Coupling-constant renormalization in unified gauge theories containing the Pati-Salam model

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The relationship between $\sin^2\theta_W$ and the renormalized quark-gluon constant is found for unified models containing the Pati-Salam model with either fractional or integral quark charges, if $SU(2)_L \times U(1) \times SU(3)$ or $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$ is preserved down to the final stage of symmetry breaking. $\sin^2\theta_W$ is reduced from a bare value of 3/8 to asymptotic values of 1/6 and 1/4, respectively, in the limit of a large quark-gluon coupling constant. The relevance of this limit is discussed for Pati-Salam models with fractional and integral quark charges. The experimental range for $\sin^2\theta_W$ is shown to be consistent with an asymptotic value of 1/4, but not 1/6, if the Weinberg-Salam constraint $m_Z{}^2 = m_W{}^2/\cos^2\theta_W$ is relaxed.

I. INTRODUCTION

The success of the Weinberg-Salam theory[1] in unifying electromagnetism with the observed weak interactions suggests the possibility that unification may extend to the strong interactions as well. One obstacle to such a program is the disparity in strengths between the strong interactions and the electromagnetic-weak interactions. This problem may be avoided by theories in which quark confinement arises from infrared divergences of a non-Abelian theory. Unified models that seek to avoid infrared mechanisms (or catastrophes) can arise provided differing charge renormalizations produced by spontaneous symmetry breaking yield a large quark-gluon coupling constant.[2]

Georgi, Quinn, and Weinberg[3] have further considered the possibility that the observed strength of the strong interactions in a unified model is a consequence of renormalization effects. The $SU(2)\times U(1)$ structure of the electromagnetic-weak interactions and the color SU(3) appropriate for the strong interactions are embedded in a single simple group. As a consequence, all interactions have the same bare coupling constant. Georgi, Quinn, and Weinberg then use a decoupling theorem[4] which allows the coupling constants g_1, g_2, g_3 for the groups U(1), SU(2), SU(3) to be independently renormalized. Application of this renormalization process to the unified model of Georgi and Glashow[5] revealed that the development of a large strong-interaction constant at low energies was linked to a severe reduction in the Weinberg angle from its bare value. For large values of the quark-gluon coupling constant ($g_3{}^2/4\pi \gtrsim 0.5$), $\sin^2\theta_W$ is reduced from the bare value of $\frac{3}{8}$ to an asymptotic value of $\frac{1}{6}$. Even for $g_3{}^2/4\pi = 0.1$, an estimate consistent with calculations for massless gluon theories,[6] the predicted value of $\sin^2\theta_W$ is only 0.21.[3]

In a previous work we considered the possibility of embedding Pati-Salam-type models in unified theories.[7] We found that such models lead to $\sin^2\theta_W = \frac{3}{8}$ in the bare theory for fractionally charged quarks. For the integral-charge model discussed by Pati and Salam,[8] only the valence portion of the electromagnetic current enters the current coupled to Z; the colored portion is not so coupled. Since the uncolored portion of the electromagnetic current is the same for fractional or integral quark charges, the structure of the Z is also the same. Thus, the bare value for $\sin^2\theta_W$ is $\frac{3}{8}$, independent of whether the quarks are fractionally or integrally charged.[9]

The Pati-Salam model with integrally charged quarks (and consequent breaking of exact color symmetry) is able to accommodate the nonobservation of color and isolated quarks. The model also explains the apparent consistency of electroproduction results with fractional quark charges.[10] However, the question of the strength of the strong interactions in the model is even more crucial than in models with fractional quark charges and exact color symmetry. Those estimates for $g_3{}^2/4\pi$ of order 0.1 have been developed only for the latter-type model[3,6] and may be too small for models without exact color symmetry.

In this paper we address the question of whether embedded Pati-Salam-type models with fractionally or integrally charged quarks can develop large quark-gluon couplings through renormalization and still retain physically acceptable values of $\sin^2\theta_W$. The mass scales and identity of normal and superheavy particles appropriate to such theories will also be discussed.

In the section that follows we review the Pati-Salam model for fractional and integral quark charge assignments.

In the third section we use and extend the Georgi-Quinn-Weinberg prescription to obtain relationships between $\sin^2\theta_W$ and quark-gluon coupling constant in unified theories containing the Pati-Salam model. The $SU(2)_L\times U(1)\times SU(3)$ subgroup of the Pati-Salam group $SU(2)_L\times SU(2)_R\times SU(4)$ is assumed to be preserved to the final stage of symmetry breaking. Both fractional- and integral-quark-charge models are shown to lead to $\sin^2\theta_W \to \frac{1}{6}$ as the quark-gluon coupling constant becomes large.

Empirical mass limits and models for CP violation suggest the possibility that gauge bosons coupling to the $SU(2)_R$ also present in the Pati-Salam model may have masses not much larger than those of the W and Z.[11] In the fourth section we consider the possibility that the $SU(2)_R$ bosons do not acquire mass until the final stage of symmetry breaking [the preserved group now being $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$]. We find that $\sin^2\theta_W$ now has an asymptotic value of $\frac{1}{4}$ as the quark-gluon coupling constant becomes large, and we discuss whether the additional gauge bosons can be sufficiently light to justify the extra symmetry.

In the fifth section we relate the range of $\sin^2\theta_W$ obtained in the previous two sections to the experimental range obtained from inclusive neutrino-nucleon scattering. The value of $\frac{1}{4}$ is found to be within the lower bound of this range provided the Weinberg-Salam constraint (depending on the Higgs structure of symmetry breaking) $m_Z{}^2 = m_W{}^2/\cos^2\theta_W$ is relaxed. The value of $\frac{1}{6}$ is not within this range regardless of the choice for m_Z.

II. REVIEW OF THE PATI-SALAM MODEL

The Pati-Salam model[8] contains two fermion multiplets ψ_L and ψ_R, which transform respectively as $(2+2, 1, \overline{4})$ and $(1, 2+2, \overline{4})$ under $SU(2)_L \times SU(2)_R \times SU(4)$.[12] Denoting flavor indices by i and color indices by α, we have

$$\psi^{L,R}_{i\alpha} = \tfrac{1}{2}(1 \pm \gamma_5) \begin{bmatrix} \mathcal{P}_r & \mathcal{P}_w & \mathcal{P}_b & \nu_e \\ \mathfrak{N}_r & \mathfrak{N}_w & \mathfrak{N}_b & e^- \\ \lambda_r & \lambda_w & \lambda_b & \mu^- \\ \mathcal{P}'_r & \mathcal{P}'_w & \mathcal{P}'_b & \nu_\mu \end{bmatrix}, \qquad (1)$$

where red, white, and blue denote the SU(3) triplet of color. In the fractional-quark-charge model $\mathcal{P}$ and $\mathcal{P}'$ have charge $\frac{2}{3}$, $\mathfrak{N}$ and λ have charge $-\frac{1}{3}$, and the photon has no SU(3) color components. In the integral-quark-charge model, the red column has the same charge assignments as the lepton column, the white and blue columns have one more unit of charge in each flavor, and the photon has SU(3) color components. The integral-charge model cannot, therefore, preserve electromagnetism and color as distinct exact symmetries.

The interaction of fermions with gauge bosons in the theory is given by ($\psi \equiv \psi^L + \psi^R$)

$$\mathcal{L}_I = i g_L \overline{\psi}^L_{i\alpha} \gamma_\mu (\vec{K}\cdot\vec{W}^\mu_L)_{ij} \psi^L_{j\alpha} + i g_R \overline{\psi}^R_{i\alpha}\gamma_\mu(\vec{K}\cdot\vec{W}^\mu_R)_{ij}\psi^R_{j\alpha} - if\overline{\psi}_{i\alpha}\gamma_\mu(L^A V^{A\mu})_{\beta\alpha}\psi_{i\beta}, \qquad (2)$$

where the gauge bosons associated with flavor are given by

$$\vec{K}\cdot\vec{W} = \tfrac{1}{2}\begin{bmatrix} N^1 & W^+ & & \\ W^- & -N^1 & & 0 \\ & 0 & -N^1 & W^+ \\ & & W^+ & N^1 \end{bmatrix}, \qquad (3)$$

and the gauge bosons associated with color and lepton number are given by

$$L^A V^A = \frac{1}{\sqrt{2}} \begin{bmatrix} V(8) + V^{15}\hat{I}/\sqrt{12} & & & \begin{matrix}\overline{X}_1 \\ \overline{X}_2 \\ \overline{X}_3\end{matrix} \\ X_1 & X_2 & X_3 & -\frac{1}{2}\sqrt{3}\,V^{15} \end{bmatrix}. \qquad (4)$$

The X's are leptoquark bosons that allow hadrons to decay directly into leptons. Limits on $K^0 \to e^-\mu^+$ (see Refs. 7 and 8) require these bosons to be superheavy relative to other gauge bosons in the theory. We shall assume that the symmetries coupling to the X bosons are badly broken and that at most the $SU(3)\times U(1)$ subgroup of SU(4) is preserved down to the final stage of symmetry breaking.[13] As a result, we will need to distinguish between the coupling to fermions of the color gluons $V^1 - V^8$ and the color-singlet neutral boson V^{15}. We call the former coupling constant f_s and the latter f_{15}; they will subsequently be identified with coupling constants g_3 and g_1, respectively, of appropriate (SU(3) and U(1) subgroups of a unified theory.[14]

In the fractional-quark-charge model discussed explicitly in Ref. 7, the five neutral gauge bosons of the theory after spontaneous breakdown are given as follows ($\omega \equiv g_L/g_R$, $\lambda \equiv (\frac{3}{2})^{1/2} f_{15}/g_R$):

$$A(\equiv \text{photon}) = e\left[\frac{N^1_L}{g_L} + \frac{N^1_R}{g_R} - \left(\frac{2}{3}\right)^{1/2}\frac{V^{15}}{f_{15}}\right], \quad V^3, \quad V^8, \quad m=0,$$

$$Z = \frac{\omega(1+\lambda^2)N^1_L - \lambda^2 N^1_R + \lambda V^{15}}{\{(1+\lambda^2)[\lambda^2 + \omega^2(1+\lambda^2)]\}^{1/2}}, \quad m_Z \simeq m_W, \qquad (5)$$

$$S = \frac{N^1_R + \lambda V^{15}}{(\lambda^2+1)^{1/2}}, \quad m_S{}^2 \gtrsim f_{15}{}^2 G_F{}^{-1}.$$

From the expression for the photon in (5) and an analysis of the currents coupling to Z,[15] we see that

$$e^{-2} = g_L{}^{-2} + g_R{}^{-2} + \tfrac{2}{3} f_{15}{}^{-2}, \qquad (6a)$$

$$\sin^2\theta_W = \frac{\lambda^2}{\lambda^2+\omega^2(1+\lambda^2)} = \frac{3f_{15}{}^2g_R{}^2}{2g_L{}^2g_R{}^2+3f_{15}{}^2(g_L{}^2+g_R{}^2)}. \qquad (6b)$$

In the integral-quark-charge model the Z and S bosons have the same structure as in (5) if we neglect the gluon masses.[16] Hence the lower bound on the mass of S in (5) and the expression for $\sin^2\theta_W$ in (6b) remain valid.[9] The photon and neutral gluons are given by

$$A = e\left[\frac{N_L^1}{g_L}+\frac{N_R^1}{g_R}-\left(\frac{2}{3}\right)^{1/2}\frac{V_{15}}{f_{15}}+\frac{V^3+V^8/\sqrt{3}}{f_s}\right], \quad m_A=0,$$

$$U^0 = -\tfrac{4}{3}f_s{}^{-2}\left[\frac{N_L^1}{g_L}+\frac{N_R^1}{g_R}-\left(\frac{2}{3}\right)^{1/2}\frac{V^{15}}{f_{15}}\right] + (e^{-2}-\tfrac{4}{3}f_s{}^{-2})\left(\frac{V^3+V^8/\sqrt{3}}{f_s}\right), \quad m_{U^0}\simeq 1\text{ GeV}, \qquad (7)$$

$$V^0=\tfrac{1}{2}(V^3-\sqrt{3}\,V^8), \quad m_{V^0}\simeq 1\text{ GeV}.$$

From (7) we see that for the integral-quark-charge model

$$e^{-2}=g_L{}^{-2}+g_R{}^{-2}+\tfrac{2}{3}f_{15}{}^{-2}+\tfrac{4}{3}f_s{}^{-2}. \qquad (8)$$

When the above models are embedded in a unified theory, we obtain the constraint $g_L=g_R=f_s=f_{15}$.[17] With this constraint, (6b) predicts a bare Weinberg angle given by $\sin^2\theta_W=\frac{3}{8}$. Moreover, (6a) and (8) imply that the bare quark-gluon coupling is given by $f_s{}^2=\frac{8}{3}e^2$ in the fractional-quark-charge model and by $f_s{}^2=4e^2$ in the integral-quark-charge model.

III. RENORMALIZATION OF COUPLING CONSTANTS OF SU(2) × U(1) × SU(3)

We assume the existence of a hierarchy of symmetry breaking in which all gauge bosons of the unified theory containing the Pati-Salam model acquire superheavy masses except for those bosons coupling to an appropriate choice of SU(2)×U(1)×SU(3). These bosons are specifically the three bosons $W_L^{\pm}, N_L^1$ coupling to $SU(2)_L$, the eight gluons V^1-V^8 of color SU(3), and the single neutral boson combination $(\lambda N_R^1 - V^{15})/(\lambda^2+1)^{1/2}$ necessary for the construction of both the photon in (5) and (7) and the Z in (5). There is no U(1) coupling-constant ambiguity here as $g_R=f_{15}$. Alternatively, two commuting U(1) groups associated with V^{15} and N_R^1 may be preserved if S is sufficiently light (the limit on the mass of S will be discussed in the next section).

We denote the coupling of fermions to $W_L^{\pm}$ and N_L^1 by g_2, to V^1–V^8 by g_3, and to the U(1) boson (or bosons) by g_1. Hence we identify the Pati-Salam coupling constants to be

$$f_s=g_3, \quad g_L=g_2, \quad g_R=f_{15}=g_1, \qquad (9)$$

where g_3, g_2, and g_1 equal the bare coupling constant g_b in the unified limit.

Equations (6), (8) and (9) allow us to follow the procedure of Georgi, Quinn, and Weinberg[3] and find the renormalized value of $\sin^2\theta_W$. The coupling constants g_1, g_2, and g_3 are functions of the momentum scale μ:

$$g_i{}^{-2}(\mu)=g_b{}^{-2}+2b_i\ln\left(\frac{M}{\mu}\right), \qquad (10)$$

where M characterizes the superheavy mass scale. The decoupling theorem of Appelquist and Carazzone[4] implies that the b_i receive no contributions from superheavy multiplets of the unified theory. Since equal contributions to the b_i are obtained from multiplets containing no superheavy particles,[3] differential contributions to the b_i are acquired only from those multiplets of the unified group containing both ordinary and superheavy particles. If the gauge multiplet of the unified theory is the only such multiplet, we have[3,18]

$$b_2-b_1=2[-\tfrac{11}{3}(4\pi)^{-2}],$$
$$b_3-b_1=3[-\tfrac{11}{3}(4\pi)^{-2}]. \qquad (11)$$

Upon substitution of (9) into (6a) and (6b), we find in the fractional-charge model that

$$\sin^2\theta_W=\frac{g_1{}^2(\mu)}{g_1{}^2(\mu)+\frac{5}{3}g_2{}^2(\mu)}, \qquad (12a)$$

$$\frac{1}{e^2}=\frac{1}{g_2{}^2(\mu)}+\frac{5}{3g_1{}^2(\mu)}. \qquad (12b)$$

These relations are the same as those obtained for the model of Georgi and Glashow,[5] and, after application of (10) and (11), they lead to the following relations[3]:

$$\sin^2\theta_W(\mu)=\frac{1}{6}+\frac{5}{9u}, \qquad (13a)$$

$$\ln\left(\frac{M}{\mu}\right)=\frac{\pi}{11\alpha}\left(1-\frac{8}{3u}\right), \qquad (13b)$$

where $u\equiv g_3{}^2(\mu)/4\pi\alpha$ at physically appropriate values of μ. The value of μ appropriate for the observed values of electromagnetic and weak gauge coupling is of order 10 GeV.[3]

Equations (13a) and (13b) imply that as u increases from its bare value of $\frac{8}{3}$, $\sin^2\theta_W$ is seen to decrease dramatically even for low values of u, as has been remarked earlier.

To analyze the integral-charge model, we substitute (9) into (6b) and (8). Equation (12b) is replaced by

$$\frac{1}{e^2}=\frac{1}{g_2^{\,2}(\mu)}+\frac{5}{3g_1^{\,2}(\mu)}+\frac{4}{3g_3^{\,2}(\mu)}\,. \tag{14}$$

Utilizing (14), (12a), and (10), we obtain relations independent of the bare coupling constant:

$$\frac{1}{g_2^{\,2}(\mu)}+\frac{5}{3g_1^{\,2}(\mu)}-\frac{8}{3g_3^{\,2}(\mu)}=\frac{1}{e^2}\left(1-\frac{4}{u}\right)=2[(b_2-b_1)-\tfrac{8}{3}(b_3-b_1)]\ln(M/\mu), \tag{15}$$

$$\frac{1}{g_2^{\,2}(\mu)}-\frac{1}{g_1^{\,2}(\mu)}=\frac{1}{5e^2}\left(1-\frac{4}{3u}\right)(8\sin^2\theta_W-3)=2(b_2-b_1)\ln(M/\mu). \tag{16}$$

Substitution of (11) into (15) and (16) leads to the following expressions:

$$\sin^2\theta_W=\frac{u+2}{6u-8}, \tag{17a}$$

$$\ln\left(\frac{M}{\mu}\right)=\frac{\pi}{11\alpha}\left(1-\frac{4}{u}\right). \tag{17b}$$

Equations (17a) and (17b) show that as u increases above its bare value of 4, $\sin^2\theta_W$ decreases once again to an asymptotic value of $\frac{1}{6}$. This dependence of $\sin^2\theta_W$ with u is displayed in Fig. 1; for $u\gtrsim 7$ the value of $\sin^2\theta_W$ for integral quark charges is $\lesssim 5\%$ larger than the value for fractional quark charges.

It should be noted that the decision not to include the leptoquark bosons X_i in the final state of symmetry breaking reduces the upper bound for $\sin^2\theta_W$ from its bare value. In the Pati-Salam model suppression of $K^0\to e^-\mu^+$ leads us to expect $m_X^{\,2}\gtrsim f^2G_F^{\,-1}\alpha^{-2}$.[8] Assuming that the leptoquark coupling is large ($f^2/4\pi\approx 1$), then $m_X\gtrsim 3\times 10^4$ GeV.[19] Treating 3×10^4 GeV as a lower bound for the superheavy mass scale M and choosing $\mu\simeq 10$ GeV, we obtain maximum values for $\sin^2\theta_W$ of 0.34 for the fractional-quark-charge model, and of 0.32 for the integral-quark-charge model. Of course, values of M for which u is in a realistic range ($u>10$) are several orders of magnitude higher than the above estimate, as is seen from the substitution of such values for u into (13b) and (17b).

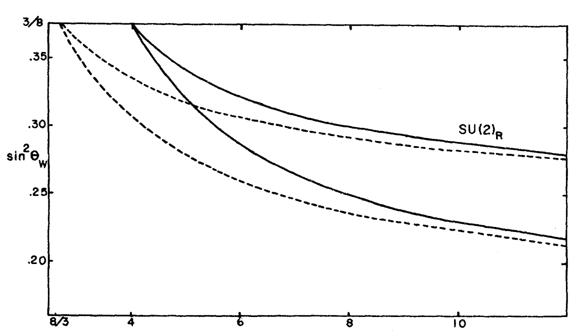

FIG. 1. $\sin^2\theta_W$ is plotted against the quark-gluon coupling constant parameter $u=g_3^{\,2}/4\pi\alpha$. The dashed curves are for fractional quark charges, and the solid curves are for integral quark charges. The pair of curves obtained assuming the preservation of an additional $SU(2)_R$ [$SU(2)_L\times SU(2)_R\times U(1)\times SU(3)$] are so labeled. The other pair of curves are obtained for the preservation of $SU(2)\times U(1)\times SU(3)$.

IV. RENORMALIZATION OF COUPLING CONSTANTS OF $SU(2)_L\times SU(2)_R\times U(1)\times SU(3)$

In the previous section we assumed the $SU(2)_R$ present in the Pati-Salam model was coupled to superheavy gauge bosons, consistent with the symmetry-breaking procedure of Refs. 2 and 7. Such as assumption is not, however, necessary to suppress anomalous reactions; the empirical lower bounds on the masses of the $SU(2)_R$ gauge bosons are much lower than those for the leptoquark bosons.[8] Moreover, models for CP violation suggest that those bosons may not be much more massive than $W_L^{\pm}$ or Z.[11,20]

In this section we shall assume that the $SU(2)_R$ coupled to $W_R^{\pm}$ and N_R^{1} is preserved down to the final stage of symmetry breaking. Such preservation is possible only if the S and $W_R^{\pm}$ bosons have masses on the same scale as those of the $W_L^{\pm}$ bosons, a requirement we will discuss later in this section.

Preservation of this additional $SU(2)_R$ leads us to identify the Pati-Salam coupling constants with those of the unbroken groups as follows:

$$f_s=g_3,\quad g_L=g_R=g_2,\quad f_{15}=g_1\,. \tag{18}$$

The unbroken U(1) generator is no longer the rotated generator of the previous section. It couples only to V^{15}.

The preservation of this U(1) symmetry is still necessary to construct the photon and the Z, despite the presence of a distinct neutral boson coupled to the neutral generator of $SU(2)_R$. Rotation of this generator to couple to the boson $(\lambda N_R^{1}-V^{15})/(\lambda^2+1)^{1/2}$ cannot be done without violating the commutation relations of $SU(2)_R$, as $0=[L^{15},K^{\pm}]\neq[K^{N^1},K^{\pm}]$.

For the fractional-charge model, we substitute (18) into (6a) and (6b) and find that

$$\sin^2\theta_W=\frac{3g_1^{\,2}(\mu)}{6g_1^{\,2}(\mu)+2g_2^{\,2}(\mu)}, \tag{19a}$$

$$\frac{1}{e^2}=\frac{2}{g_2^{\,2}(\mu)}+\frac{2}{3g_1^{\,2}(\mu)}, \tag{19b}$$

where $g_i^2(\mu)$ is given by (10) and (11). Following the procedure of Georgi, Quinn, and Weinberg, we obtain

$$\sin^2\theta_W = \frac{1}{4} + \frac{1}{3u}, \tag{20a}$$

$$\ln\left(\frac{M}{\mu}\right) = \frac{3\pi}{22\alpha}\left(1 - \frac{8}{3u}\right). \tag{20b}$$

As u increases from its bare value of $\frac{8}{3}$, $\sin^2\theta_W$ now decreases to an asymptotic value of $\frac{1}{4}$.

This asymptotic value also occurs in the integral-quark-charge model. Substitution of (18) into (8) allows us to replace (19b) with

$$\frac{1}{e^2} = \frac{2}{g_2^2(\mu)} + \frac{2}{3g_1^2(\mu)} + \frac{4}{3g_3^2(\mu)}. \tag{21}$$

Utilizing (21), (19a), (11), and (10), we extend the procedure of the previous section for the integral-charge model to obtain

$$\sin^2\theta_W = \frac{3u}{4(3u-4)}, \tag{22a}$$

$$\ln\left(\frac{M}{\mu}\right) = \frac{3\pi}{22\alpha}\left(1 - \frac{4}{u}\right). \tag{22b}$$

The behavior of $\sin^2\theta_W$ with u given in (20a) and (22a) is shown graphically in Fig. 1. Once again the integral-charge value of $\sin^2\theta_W$ is seen to be larger than the fractional-charge value of $\sin^2\theta_W$ for a given value of u, but the difference becomes very small as u increases. The question of whether the new lower bound of $\frac{1}{4}$ for $\sin^2\theta_W$ is consistent with experiment is discussed in the next section. The leptoquark limit of $M >> 3\times10^4$ GeV reduces the upper bound on $\sin^2\theta_W$ from $\frac{3}{8}$ to 0.36 for fractional quark charges, and to 0.35 for integral quark charges.

Central to the assumption that $SU(2)_R$ is preserved down to the final stage of symmetry breaking is the claim that $W_R^\pm$ and S have masses on the same order as $W_L^\pm$ and Z. The observed left-handedness of the weak-interaction charged current imposes the condition that $M_{W_R} \gtrsim 3m_{W_L}$ if g_L and g_R are equal.[8] The assumed nonobservation of S in neutral-current phenomena implies that $m_S^2 > f_{15}^2 G_F^{-1}$ as stated previously in (5). Had we not distinguished between f_{15} and f_s, this inequality would imply a mass for S of order 1000 GeV.[8] Identifying f_{15} with g_1 and utilizing (19a) and (19b), we find that $f_{15}^2 = 2e^2/[3(1-2\sin^2\theta_W)]$ for fractional quark charges. Since $\sin^2\theta_W$ ranges between $\frac{3}{8}$ and $\frac{1}{4}$, we find that

$$\tfrac{4}{3}e^2 < f_{15}^2 < \tfrac{8}{3}e^2. \tag{23}$$

A similar analysis appropriate for integral quark charges yields the range

$$\tfrac{4}{3}e^2 < f_{15}^2 < 4e^2. \tag{24}$$

The values for f_{15} given by (23) and (24) are sufficiently small for m_S to be $\approx 3m_{W_L}$.

Assuming m_S and m_{W_R} are $\approx 3m_{W_L}$, the question still remains as to whether (10) is valid for values of $\mu \simeq 10$ GeV. Strictly speaking, the range of applicability of the renormalization-group equation leading to (10) does not extend to values of μ below the ordinary gauge boson masses, but the insensitivity of (10) to the choice for μ makes such extrapolation reasonable.[3] To illustrate, we use (20a) and (20b) to calculate the difference between $\sin^2\theta_W$ obtained with $\mu = 200$ GeV, the assumed mass of the $SU(2)_R$ bosons, and $\mu = 10$ GeV:

$$\sin^2\theta_W(200) - \sin^2\theta_W(10) = (11\alpha/12\pi)\ln(20) \approx 0.006. \tag{25}$$

For the integral-quark-charge model, (22a) and (22b) imply that

$$\sin\theta_W(200) - \sin^2\theta_W(10) \underset{u\gg4}{\rightarrow} 0.004. \tag{26}$$

Values of u estimated at $\mu = 200$ GeV differ significantly from those at $\mu = 10$ GeV only if both values are in the asymptotic region.

We conclude from these arguments that an additional $SU(2)_R$ may indeed be present at the final stage of symmetry breaking; the preserved group is now $SU(2)_L\times SU(2)_R\times U(1)\times SU(3)$. This group contains all gauge bosons in the Pati-Salam model except the leptoquark bosons X_i. If this is the case, we find that $\sin^2\theta_W$ approaches $\frac{1}{4}$ as the quark-gluon coupling constant becomes large, independent of whether the quarks are fractionally or integrally charged.

V. THE EMPIRICAL RANGE OF $\sin^2\theta_W$

In this section we investigate whether values for $\sin^2\theta_W$ obtained in the previous two sections for the large quark-gluon coupling-constant limit are included in the empirical range of $\sin^2\theta_W$. Since the mass of the Z depends on the method of spontaneous symmetry breaking rather than on the current structure, we will allow this mass to be arbitrary.

The expression for Z given in (5) implies a neutral current whose structure is given by[7]

$$J_Z^\lambda = \bar\nu_e\gamma^\lambda\frac{1+\gamma_5}{2}\nu_e + \bar e\gamma^\lambda\left[\left(\frac{-1+4\sin^2\theta_W}{2}\right) - \frac{\gamma_5}{2}\right]e + (e\to\mu)$$
$$+ \sum_{\text{colors}}\left\{\bar{\mathcal{P}}\gamma^\lambda\left[\left(\frac{1-8\sin^2\theta_W/3}{2}\right)+\frac{\gamma_5}{2}\right]\mathcal{P} - \bar{\mathcal{N}}\gamma^\lambda\left[\left(\frac{1-4\sin^2\theta_W/3}{2}\right)+\frac{\gamma_5}{2}\right]\mathcal{N} + (\mathcal{P}\to\mathcal{P}';\mathcal{N}\to\lambda)\right\}. \tag{27}$$

The above current applies for both fractional and integral quark charges, as has been noted earlier. We define $N \equiv m_Z^2 \cos^2\theta_W / m_W^2$ and note that $N=1$ represents the "orthodox" Weinberg-Salam model constraint, a constraint also obtained for the Pati-Salam model in Ref. 7.

Following the conventions of Wolfenstein,[21] we denote inclusive-cross-section ratios by $R \equiv \sigma(\nu N \to \nu X)/\sigma(\nu N \to \mu^- X)$, $\overline{R} \equiv \sigma(\overline{\nu} N \to \overline{\nu} X)/\sigma(\overline{\nu} N \to \mu^+ X)$, and $r \equiv \sigma(\overline{\nu} N \to \mu^+ X)/\sigma(\nu N \to \mu^- X)$. Using (27) we identify the Wolfenstein neutral-current parameters:

$$\begin{aligned} g_V &= (1 - 2\sin^2\theta_W)/N, \\ g_A &= 1/N, \\ g'_V &= -2\sin^2\theta_W/3N, \\ g'_A &= 0. \end{aligned} \tag{28}$$

Assuming the equality of contributions arising from vector and axial-vector isoscalar and isovector currents,[22] we find that

$$\begin{aligned} R &= \tfrac{1}{8}\{(g_A+g_V)^2 + (g'_A+g'_V)^2 \\ &\quad + r[(g_A-g_V)^2 + (g'_A-g'_V)^2]\} \\ &= [\tfrac{1}{2} - \sin^2\theta_W + \tfrac{5}{9}(1+r)\sin^4\theta_W]/N^2, \end{aligned} \tag{29a}$$

$$\begin{aligned} \overline{R} &= \tfrac{1}{8}\{(g_A+g_V)^2 + (g'_A+g'_V)^2 \\ &\quad + r^{-1}[(g_A-g_V)^2 + (g'_A-g'_V)^2]\} \\ &= (\tfrac{1}{2} - \sin^2\theta_W + \tfrac{5}{9}(1+r^{-1})\sin^4\theta_W)/N^2. \end{aligned} \tag{29b}$$

For $r=\frac{1}{3}$ and $N=1$, (29a) and (29b) become identical to expressions obtained in a parton-model calculation by Sehgal.[23] Defining $\overline{R}/R \equiv \rho$, we find that

$$\sin^2\theta_W = \frac{(\rho-1) - (\rho-1)^{1/2}\{\rho - 1 - \frac{10}{9}[\rho(1+r) - (1+r^{-1})]\}^{1/2}}{\frac{10}{9}[\rho(1+r) - (1+r^{-1})]}. \tag{30}$$

This expression is independent of N.

The neutral-current data from Gargamelle yields $R=0.217\pm0.026$, $\overline{R}=0.43\pm0.12$,[21] implying a range for ρ of $1.28 \lesssim \rho \lesssim 2.88$. We see that if $r=0.26$, then $0.212<\sin^2\theta_W<0.412$; if $r=\frac{1}{3}$, then $0.238<\sin^2\theta_W<0.488$. The value $\sin^2\theta_W=\frac{1}{4}$, which was obtained in the large quark-gluon coupling-constant limit for preservation of $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$, is within these ranges for $\sin^2\theta_W$. The value $\sin^2\theta_W=\frac{1}{6}$, obtained assuming the preservation of $SU(2)_L \times U(1) \times SU(3)$, is not within these ranges.

The question remains as to how much deviation from $N=1$ is necessary to accommodate $\sin^2\theta_W=\frac{1}{4}$. It is straightforward to obtain upper and lower limits on values of $\sin^2\theta_W$ for a given value of N by insisting that R and $\overline{R}$ individually remain within their empirical ranges. Upper and lower bounds of $\sin^2\theta_W$ so obtained are displayed in Fig. 2 for $r=0.26$ and $r=0.33$. The range of $\sin^2\theta_W$ able to accommodate the orthodox value of $N=1$ is seen from the figure to be $0.33<\sin^2\theta_W<0.41$ for $r=0.26$ and $0.35<\sin^2\theta_W<0.48$ for $r=0.33$. We see, however, that relaxing the requirement of $N=1$ by a factor of 10% is sufficient to allow values of $\sin^2\theta_W$ as low as 0.25 for either value of r.

VI. CONCLUSIONS

We have shown that if $SU(2) \times U(1) \times SU(3)$ is preserved down to the final stage of symmetry breaking in a unified model containing the Pati-Salam model, then $\sin^2\theta_W$ decreases from $\frac{3}{8}$ to $\frac{1}{6}$ as the quark-gluon coupling becomes large. This result is independent of whether quarks are fractionally or integrally charged. For a given value of the quark-gluon coupling constant, the integral-charge model yields a larger value of $\sin^2\theta_W$ then does the fractional-charge model, but this difference de-

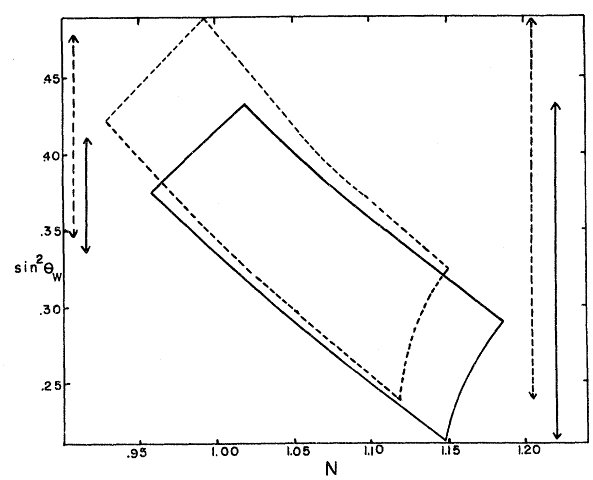

FIG. 2. Upper and lower bounds for $\sin^2\theta_W$ are plotted against $N = m_Z^2\cos^2\theta_W/m_W^2$. Values for these bounds are obtained by requiring R and $\overline{R}$ to be within their repective experimental ranges as given by the 1974 Gargamelle data. The solid closed curve is obtained for the experimental value $r=0.26$; the dashed closed curve is obtained for the quark-model value $r=\frac{1}{3}$. The arrows on the left side of the figure project the range of $\sin^2\theta_W$ if $N=1$. The arrows on the right side of the figure show the range of $\sin^2\theta_W$ for arbitrary N. Physically allowed values of ($\sin^2\theta_W$, N) are inside the closed curves.

creases as the quark-gluon coupling constant increases and is not significant for large values of this constant. The asymptotic value of $\frac{1}{6}$ cannot be brought within the experimental range for $\sin^2\theta_W$ by allowing the mass of the Z to be arbitrary.

If $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$ is preserved down to the final stage of symmetry breaking, $\sin^2\theta_W$ decreases from $\frac{3}{8}$ to $\frac{1}{4}$ as the quark-gluon coupling becomes large. This result is also independent of the choice for quark charges. The value $\frac{1}{4}$ can be brought within the experimental range for $\sin^2\theta_W$ by relaxing the Weinberg-Salam constraint $m_Z{}^2 = m_W{}^2/\cos^2\theta_W$ by a factor of 10%.

Values of $\sin^2\theta_W$ obtained for large values of the quark-gluon coupling must be taken seriously for the integral-quark-charge model, since this model does not utilize infrared divergences to make the strong interactions strong. Although the fractional-quark-charge model may not require large values of the quark-gluon coupling, the larger values for $\sin^2\theta_W$ obtained by preserving the $SU(2)_R$ in the Pati-Salam model make that preservation an attractive possibility.

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[9]Since the current coupled to Z is the same in the fractional-and integral-quark-charge model, we define the Weinberg angle in terms of that current [Eq. (27)]. This allows unambiguous relations between $\sin^2\theta_W$ and neutral-current data despite the incomplete correspondence of the integral-charge Pati-Salam model to the Weinberg-Salam model with integral quark charges.

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[12]Mirror fermion representations may also be present to cancel anomalies. Such fermions provide alternative explanations for heavy-lepton and ψ-particle phenomena, as discussed in J. C. Pati and A. Salam, Phys. Lett. 58B, 333 (1975).

[13]If the full SU(4) is preserved, the X's must be light, and the quark-gluon and fermion-X coupling constants are the same. This would remove suppression of anomalous interactions mediated by X, as the quark-gluon coupling constant becomes large at lower energies.

[14]The idea that different gauge bosons in the SU(4) multiplet may couple to fermions with different strengths upon renormalization is proposed in note 38, of J. C. Pati and A. Salam, Phys. Rev. D 11, 1137 (1975).

[15]This analysis is performed in Ref. 7. Eventual embedment in a unified group compels a choice of $\lambda = (\frac{3}{2})^{1/2}$ if g_L, g_R, f_{15}, and f_s all go to a single bare value g_b upon embedment [See Eq. (37) of Ref. 7]. The expression for $\sin^2\theta_W$ in (6b) reflects this choice.

[16]These results are obtained largely in Ref. 8. We have not, however, kept the assumption that $f \gg g$, especially since the relvant coupling is in fact f_{15} rather than f_s. There is no reason for f_{15} to become large, and it does not in fact do so, as in shown in Sec. IV.

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[19]The renormalized value of the leptoquark coupling may not have to be large, as is pointed out in Ref. 14. Even so we require $m_X \gg m_W$ to have a hierarchy of symmetry breaking.

[20]Models for CP violating utilizing $SU(2)_L \times SU(2)_R$ symmetry are discussed by R. N. Mohapatra, in *Particles and Fields–1974*, proceedings of the 1974 Williamsburg meeting of the Division of Particles and Fields of the American Physical Society, edited by C. Carlson (A. I. P., New York, 1975), p. 127. One such model (Ref. 11) requires relatively light right-handed bosons such that $(m_{W_R}/m_{W_L})^2 \lesssim 10^3$. An alternative model has also been developed, however, in which the gauge bosons coupled to $SU(2)_R$ are superheavy [R. N. Mohapatra, J. C. Pati, and L. Wolfenstein, Phys. Rev. D 11, 3319 (1975)].

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