leptons but *couples to the strangeness-changing hadronic neutral current with a small coefficient* and the strangeness-conserving isosinglet current with a large coefficient

$$J_\mu^x = A\,J_\mu(\Delta S=0,\ I=0)+\epsilon J_\mu(\Delta S\neq 0)\,, \qquad (2)$$

$$\frac{\epsilon}{A}\ll 1\,. \qquad (3)$$

The $\Delta I=\frac{1}{2}$ enhancement is viewed as due to a significant amplitude for the tree-diagram process, $s\to d+$ (isosinglet), which changes the isospin by $\frac{1}{2}$. A similar mechanism through a loop diagram, $s\to d$, has been adopted by DGG.[4]

Since we introduce a strangeness-changing neutral current $J_\mu(\Delta S\neq 0)$ at the lowest order, we have to check the $\Delta S=2$ processes which should be suppressed sufficiently. The smallness of ϵ together with some suggestions which will be discussed in Secs. V and VI provide a way out of the $\Delta S=2$ difficulty. Our assertion is that the $\Delta I=\frac{1}{2}$ enhancement and the $\Delta S=2$ suppression are possible for appropriate values of M_x (mass of X_μ) and GA (coupling constant of X_μ).

It is noted that all the other aspects of the present model except the X_μ contributions are exactly the same as in the WS theory involving g and Weinberg angle θ_W only.

In Sec. II the gauge model based on the group $SU(2)\otimes U(1)\otimes U(1)'$ is built by incorporating the leptons and then by breaking the symmetry spontaneously. In Sec. III the hadrons are included through quarks, and it is shown that the new particle X_μ comes into play in the hadronic sector. In Secs. IV and V we solve the problem of the $\Delta I=\frac{1}{2}$ enhancement through X_μ exchange subject to the condition of the small K_L-K_S mass difference, and in Sec. VI several implications on X_μ are discussed. Especially, we call attention to the possibility that X_μ may be a manifestation of gluon bound states whose mass is estimated to be 1–3 GeV. In the Appendix the calculation of the one-pion contribution to the K_L-K_S mass difference is briefly sketched.

II. SU(2) ⊗ U(1) ⊗ U(1)′ MODEL IN LEPTONIC SECTOR

The gauge fields are a triplet $\vec{A}_\mu^\alpha$ $(\alpha=1,2,3)$ and singlets B_μ and C_μ are coupled to $\vec{I}$, Y, and C currents, respectively. To break the gauge symmetry spontaneously, we introduce a doublet ϕ,

$$\phi=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \qquad (4)$$

with $Y=Y_S$ and $C=1-Y_S$ and a singlet ϕ'

$$\phi'=(\phi^{0\prime})\,, \qquad (5)$$

with $Y=-C=1$. They have nonvanishing vacuum expectation values

$$\langle\phi\rangle_0=\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix},\quad \langle\phi'\rangle_0=\frac{v'}{\sqrt{2}}. \qquad (6)$$

As usual, only the known leptons are grouped in left-handed SU(2) doublets and right-handed singlets:

$$L_e=\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix}=\begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}, \qquad (7)$$

with $Y=Y_L$ and $C=-1-Y_L$ and

$$e_R=\tfrac{1}{2}(1-\gamma_5)e\,, \qquad (8)$$

with $Y=Y_R$ and $C=-2-Y_R$ and likewise for the muon and its neutrino.

With these gauge fields A_μ^α, B_μ, C_μ, scalars ϕ, ϕ', and leptons L_e and e_R, we can write straightforwardly a gauge-invariant renormalizable Lagrangian,

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\tfrac{1}{4}(\partial_\mu C_\nu-\partial_\nu C_\mu)^2\\ &-\vec{L}_e\gamma_\mu[\partial_\mu-ig\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu-\tfrac{1}{2}ig'Y_LB_\mu+\tfrac{1}{2}ig''(1+Y_L)C_\mu]L_e-\bar{e}_R\gamma_\mu[\partial_\mu-\tfrac{1}{2}ig'Y_RB_\mu+\tfrac{1}{2}ig''(2+Y_R)C_\mu]e_R\\ &-G_e(\bar{e}_R\phi^+L_e+\bar{L}_e\phi e_R)+(e\to\mu)\\ &-|\partial_\mu\phi-\tfrac{1}{2}ig\vec{\tau}\cdot\vec{A}_\mu-\tfrac{1}{2}ig'Y_SB_\mu\phi-\tfrac{1}{2}ig''(1-Y_S)C_\mu\phi|^2-|\partial_\mu\phi'-\tfrac{1}{2}ig'B_\mu\phi'+\tfrac{1}{2}ig''C_\mu\phi'|^2\\ &-\mu_\phi{}^2\phi^+\phi-\lambda_\phi(\phi^+\phi)^2-\mu_{\phi'}{}^2\phi'^+\phi'-\lambda_{\phi'}(\phi'^+\phi')^2\,.\end{aligned} \qquad (9)$$

The Lagrangian (9) has an exact gauge symmetry for $\mu_\phi{}^2 \geq 0$ and $\mu_\phi{}^2 \geq 0$. However, for $\mu_\phi{}^2 < 0$ and $\mu_{\phi'}{}^2 < 0$ the gauge symmetry is spontaneously broken as the vacuum expectation values of ϕ and ϕ' are nonvanishing as defined in Eq. (6). Then the Higgs mechanism[9] is operative to generate the vector-boson masses. The four real scalars combine with corresponding vector fields to make them massive and two remaining real scalars (Higgs scalars) with masses $-2\mu_\phi{}^2$ and $-2\mu_{\phi'}{}^2$ decouple completely from any of the gauge fields. (Note that we had six real scalars or three complex scalars.) The photon field A_μ remains massless while the other four vector fields $W_\mu^\pm$, Z_μ, and X_μ acquire masses. Specifically, the W_μ defined by

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu{}^2) \tag{10}$$

has a mass

$$M_W = \tfrac{1}{2} g v , \tag{11}$$

and Z_μ and X_μ have a mass relation

$$\tfrac{1}{2}(M_Z{}^2 Z_\mu{}^2 + M_X{}^2 X_\mu{}^2) = \frac{v^2}{8}[gA_\mu{}^3 - g' Y_S B_\mu - g''(1-Y_S)C_\mu]^2 + \frac{v'^2}{8}(g' B_\mu - g'' C_\mu)^2 , \tag{12}$$

where A_μ, Z_μ, and X_μ are related to the original $A_\mu{}^3$, B_μ, and C_μ by an orthogonal transformation

$$\begin{pmatrix} A_\mu{}^3 \\ B_\mu \\ C_\mu \end{pmatrix} = M \begin{pmatrix} A_\mu \\ Z_\mu \\ X_\mu \end{pmatrix} . \tag{13}$$

It should be noted that the choice $Y = -C - 1$ for ϕ' is general for the present purpose.[10] The additional X_μ should not interfere with the successful phenomenology of leptonic decays of K mesons in the WS–GIM scheme. *Hence we require that X_μ does not interact with leptons* (the X_μ will be coupled to the strangeness-changing neutral current in Sec. III).

This requirement simplifies the arguments and leads to a nice result that the leptonic and semileptonic interactions do not depart from those of the WS–GIM scheme. Then from the leptonic Lagrangian in (9), the orthogonal matrix M can be simply parametrized by

$$M = \begin{pmatrix} \sin\theta_W & -\cos\theta_W & 0 \\ \cos\theta_W \sin\zeta & \sin\theta_W \sin\zeta & -\cos\zeta \\ \cos\theta_W \cos\zeta & \sin\theta_W \cos\zeta & \sin\zeta \end{pmatrix}, \tag{14}$$

and two contraints are obtained:

$$Y_R = 2Y_L , \qquad g' Y_L \cos\zeta = -g''(1+Y_L)\sin\zeta . \tag{15}$$

From (12), (13), and (14), we have three more constraints:

$$\tan\zeta = g''/g' , \qquad \tan\theta_W = \frac{g''}{g}\cos\zeta , \qquad Y_S = -Y_L = \sin^2\zeta . \tag{16}$$

The mass of Z_μ is given by

$$M_Z = \tfrac{1}{2} g v \left| \cos\theta_W + \frac{g''}{g}\sin\theta_W \cos\zeta \right| = M_W |\sec\theta_W| , \tag{17}$$

and the mass of X_μ is given by

$$M_X = \tfrac{1}{2} G v' , \tag{18}$$

where G is defined to be a positive number,

$$G \equiv |g' \cos\zeta + g'' \sin\zeta| = (g'^2 + g''^2)^{1/2} . \tag{19}$$

To have the correct electromagnetic interactions of electrons, we get

$$-e = g\sin\theta_W = g'\cos\theta_W \sin\zeta = g''\cos\theta_W\cos\zeta , \qquad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} + \frac{1}{g''^2} . \tag{20}$$

As usual, the Fermi theory of weak interaction (μ decay) is recovered in the low-momenta region provided that

$$\frac{g^2}{8M_W{}^2} = \frac{G_F}{\sqrt{2}} . \tag{21}$$

With these relations, the lepton–gauge-boson coupling is exactly the same as in the WS model,

$$\mathcal{L}^{\text{int}}_{\text{lepton-gauge}} = \frac{ig}{\sqrt{2}}\bar{L}_e \gamma_\mu (\tau^+ W_\mu^+ + \tau^- W_\mu^-) L_e + ie\bar{e}\gamma_\mu e A_\mu - \tfrac{1}{2} i g \sec\theta_W Z_\mu [\bar{\nu}_{eL}\gamma_\mu \nu_{eL} + (2\sin^2\theta_W - \tfrac{1}{2})\bar{e}\gamma_\mu e + (-\tfrac{1}{2})\bar{e}\gamma_\mu\gamma_5 e] + (e \to \mu) . \tag{22}$$

Especially, we note that the Z_μ coupling to the neutrino current is the same form as in the WS theory. As required, X_μ is absent from the lepton–gauge-boson interaction Lagrangian, which ensures that no new neutrino-induced neutral-current effect is added to the SU(2)⊗U(1) model.

III. HADRONIC SECTOR

In this section we include hadrons for the SU(2) ⊗U(1)⊗U(1)′ gauge model. To construct a hadronic Lagrangian, let us first group the GIM quarks into two doublets

$$L_1=\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}u\\ d_\theta\end{pmatrix}=\begin{pmatrix}u_L\\ d_{\theta L}\end{pmatrix},$$

$$L_2=\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}c\\ s_\theta\end{pmatrix}=\begin{pmatrix}c_L\\ s_{\theta L}\end{pmatrix}, \tag{23}$$

$$d_\theta=d\cos\theta_C+s\sin\theta_C,\quad s_\theta=s\cos\theta_C-d\sin\theta_C,$$

and four singlets

$$R_i=\tfrac{1}{2}(1-\gamma_5)q_i\quad(i=1,2,3,4), \tag{24}$$

where $q_1=u$, $q_2=d$, $q_3=s$, and $q_4=c$. Note that we introduced the Cabibbo-mixed states d_θ and s_θ in the left-handed doublets and unmixed quarks in the right-handed singlets.[11]

The quark q_i has charge Q_i given by

$$Q_1=Q_4=a=\tfrac{2}{3},\quad Q_2=Q_3=a-1. \tag{25}$$

Hence Y and C for different multiplets can be assigned by

$$\begin{aligned}&Y_{L_i}=\beta_i,\quad C_{L_i}=2a-1-\beta_i\quad(i=1,2),\\ &Y_{R_i}=\alpha_i,\quad C_{R_i}=2Q_i-\alpha_i\quad(i=1,2,3,4).\end{aligned} \tag{26}$$

Now the gauge-invariant Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{\text{quark}}=&-\sum_{j=1}^{2}\bar{L}_j\gamma_\mu[\partial_\mu-\tfrac{1}{2}ig\vec{\tau}\cdot\vec{A}_\mu-\tfrac{1}{2}ig'\beta_jB_\mu\\ &\qquad-\tfrac{1}{2}ig''(2a-1-\beta_j)C_\mu]L_j\\ &-\sum_{j=1}^{4}\bar{R}_j\gamma_\mu(\partial_\mu-\tfrac{1}{2}ig'\alpha_jB_\mu-\tfrac{1}{2}ig''C_{R_j}C_\mu)R_j\\ &-\mathcal{L}_{\text{mass}},\end{aligned} \tag{27}$$

where $\mathcal{L}_{\text{mass}}$ contains the invariant interaction of quarks with the scalar fields and generates quark masses by the spontaneous symmetry breakdown. We keep our attention only on the quark–gauge-field interaction Lagrangian. From (14), (20), and (27) it is easily checked that the correct em interaction is obtained,

$$-eA_\mu J_\mu^{\text{em}} \tag{28}$$

with

$$J_\mu^{\text{em}}=i\bar{q}\gamma_\mu\tfrac{1}{2}\left[\lambda^3+\frac{1}{\sqrt{3}}\lambda^8-(\tfrac{2}{3})^{1/2}\lambda^{15}+\frac{\sqrt{2}}{3}\lambda^0\right]q, \tag{29}$$

where λ^α ($\alpha=0,1,\ldots,15$) are the usual U(4) matrices[12] and q denotes the column matrix of quarks with the element q_i. As expected we have six hadronic parameters β_i ($i=1,2$), α_i ($i=1,2,3,4$) due to our introduction of six hadronic multiplets. In terms of $W_\mu^\pm$, A_μ, Z_μ, and X_μ the quark–gauge-field interaction Lagrangian is

$$\begin{aligned}\mathcal{L}'=&g[W_\mu^-(L_\mu^{\pi^+}+L_\mu^{F^+})+\text{H.c.}]-eA_\mu J_\mu^{\text{em}}\\ &-g\sec\theta_W Z_\mu\left(L_\mu^{\text{em}}-\frac{\sqrt{2}}{3}L_\mu^0-\sin^2\theta_W J_\mu^{\text{em}}\right)\\ &-GX_\mu J_\mu^X,\end{aligned} \tag{30}$$

where

$$\begin{aligned}J_\mu^X=&\frac{1}{2}\left(\sum_{i=1}^{4}\alpha_i\right)R_\mu^0\Big/\sqrt{2}+\tfrac{1}{2}(\alpha_1-\alpha_2)R_\mu^3\\ &+\tfrac{1}{2}(\alpha_1+\alpha_2-2\alpha_3)R_\mu^8/\sqrt{3}\\ &+\tfrac{1}{2}(\alpha_1+\alpha_2+\alpha_3-3\alpha_4)R_\mu^{15}/\sqrt{6}\\ &+(\beta_1+\beta_2)L_\mu^0/\sqrt{2}+\frac{1}{\sqrt{3}}(\beta_1-\beta_2)(L_\mu^8+L_\mu^{15}/\sqrt{2})\\ &-\sin^2\xi\left(R_\mu^{\text{em}}+\frac{\sqrt{2}}{3}L_\mu^0\right),\end{aligned} \tag{31}$$

$$L_\mu^{\pi^\pm}=(L_\mu^1\mp iL_\mu^2)/\sqrt{2},\quad L_\mu^{F^\pm}=(L_\mu^{13}\pm iL_\mu^{14})/\sqrt{2}, \tag{32}$$

and the left- and right-handed currents are defined as

$$\begin{aligned}L_\mu^\alpha&=i(\bar{L}_1,\bar{L}_2)\gamma_\mu\frac{\lambda^\alpha}{2}\begin{pmatrix}L_1\\ L_2\end{pmatrix},\\ R_\mu^\alpha&=i\bar{q}\gamma_\mu\frac{1-\gamma_5}{2}\frac{\lambda^\alpha}{2}q.\end{aligned} \tag{33}$$

Note that the charged, neutral, and em currents coupled to $W_\mu^\pm$, Z_μ, and A_μ respectively are the same as those of the WS model with the GIM scheme.

IV. THE NEW HADRONIC CURRENT

In order to look at the additional hadronic current J_μ^X more closely, let us explicitly write Eq. (31) in terms of quark fields,

$$J_\mu^X=J_\mu(\Delta S=0,I=0)+J_\mu(\Delta S=0,I=1)+J_\mu(\Delta S\neq0), \tag{34}$$

where

$$-iJ_\mu(\Delta S=0,I=0)=\left(\frac{\alpha_1+\alpha_2}{4}-\tfrac{1}{6}\sin^2\zeta\right)(\bar{u}_R\gamma_\mu u_R+\bar{d}_R\gamma_\mu d_R)+\left(\frac{\alpha_3}{2}+\tfrac{1}{3}\sin^2\zeta\right)\bar{s}_R\gamma_\mu s_R+\left(\frac{\alpha_4}{2}-\tfrac{2}{3}\sin^2\zeta\right)\bar{c}_R\gamma_\mu c_R$$
$$+\tfrac{1}{4}[\beta_1(1+\cos^2\theta_C)+\beta_2\sin^2\theta_C-\tfrac{2}{3}\sin^2\zeta](\bar{u}_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu d_L)$$
$$+\left(\frac{\beta_1}{2}\sin^2\theta_C+\frac{\beta_2}{2}\cos^2\theta_C-\tfrac{1}{6}\sin^2\zeta\right)s_L\gamma_\mu s_L+\left(\frac{\beta_2}{2}-\tfrac{1}{6}\sin^2\zeta\right)\bar{c}_L\gamma_\mu c_L\ , \quad (35)$$

$$-iJ_\mu(\Delta S=0,I=1)=\left(\frac{\alpha_1-\alpha_2}{4}-\tfrac{1}{2}\sin^2\zeta\right)(\bar{u}_R\gamma_\mu u_R-\bar{d}_R\gamma_\mu d_R)+\left(\frac{\beta_1-\beta_2}{4}\right)\sin^2\theta_C(\bar{u}_L\gamma_\mu u_L-\bar{d}_L\gamma_\mu d_L)\ , \quad (36)$$

$$-iJ_\mu(\Delta S\neq 0)=\tfrac{1}{2}(\beta_1-\beta_2)\sin\theta_C\cos\theta_C(\bar{s}_L\gamma_\mu d_L+\bar{d}_L\gamma_\mu s_L)\ . \quad (37)$$

While in other works[13] one makes $J_\mu(\Delta S\neq 0)$ vanish to meet the condition of highly suppressed neutral K° decays, in our scheme this $J_\mu(\Delta S\neq 0)$ term is just what is needed for the $\Delta I=\frac{1}{2}$ enhancement. Actually it turns out that the $J_\mu(\Delta S\neq 0)$ term should be small so that it gives a small contribution to the K_L-K_S mass difference. Then the second term in Eq. (36) becomes very small as it is proportional to $(\beta_1-\beta_2)\sin^2\theta_C$. Also to have a small $\Delta I=\frac{3}{2}$ amplitude compared to the $\Delta I=\frac{1}{2}$ amplitude via X_μ exchange, $J_\mu(\Delta S=0,I=1)$ is necessarily small and negligible compared to $J_\mu(\Delta S=0,I=0)$, which will be the case if

$$\alpha_1-\alpha_2=2\sin^2\zeta\ . \quad (38)$$

In general, $J_\mu(\Delta S=0,I=0)$ can have arbitrary mixtures of vector and axial-vector components in our model, though $J_\mu(\Delta S\neq 0)$ is always a $V-A$ type.

(i) Pure V type $J_\mu(\Delta S=0,I=0)$. The hadronic parameters α's and β's should satisfy the following relations to have the pure vector type $J_\mu(\Delta S=0,I=0)$:

$$\tfrac{1}{2}(\alpha_1+\alpha_2)=\beta+\epsilon\cos^2\theta_C\ ,$$
$$\alpha_3=\beta-\epsilon\cos 2\theta_C-\sin^2\zeta\ , \quad (39)$$
$$\alpha_4=\beta-\epsilon+\sin^2\zeta\ .$$

(ii) Pure A type $J_\mu(\Delta S=0,I=0)$. The following relations are required for the $J_\mu(\Delta S=0,I=0)$:

$$-\tfrac{1}{2}(\alpha_1+\alpha_2)=\beta+\epsilon\cos^2\theta_C-\tfrac{2}{3}\sin^2\zeta\ ,$$
$$-\alpha_3=\beta-\epsilon\cos 2\theta_C+\tfrac{1}{3}\sin^2\zeta\ , \quad (40)$$
$$-\alpha_4=\beta-\epsilon-\tfrac{5}{3}\sin^2\zeta\ .$$

(iii) V−A type $J_\mu(\Delta S=0,I=0)$. The following relations should be satisfied:

$$\alpha_1=\alpha_4=\tfrac{4}{3}\sin^2\zeta,\quad \alpha_2=\alpha_3=-\tfrac{2}{3}\sin^2\zeta\ . \quad (41)$$

In Eqs. (39) and (40), β and ϵ are

$$\beta=\tfrac{1}{2}(\beta_1+\beta_2),\quad \epsilon=\tfrac{1}{2}(\beta_1-\beta_2)\ . \quad (42)$$

For cases (i), (ii), and (iii) we could eliminate the parameters α_1, α_2, α_3, and α_4, leaving only β and ϵ in $J_\mu(\Delta S=0,I=0)$:

$$-iJ_\mu(\Delta S=0,\ I=0)=\left(\frac{\beta}{2}+\frac{\epsilon}{2}\cos^2\theta_C-\frac{\sin^2\zeta}{6}\right)(\bar{u}\Gamma_\mu u+\bar{d}\Gamma_\mu d)+\left(\frac{\beta}{2}-\frac{\epsilon}{2}\cos 2\theta_C-\frac{\sin^2\zeta}{6}\right)\bar{s}\Gamma_\mu s+\left(\frac{\beta}{2}-\frac{\epsilon}{2}-\frac{\sin^2\zeta}{6}\right)\bar{c}\Gamma_\mu c\ , \quad (43)$$

where

$$\Gamma_\mu=\gamma_\mu \quad \text{for pure } V \text{ type}\ ,$$
$$\Gamma_\mu=\gamma_\mu\gamma_5 \quad \text{for pure } A \text{ type}\ , \quad (44)$$
$$\Gamma_\mu=\gamma_\mu\tfrac{1}{2}(1+\gamma_5) \quad \text{for pure } V-A \text{ type}\ .$$

Since ϵ is considered to be very small, Eq. (43) takes approximately an SU(4)-singlet structure,

$$-iJ_\mu(\Delta S=0,I=0)\cong -i(\tfrac{1}{2}\beta-\tfrac{1}{6}\sin^2\zeta)2\sqrt{2}\,J_\mu^0=\left(\frac{\beta}{2}-\tfrac{1}{6}\sin^2\zeta\right)(\bar{u}\Gamma_\mu u+\bar{d}\Gamma_\mu d+\bar{s}\Gamma_\mu s+\bar{c}\Gamma_\mu c)\ . \quad (45)$$

(iv) V+A type $J_\mu(\Delta S=0, I=0)$. If β_1 and β_2 satisfy

$$\beta_1=\beta_2=\tfrac{1}{3}\sin^2\zeta\,, \tag{46}$$

we have

$$-iJ_\mu(\Delta S=0,I=0)=\left(\frac{\alpha_1+\alpha_2}{4}-\tfrac{1}{6}\sin^2\zeta\right)(\bar u_R\gamma_\mu u_R+d_R\gamma_\mu d_R)+\left(\frac{\alpha_3}{2}+\tfrac{1}{3}\sin^2\zeta\right)\bar s_R\gamma_\mu s_R+\left(\frac{\alpha_4}{2}-\tfrac{2}{3}\sin^2\zeta\right)\bar c_R\gamma_\mu c_R\,. \tag{47}$$

However, we shall restrict ourselves to the *pure V type* $J_\mu(\Delta S=0)$ to avoid parity nonconservation in the nonweak interactions.

V. RELAIZATION OF $\Delta I=\frac{1}{2}$ ENHANCEMENT AND $\Delta S=2$ SUPPRESSION

The current-current interaction of J^X_μ is taken as the additional nonleptonic interaction Lagrangian. This is possible for an appropriately large value of M_X [i.e., a reasonably large value for taking a local limit, $M_X^2\gg$ typical momentum transfer squared in nonleptonic decays, $(m_K-m_\pi)^2$]. We then get six different nonleptonic terms. Among these, three $\Delta S=0$ terms are difficult to test due to the competing strong-interaction effects and hence are neglected at least for the moment. The remaining three terms are

$$\begin{aligned}\mathcal{L}^X(\Delta I=\tfrac{1}{2})&=2\frac{G^2}{M_X^{\,2}}J_\mu(\Delta S=0,\ I=0)\,J_\mu(\Delta S\neq 0)\,,\\ \mathcal{L}^X(\Delta I\leqslant\tfrac{3}{2})&=2\frac{G^2}{M_X^{\,2}}J_\mu(\Delta S=0,\ I=1)\,J_\mu(\Delta S\neq 0)\,,\\ \mathcal{L}^X(\Delta S=2)&=\frac{G^2}{M_X^{\,2}}J_\mu(\Delta S\neq 0)\,J_\mu(\Delta S\neq 0)\,.\end{aligned} \tag{48}$$

As we have mentioned repeatedly, $\mathcal{L}^X(\Delta S=2)$ should be sufficiently small [i.e., $(\beta_1-\beta_2)^2\sin^2\theta_C\cos\theta_C\ll 1$] so as to have the small K_L-K_S mass difference. Then the $\mathcal{L}^X(\Delta I\leqslant\frac{3}{2})$ term is *also* small [to the order of $(\beta_1-\beta_2)^2\cos\theta_C\sin^3\theta_C$] if the condition (38) is met. Under these conditions the bulk of the $\Delta I=\frac{3}{2}$ amplitudes comes from $W^\pm_\mu$ exchanges which also give the $\Delta I=\frac{1}{2}$ amplitude.

Though $\mathcal{L}^X(\Delta I\leqslant\frac{3}{2})$ and $\mathcal{L}^X(\Delta S=2)$ are small, $\mathcal{L}^X(\Delta I=\frac{1}{2})$ should be comparable to the other nonleptonic Lagrangian coming from $W^\pm_\mu$ and Z_μ exchanges. Thus while $AG^2\epsilon/M_X^{\,2}$ is of the order of G_F, $G^2\epsilon^2/M_X^{\,2}$ should fall within the order of $G_F\alpha^2$ where $\alpha=\frac{1}{137}$. Here AG is the typical strength of the X_μ coupling to the $J_\mu(\Delta S=0,I=0)$ isosinglet current, e.g., $A=\frac{1}{2}\beta-\frac{1}{6}\sin^2\zeta$ for the pure V type $J_\mu(\Delta S=0,I=0)$.

Since the experimental strength of the $\Delta I=\frac{1}{2}$, $S\neq 0$ nonleptonic amplitude is more than 20 times the $\Delta I=\frac{3}{2}$ amplitude, we have approximately

$$\frac{2A(G^2/8M_X^{\,2})\,|2\epsilon|+g^2/8M_W^{\,2}}{\frac{1}{5}g^2/8M_W^{\,2}}\simeq 20 \tag{49}$$

subject to the constraint

$$\frac{G^2}{m_K^{\,2}M_X^{\,2}}\langle\bar K^0|-J^+_\mu(\Delta S\neq 0)\,J_\mu(\Delta S\neq 0)|K^0\rangle<\frac{m_L-m_S}{m_K}\,, \tag{50}$$

where in Eq. (49) use has been made of the fact that the relative enhancement of an octet piece compared to the 27-plet piece is about 5.[14] Assuming that most of the K_L-K_S mass difference comes from the X_μ contribution, we replace the inequality (50) by an equality and equate it with the experimental value $(m_L-m_S)/m_K\cong 0.7\times 10^{-14}$:

$$\frac{G^2}{m_K^{\,2}M_X^{\,2}}\langle\bar K^0|-J^+_\mu(\Delta S\neq 0)\,J_\mu(\Delta S\neq 0)|K^0\rangle\cong 0.7\times 10^{-14}\,. \tag{50$'$}$$

We look for a simultaneous solution of (49) and (50′) for M_X and $\epsilon(=(\beta_1-\beta_2)/2)$. Certainly there is an ambiguity in the evaluation of the matrix element $\langle\bar K^0|-J^+_\mu(\Delta S\neq 0)J_\mu(\Delta S\neq 0)|K^0\rangle$. We can at best give an order of magnitude estimate at the moment. The contribution to this matrix element coming from vacuum insertions is calculable, and it is hoped that it will give a dominant contribution. All the other contributions are much more dependent on the assumption of hadronic wave functions. We parametrize the latter by a single number δ defined as the ratio to the vacuum contribution. However, it is well known that CP-even (or CP-odd) states contribute to this matrix element positively (or negatively).[15] As shown in the Appendix the one-pion contribution to δ can be as large as that of the vacuum contribution depending upon the form of the hadronic matrix assumed. Then Eq. (50′) can be simplified by making use of partial conservation of axial-vector current (PCAC),

$$\tfrac{2}{3}(4)\frac{G^2\epsilon^2 f_K^{\,2}\sin^2\theta_C\cos^2\theta_C}{4M_X^{\,2}}(1-\delta)\cong 0.7\times 10^{-14}\,, \tag{50$''$}$$

where the factor 4 is due to the four ways of inserting the vacuum in the case of single-color scheme, and the reduction factor ($\frac{2}{3}$) is due to the fact that any physical states should be color singlets of three colors. Using the numerical values $f_K = 0.122$ GeV and $v = 248$ GeV, a simultaneous solution of Eqs. (49) and (50″) is found

$$\epsilon = \left(\frac{A}{1-\delta}\right)(0.31\times 10^{-6}), \tag{51}$$

$$M_X = GA(1-\delta)^{-1/2}(0.08)\ \text{GeV}. \tag{52}$$

The relation between GA and $(1-\delta)$ is plotted in Fig. 1 for several values of M_X. From this we can easily see that $M_X \geq 1$ GeV implies generally that $GA > 1$ for all reasonable values of $(1-\delta)$. For example, for $M_X = 1$ GeV, a typical value of $(1-\delta)$ is 0.1 for $GA \cong 4$ while $(1-\delta)$ is 0.5 for $GA \simeq 9$. Thus we may say that the assumption of the mass M_X large enough to take a local current-current interaction leads to a large value of GA compared to g for a wide range of $(1-\delta)$. Noting that GA characterizes the coupling constant of X_μ to the isosinglet current, one obtains that X_μ *provides strong or medium-strong interactions* [$GA = 4$ implies that $(GA)^2/4\pi = 1.3$] unless the one-pion and higher intermediate states cancel the contribution to the matrix element (50) coming from the vacuum insertions. If this is the case, $J_\mu(\Delta S = 0, I = 0)$ should be pure V or A type not to violate the observed parity conservation in strong interactions. [However, pure A type $J_\mu(\Delta S = 0, I = 0)$ is not favored due to its contribution to the triangle anomaly.] Then the SU(2)⊗U(1) group gets almost decoupled from the U(1)′ group when $A \approx 1$. This is because $G \gg g$ implies that $\sin^2\xi \approx 1$ for comparable values of g and g' and the matrix M given in Eq. (14) is reduced to

$$M \xrightarrow[G\gg g]{} \begin{pmatrix} \sin\theta_W & -\cos\theta_W & 0 \\ \cos\theta_W & \sin\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{53}$$

VI. THE NEW BOSON X_μ

In Sec. V, we gave some specific numerical examples for the mass of X_μ. In this section we discuss several possibilities for the new particle X_μ.

(i) X_μ as a weak gauge boson. From Fig. 1 we see that $(1-\delta)$ should be very small if GA is to be a weak coupling constant. This is equivalent to the fact that the vacuum contribution to the K_L-K_S mass difference is almost completely canceled by the other contributions. In this case $J_\mu(\Delta S = 0, I = 0)$ can be any mixture of vector and axial-vector isosinglet currents. Though the mass of X_μ falls in the low-energy range (around 1 GeV or more), the strong interactions may shield a direct observation of it.

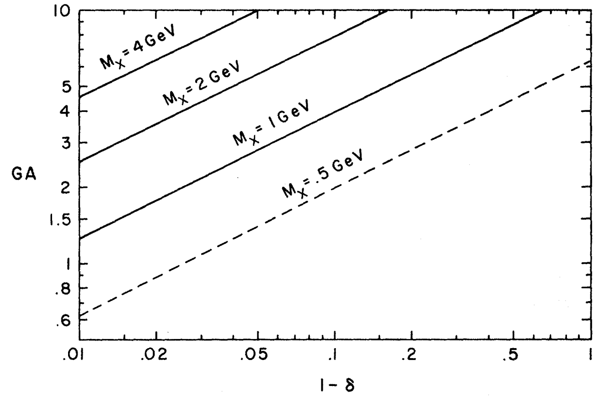

FIG. 1. The solutions of Eqs. (49) and (50″) are drawn as GA vs $(1-\delta)$ for the values of $M_X = 1$, 2, and 4 GeV. Also the case $M_X = 0.5$ is shown as a comparison.

(ii) X_μ as a strongly interacting gluon. For a wide range of $(1-\delta)$, we have solutions with $GA > 1$, which imply that X_μ is a strongly interacting boson. Then we are permitted to choose only a pure vector isosinglet such as $J_\mu(\Delta S = 0, I = 0)$ from considerations of the parity conservation in nonweak interactions and the removal of the triangle anomaly. This X_μ boson may be a gauge boson corresponding to another strong-interaction gauge group U(1)′, and the strong interactions are mediated by the singlet gluon X_μ along with the conventional octet gluons $G_\mu^\alpha(\alpha = 1, 2, \ldots, 8)$ which transforms like an adjoint representation of $SU(3)_{\text{color}}$. Namely, we are led to a Weinberg-type model for strong interactions with gauge group $SU(3)_{\text{color}} \otimes U(1)'$.

However, this possibility faces a serious difficulty which has to do with theoretical reasons, as has recently been pointed out by Fritzsch, Gell-Mann, and Leutwyler.[16] Though it has not been ruled out by experiment so far, it is argued that the light-cone algebra breaks down for the axial-vector current in the color-singlet-gluon picture due to the unavoidable anomalous divergence term.

(iii) X_μ as a gluon bound state—phenomenological description. If $(1-\delta)$ falls in the range of 0.1–1 and $J_\mu(\Delta S = 0, I = 0)$ is a pure vector or axial-vector, the gluon-bound-state (gluonic matter not containing any quark contents) picture[17] is another possibility for X_μ. In this case our description is only phenomenological in the sense that in principle the X_μ interaction is derivable from the basic strong-interaction dynamics.

Let us assume that strong interactions are mediated by the octet gluons corresponding to $SU(3)_{\text{color}}$. Normally any single-gluon state is not supposed to be a physical state since *all the physical states are color singlets*.[18] However, there can be color-singlet states which are resonances or bound states of two or more gluons. Consider the simplest color-singlet bound states that are made of two and three gluons. Since the direct products of two and three adjoint representations of color SU(3) group are

$$8\otimes 8 = 1\oplus 8\oplus 8\oplus 10\oplus 10^*\oplus 27\,, \tag{54}$$

$$8\otimes 8\otimes 8 = 1\oplus 1\oplus \text{eight}8\oplus\text{four}10 \oplus\text{four}10^*\oplus\text{six}27\oplus\text{two}35 \oplus\text{two}35^*\oplus 64\,, \tag{55}$$

there are three color-singlet gluon bound states with all the possible spin and other quantum-number classifications up to three-gluon states. Note that these states do not change the isospin. Among various gluon bound states, spin-1 states can be identified as X_μ.

Though both the two- and three-gluon states can contribute to the $\Delta I=\frac{1}{2}$ enhancement, it is reasonable to assume that the two-gluon-state contribution will dominate the others as the masses of three-gluon states are expected to be much larger than those of two-gluon states. If we identify X_μ as the spin-1 two-gluon state, the present mechanism alone will clearly underestimate the $\Delta I=\frac{1}{2}$ amplitude, leaving the possibility that the mass range of the gluon bound state X_μ may be as large as three times the value considered in this paper. (We should include the contributions of spin-0 and spin-2 states.)

We understand in this gluon-bound-state picture why ϵ is so small and yet GA is so big. Large GA entails simply that the gluon bound state interacts strongly. On the other hand, ϵ is so small because of the phenomenological effect that X_μ changes the s quark to a d quark. Somehow the weak interaction of $W_\mu^\pm$ exchange changes the s quark to a d quark, to which X_μ is coupled as $g^2 d\Gamma_\mu s X_\mu$. Hence we expect a small number ϵ to be of the $G_F M_P{}^2$ order. [Note that $\epsilon\approx 0.3\times 10^{-6} A/(1-\delta)$.] For a better understanding of this point, several Feynman diagrams are drawn in Fig. 2.

The asymptotically free theory[19] predicts that the coupling constant decreases logarithmically as energy increases. Hence we naively expect that the mass of X_μ is not greater than 3–4 GeV for large GA, so that

$$1\ \text{GeV} \lesssim M_X < 3\text{–}4\ \text{GeV}\,. \tag{56}$$

This case can be tested in hadronic reactions since color-singlet states should be observable.

VII. CONCLUSION

Enlarging the weak and em gauge group to $SU(2)\otimes U(1)\otimes U(1)'$ and introducing the heavy boson X_μ, it is possible to construct a mechanism which gives the $\Delta I=\frac{1}{2}$ enhancement in nonleptonic decays. As pointed out in Sec. VI, there can be several explanations for this X_μ. The gluon-bound-state picture for X_μ is especially fascinating owing to the fact that a natural explanation for the $\Delta I=\frac{1}{2}$ enhancement exists in the minimal weak and em gauge theory, namely in the WS model, not introducing any more currents or particles except those necessary for the WS–GIM scheme. The gluon bound states also affect the strong interactions. It is noted that the gluon-bound-state mass is around 1–3 GeV if it really acts for the $\Delta I=\frac{1}{2}$ enhancement.

Whatever the X_μ turns out to be, the present scheme for nonleptonic weak interactions may give an insight into the $\Delta I=\frac{1}{2}$ enhancement and the $\Delta S=2$ suppression, but without departing from the WS theory for leptonic and semileptonic weak interactions.

APPENDIX: ONE-PION CONTRIBUTION

To evaluate the matrix element

$$\langle \overline{K}^0 | -\mathcal{L}(\Delta S=2) | K^0\rangle = \left\langle \overline{K}^0 \left| -\frac{G^2}{M_X{}^2}\epsilon^2 \sin^2\theta_C\cos^2\theta_C \overline{s}_L\gamma_\mu d_L \overline{s}_L\gamma_\mu d_L \right| K^0\right\rangle\,, \tag{A1}$$

let us insert a complete set of states in every pos-

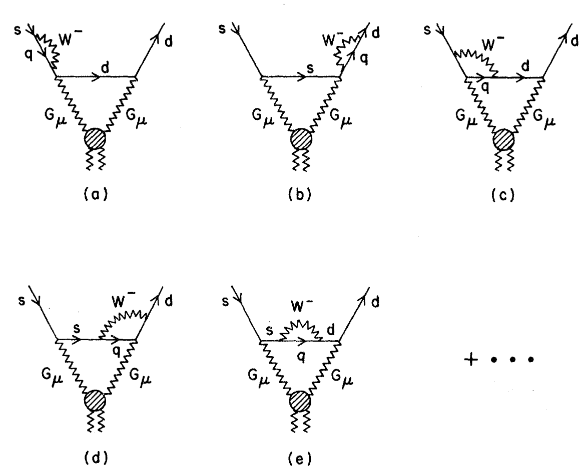

FIG. 2. The WS and quark-gluon theory predicts the $\Delta I=\frac{1}{2}$ enhancement through gluon bound states, whose mechanism is approximated at our $SU(2)\otimes U(1)\otimes U(1)'$ model. Several diagrams for this mechanism are shown above where q is the u or c quark and ● represents the gluon bound state X_μ. The dots denote the other contributions.

sible way between the current operators

$$1=|0\rangle\langle 0|+\int \frac{d^3p_\pi}{(2\pi)^3 2E_\pi}|\pi\rangle\langle\pi|+\cdots . \quad (A2)$$

The vacuum insertion is evaluated in the text in the three-color scheme. In this Appendix we are interested in the next term in Eq. (A2). The one-pion term gives a relevant matrix element

$$-\frac{G^2}{4M_X^2}\epsilon^2\sin^2\theta_C\cos^2\theta_C\int\frac{d^3p_\pi}{(2\pi)^3 2E_\pi}|\langle\pi|V_\mu^{6-i7}|K^0\rangle|^2 , \quad (A3)$$

which can be compared to the vacuum term in the PCAC approximation,

$$\frac{G^2}{4M_X^2}\epsilon^2\sin^2\theta_C\cos^2\theta_C|\langle 0|A_\mu^{6-i7}|K^0\rangle|^2 = \frac{G^2}{4M_X^2}\epsilon^2\sin^2\theta_C\cos^2\theta_C f_K{}^2 M_K{}^2 . \quad (A4)$$

To calculate (A3), let us define

$$\langle\pi^0(k)|V_\mu^{6-i7}|K^0(P)\rangle = -\frac{1}{\sqrt{2}}[(P+k)_\mu g_0(q^2)+(P-k)_\mu g_0'(q^2)] \simeq -\frac{1}{\sqrt{2}}(P+k)_\mu g_0(q^2) , \quad (A5)$$

where $q=P-k$, with

$$g_0(0)=1 . \quad (A6)$$

Experimentally $g_0(q^2)$ at a low q^2 is given by

$$g_0(q^2)=1-\frac{2}{M_0{}^2}q^2 \quad (A7)$$

with $M_0{}^2=1.8\pm 0.3$ GeV^2. If we integrate (A3) over the entire $q^2\geq 0$ using the dipole fit of Eq. (A7), we obtain $\delta_{1\pi}=8.4^{+4.5}_{-3.3}$ for the ratio of the one-pion contribution to the vacuum one. This is clearly unreasonable. However, if we integrate Eq. (A7) over the physically allowed q^2 in $K^0_{\mu 3}$ decay, namely up to the maximum pion momenta

$$|\vec{k}|_{\max}=[(m_K-m_\mu+m_\pi)(m_K-m_\mu-m_\pi)]^{1/2} , \quad (A8)$$

we obtain $\delta_{1\pi}=0.13$. Thus the K_L-K_S mass difference is very sensitive to the hadronic wave functions assumed. Even if the three-color model is considered, this result is not changed, since the vacuum contribution also undergoes a change by the same factor.

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