Hadronic decays and soft-pion theorems in the charmonium model*

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Using the charmonium model for ψ and ψ', we show that the partial widths for decays into any "normal" hadronic final state n (i.e., any state including no particles which contain charmed quarks) must approximately satisfy $\Gamma(\psi'\to n)/\Gamma(\psi\to n) \approx \Gamma(\psi'\to e^+e^-)/\Gamma(\psi\to e^+e^-)$. So far this relation has been tested for $n = p\bar{p}$, $2\pi^+2\pi^-\pi^0$, and $K^+K^-\pi^+\pi^-$. We also derive the soft-pion theorem $\Gamma(\psi\to n)/\Gamma(\psi\to e^+e^-) \approx \sigma(e^+e^-\to n)/\sigma(e^+e^-\to\mu^+\mu^-)$, for any normal state n including a soft pion but no baryons. A number of similar predictions for other configurations of the charmed-quark–charmed-antiquark system are considered, and a related theorem is obtained which helps explain the pion momentum distribution observed in the decay $\psi'\to\psi\pi^+\pi^-$.

I. INTRODUCTION

An interesting characteristic of the decays of $\psi(3.1)$ and $\psi'(3.7)$ is the approximate validity of

$$\frac{\Gamma(\psi'\to n)}{\Gamma(\psi\to n)} \approx \frac{\Gamma(\psi'\to e^+e^-)}{\Gamma(\psi\to e^+e^-)} \tag{1.1}$$

for each "normal" hadronic decay channel n (i.e., each decay channel which includes no particles containing charmed quarks). So far, this relation has been tested[1] for $n=p\bar{p}$, $2\pi^+2\pi^-\pi^0$, and $K^+K^-\pi^+\pi^-$. As a phenomenological explanation for Eq. (1.1), it has been suggested[2] by one of the present authors that the effective Lagrangians for normal hadronic decays of ψ and ψ' have the form

$$\begin{aligned}\mathcal{L}_{\psi}(x) &= \psi^{\mu}(x)[f j_{\mu}(x) + gJ_{\mu}(x)],\\ \mathcal{L}_{\psi'}(x) &= \psi'^{\mu}(x)[f' j_{\mu}(x) + g'J_{\mu}(x)],\end{aligned} \tag{1.2}$$

with the restriction

$$\frac{f'}{f} = \frac{g'}{g}, \tag{1.3}$$

where $\psi^{\mu}(x)$ and $\psi'^{\mu}(x)$ are field operators for ψ and ψ', respectively, $j_{\mu}(x)$ is the electromagnetic current of normal hadrons, and $J_{\mu}(x)$, which may or may not be a local operator, is some additional unknown effective current of nonelectromagnetic origin. Equation (1.1) follows immediately from (1.2) and (1.3) if the relatively small mass difference between ψ and ψ' is neglected. It is perhaps useful to mention that the presence of a nonelectromagnetic term in (1.2) is necessary[3] to fit the observed total widths of ψ and ψ', and interference effects between $j_{\mu}(x)$ and $J_{\mu}(x)$ are, in general, not negligible.[2,4]

In the present article we will give a derivation of (1.2) and (1.3) using the charmonium model[5,6] of ψ and ψ'. In other words, we will assume that the strong interactions are generated by the exchange of SU(3)′-color gauge gluons and that ψ and ψ' are, respectively, the first and second radial excitations of the $^3S_1$ $c\bar{c}$ (charmed-quark–charmed-antiquark) system. We will also derive equations similar to (1.2) and (1.3) for higher radial excitations of the $^3S_1$ $c\bar{c}$ system and for other S, P, and D states. Then we will consider a set of predictions for decays including soft pions which follow from the result that the nonelectromagnetic currents, such as $J_{\mu}(x)$, which appear in each decay Lagrangian are effectively singlets under ordinary chiral $SU(3)_L\times SU(3)_R$. Last of all, we will briefly examine a soft-pion theorem which helps explain the pion momentum distribution of the decay $\psi'\to\psi\pi^+\pi^-$.

The arguments by which we will obtain expressions such as (1.1)–(1.3) can be adapted to heavy-lepton models[7] of the ψ spectrum and related states. However, the predictions using $SU(3)_L \times SU(3)_R$ properties of decay Lagrangians do not all carry over, and some of these might conceivably be used to distinguish experimentally between heavy-lepton models and charmonium.

II. NORMAL-HADRONIC-DECAY LAGRANGIANS

Consider first the normal hadronic decays of ψ and ψ' mediated by a virtual photon as shown in Fig. 1. These processes give rise to the electromagnetic terms in (1.2); if $\psi(r)$ and $\psi'(r)$ are nonrelativistic radial Schrödinger wave functions for ψ and ψ', respectively, we obtain immediately

$$\frac{f'}{f} = \frac{\psi'(0)}{\psi(0)}, \tag{2.1}$$

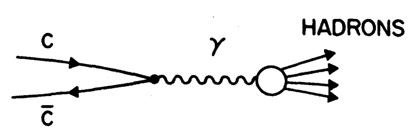

FIG. 1. Decay of a $c\bar{c}$ state into normal hadrons through a virtual photon.

neglecting, once again, the mass difference between ψ and ψ'.

In the charmonium model the only additional process which will give significant contributions to normal decays is the diagram mediated by three gluons shown in Fig. 2. A minimum of three gluons is required to construct a color singlet with odd-charge conjugation and, since the theory is asymptotically free, decays passing through more than three gluons, or with additional interactions among the gluons, will be suppressed by the (presumably) small value of the running gauge coupling constant evaluated at the mass of ψ or ψ'. Now if we call the three points at which these gluons are emitted from the charmed-quark line x, y, and z, it follows that since the charmed-quark mass is expected to be quite large (≈ 2 GeV) the main contribution to the process in Fig. 2 will come from spacings between x, y, and z which are quite small ($\approx \frac{1}{2}\ \mathrm{GeV}^{-1}$). However, the overall range of the ψ or ψ' wave function has no particular reason to be small and, for example, might be expected to be of the same order as the ranges exhibited by the electromagnetic form factors of normal hadrons ($\gtrsim 1/0.7\ \mathrm{GeV}^{-1}$). Thus to a fairly good approximation the wave function of ψ or ψ' should enter the matrix element corresponding to Fig. 2 only as an overall multiplicative factor of $\psi(0)$ or $\psi'(0)$, respectively. Neglecting the mass difference between ψ and ψ', Fig. 2 will yield the nonelectromagnetic terms in the effective Lagrangians of (1.2) with g and g' related by

$$\frac{g'}{g} \approx \frac{\psi'(0)}{\psi(0)} \tag{2.2}$$

and $J_\mu(x)$ given by an expression of the form

$$J_\mu(x) = \int d^4y_1\, d^4y_2\, d^4y_3\, K_\mu^{\lambda\nu\rho abc}(x, y_1, y_2, y_3) \times T[J_{\lambda a}(y_1) J_{\nu b}(y_2) J_{\rho c}(y_3)]\,. \tag{2.3}$$

The current $J_{\nu a}(y)$ in this expression is the SU(3)$'$-color-octet current of type a, and $K_\mu^{\lambda\nu\rho abc}(x, y_1, y_2, y_3)$ is a c-number function gotten from the three gluon and two charmed-quark propagators in Fig. 2. Equations (2.1) and (2.2) combine to yield (1.3).

The arguments we have just given can be extended immediately to any further radial excitations of the ${}^3S_1$ $c\bar{c}$ system if, as before, the masses of these states are not too far above the mass of the ψ. Similar results can also be gotten for the $J^{PC} = 1^{--}$ $c\bar{c}$ states in ${}^3D_1$ configurations if one of two different alternative assumptions is introduced. The first possibility is that the tensor force term generated by vector-gluon exchange mixes a significant ${}^3S_1$ leakage term into all ${}^3D_1$ states. Then since the processes in Figs. 1 and 2 should be strongly suppressed for the pure ${}^3D_1$ component, which vanishes at the origin of relative position space, we might expect the dominant contribution to come from the ${}^3S_1$ leakage wave function. For any such ${}^3D_1$-${}^3S_1$ mixture, ψ'', we will obtain an effective Lagrangian of the same form as those in (1.2) with electromagnetic and nonelectromagnetic coefficients f'' and g'', respectively, related to the coefficients for ψ by

$$\frac{f''}{f} \approx \frac{g''}{g} \approx \frac{\psi''(0)}{\psi(0)}\,, \tag{2.4}$$

where $\psi''(r)$ is the radial wave function of the ${}^3S_1$ component of the ${}^3D_1$-${}^3S_1$ mixture. Equation (1.1) then follows with ψ' replaced by ψ''.

Alternatively, it might happen that ${}^3D_1$-${}^3S_1$ mixing is negligible. Then for the lowest ${}^3D_1$ state, ψ_D, and any of its radial excitation, say ψ_D', we will have effective decay Lagrangians of nearly the same form as (1.2) with, however, the nonelectromagnetic current $J_\mu(x)$ replaced by a distinct nonelectromagnetic current $J_\mu^D(x)$. The coefficients in the Lagrangian for the lowest ${}^3D_1$ state ψ_D and a radial excitation ψ_D' will be related by

$$\frac{f_D'}{f_D} \approx \frac{g_D'}{g_D} \approx \left[\frac{d^2}{dr^2}\psi_D'(r) \Big/ \frac{d^2}{dr^2}\psi_D(r)\right]_{r=0}, \tag{2.5}$$

where f_D' and f_D are the electromagnetic coupling constants of ψ_D' and ψ_D, respectively, g_D' and g_D are the nonelectromagnetic coupling constants of ψ_D' and ψ_D, and $\psi_D'(r)$ and $\psi_D(r)$ are the radial Schrödinger wave functions of ψ_D' and ψ_D, respectively. Equation (2.5) cannot be used to connect the normal hadronic decays of ${}^3D_1$ states to those of ${}^3S_1$ states since $J_\mu(x) \neq J_\mu^D(x)$, but it does follow from (2.5) that we still have a relation of the form

$$\frac{\Gamma(\psi_D' \to n)}{\Gamma(\psi_D \to n)} \approx \frac{\Gamma(\psi_D' \to e^+e^-)}{\Gamma(\psi_D \to e^+e^-)} \tag{2.6}$$

for any normal hadronic channel n.

For ${}^1S_0$ $c\bar{c}$ states, electromagnetic decays leading to normal hadronic states must occur through two virtual photons and should be negligible. However, an observable branching ratio might be expected for decays to a pair of real photons. The process replacing Fig. 2 is a similar diagram with two gluons instead of three. For the lowest ${}^1S_0$ state, η_c, which may have been found at a mass of 2.8 GeV,[8] and its first radial excitation, η_c', which

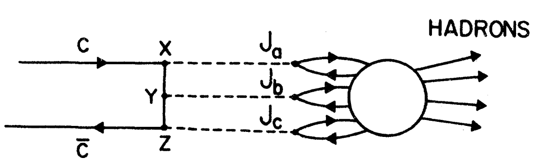

FIG. 2. Decay of a $c\bar{c}$ state into normal hadrons through three virtual gluons.

could be the state observed at 3.51 GeV,[9] we have the effective decay Lagrangians

$$\mathcal{L}_{\eta_c} = f\eta_c(x)F^{\mu\nu}(x)\,{}^*F_{\mu\nu}(x) + g\eta_c(x)J(x),$$
$$\mathcal{L}_{\eta_c'} = f'\eta_c'(x)F^{\mu\nu}(x)\,{}^*F_{\mu\nu}(x) + g'\eta_c'(x)J(x), \tag{2.7}$$

where $F^{\mu\nu}(x)$ and ${}^*F^{\mu\nu}(x)$ are, respectively, the electromagnetic field tensor and its dual, and $J(x)$ is an effective pseudoscalar current constructed from a pair of color-octet currents. As before we obtain the relation

$$\frac{f'}{f} \approx \frac{g'}{g} \tag{2.8}$$

and from (2.8) the prediction

$$\frac{\Gamma(\eta_c' \to \gamma\gamma)}{\Gamma(\eta_c \to \gamma\gamma)} \approx \frac{\Gamma(\eta_c' \to n)}{\Gamma(\eta_c \to n)} \tag{2.9}$$

for any normal hadronic final state n.

For any ${}^3P_0$ or ${}^3P_2$ $c\bar{c}$ configuration (the lowest of which have presumably been found at 3.41 and 3.53 GeV,[9] respectively) normal hadronic decays proceed again through two gluons while electromagnetic decays to a pair of photons might be observable. A pair of radial excitations of the same configuration will decay through effective Lagrangians similar to (2.7), fulfilling conditions similar to (2.8), and giving predictions of the same form as (2.9). The ${}^1P_1$ and ${}^3P_1$ $c\bar{c}$ states cannot decay to a pair of photons but require, respectively, three and four instead, which will almost certainly be unobservable. For normal hadronic decays a transition through three gluons is required. For one of these states χ and a radial excitation χ' we obtain Lagrangians similar to the nonelectromagnetic terms in (1.2) yielding the prediction

$$\frac{\Gamma(\chi' \to n)}{\Gamma(\chi \to n)} \approx \frac{\Gamma(\chi' \to m)}{\Gamma(\chi \to m)} \approx R$$

for any normal hadronic states m and n, where R is a constant independent of m and n.

Although the effective nonelectromagnetic currents we have considered so far are, in general, complicated nonlocal operators, it is conceivable that for ${}^3S_1$, ${}^3D_1$, and ${}^3P_1$ states things may actually be somewhat simpler. Suppose that the most important nonelectromagnetic decay contributions occur through three gluons coupling to a single quark line as shown in Fig. 3. Then since the running gauge coupling constant is small in the mass region we are considering, to a first approximation quarks may be treated as zero-mass free particles, and a simple kinematic argument shows that the effective current for ($J^{PC}=1^{--}$) ${}^3S_1$ or ${}^3D_1$ states will be nearly given by the local operator

$$J_\mu(x) = \bar{q}(x)\gamma_\mu q(x), \tag{2.10}$$

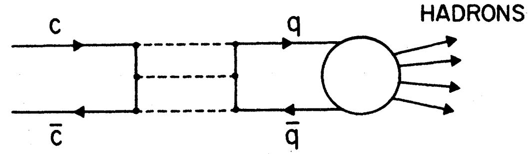

FIG. 3. Decay of a $c\bar{c}$ state into normal hadrons through three virtual gluons coupling to a single normal quark line.

while for ($J^{PC}=1^{++}$) ${}^3P_1$ states the effective current will be nearly given by

$$J_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5 q(x). \tag{2.11}$$

In these equations $q(x)$ is a quark spinor field incorporating an ordinary SU(3) index and an SU(3)′-color index both of which are summed over yielding singlet currents. It is amusing to note that (2.10) and (2.11) could be obtained if we used a short-distance operator-product expansion on expressions such as (2.3) and then had some reason to select the term with lowest dimension. Unfortunately, there does not seem to be any justification for selecting the term with lowest dimension. In any case, consequences of (1.1) and (1.2) combined with (2.10) have been discussed in Ref. 2 and will not be examined further here.

We will now briefly consider a number of specific predictions which follow from our results. Equation (1.1) obviously gives

$$\frac{\Gamma(\psi' \to \text{any } n)}{\Gamma(\psi \to \text{any } n)} \approx \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(\psi \to e^+e^-)}, \tag{2.12}$$

which enables us to compute $\Gamma(\psi' \to \text{any } n)$. This together with the observed partial widths $\Gamma(\psi' \to \psi + \text{anything})$, $\Gamma(\psi' \to \mu^+\mu^-)$, and $\Gamma(\psi' \to e^+e^-)$ can account for roughly 70% of the total width of ψ'. Therefore, as noted elsewhere,[2] the remaining 30% must be due to new channels such as

$$\psi' \to \chi + \gamma,$$
$$\psi' \to \eta_c + \text{pions}, \tag{2.13}$$

or perhaps

$$\psi' \to D + \bar{D},$$

where D is any charmed meson and χ is any one of the ${}^3P_J$ states.

An additional ${}^3S_1$ radial excitation $\psi^{(4)}$ may have been found at 4.1 GeV.[10-12] Using

$$\Gamma(\psi^{(4)} \to e^+e^-) \approx 2 \text{ keV}$$

combined with Eq. (2.12) for $\psi^{(4)}$ in place of ψ', we obtain

$$\Gamma(\psi^{(4)} \to \text{any } n) \lesssim 45 \text{ keV}. \tag{2.14}$$

Assuming μ-e universality and using the information[10]

$$\Gamma(\psi^{(4)} \to \psi + \text{anything}) + \Gamma(\psi^{(4)} \to \psi' + \text{anything}) \lesssim 15 \text{ MeV},$$

$$\Gamma(\psi^{(4)} \to \text{anything}) \approx 150 \text{ MeV},$$

we find that roughly 90% of the total width $\psi^{(4)}$ remains unexplained. Some of this might be attributed to channels similar to (2.13), but these are also open to ψ' and constitute less than 70 keV of its total width, while we still must account for 135 MeV of the width of $\psi^{(4)}$. Thus a more likely hypothesis, already discussed of course by many authors, is that $\psi^{(4)}$ decays primarily by

$$\psi^{(4)} \to D + \bar{D} + n. \tag{2.15}$$

This conclusion remains unchanged even if we assume that the upper limit in (2.14) should be increased by a factor 2 as a result of the mass difference between ψ' and $\psi^{(4)}$.

Two other states tentatively reported in[10,11] e^+e^- annihilation are $\psi^{(3)}(3.95)$ and $\psi^{(5)}(4.4)$ with the partial widths

$$\begin{aligned}\Gamma(\psi^{(3)} \to e^+e^-) &\approx 0.2 \text{ keV},\\ \Gamma(\psi^{(5)} \to e^+e^-) &\approx 0.4 \text{ keV}.\end{aligned} \tag{2.16}$$

Although the status of $\psi^{(3)}$ is still somewhat questionable, $\psi^{(5)}$ seems reasonably well established. The small values for the partial widths in (2.16) suggest identifying $\psi^{(3)}$ and $\psi^{(5)}$ either as pure ${}^3D_1$ states or as ${}^3D_1$-${}^3S_1$ mixtures. A rough calculation of the e^+e^- width for a pure ${}^3D_1$ state yields an answer more than two orders of magnitude below the observed values. Thus we will assume that these states are mixtures decaying to leptons and normal hadrons primarily through their ${}^3S_1$ components. We can then apply (2.12) with ψ' replaced by either $\psi^{(3)}$ or $\psi^{(5)}$ to obtain

$$\Gamma(\psi^{(3)} \to \text{any } n) \lesssim 5 \text{ keV},$$

$$\Gamma(\psi^{(5)} \to \text{any } n) \lesssim 10 \text{ keV}.$$

For $\psi^{(3)}$ we have in addition the observed results[10]

$$\Gamma(\psi^{(3)} \to \psi + \text{anything}) + \Gamma(\psi^{(3)} \to \psi' + \text{anything}) \lesssim 3 \text{ MeV}.$$

$$\Gamma(\psi^{(3)} \to \text{anything}) \approx 30 \text{ MeV}.$$

Again, we find that roughly 90% of the total width of $\psi^{(3)}$ is unaccounted for and must go to processes similar to (2.13) or (2.15), with (2.15) presumably the more important.

III. $SU(3)_L \times SU(3)_R$

In Eq. (2.3) each of the currents $J_{\nu a}(x)$ is a singlet under ordinary SU(3). Therefore, each is also a singlet under chiral $SU(3)_L \times SU(3)_R$. Thus if we use partial conservation of axial-vector current (PCAC) in the standard way[13] to calculate soft-pion production in ψ decays, for final states containing only normal mesons but no baryons the current $J_\mu(x)$ in (1.2) will give no contribution and instead these processes will occur entirely through the electromagnetic term in (1.2). We obtain

$$\frac{\Gamma(\psi \to \text{soft pion} + n)}{\Gamma(\psi \to e^+e^-)} \approx \frac{\sigma(e^+e^- \to \text{soft pion} + n)}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{3.1}$$

where n is any particular configuration of normal mesons and the cross section for e^+e^- annihilation should be measured just outside the resonance peak. Similar equations hold for ψ' and any higher ${}^3S_1$ radial excitations or ${}^3D_1$ states. Equation (3.1) implies in particular

$$\Gamma(\psi \to \text{soft pion} + n) = 0$$

if n consists only of an even number of pions since the processes

$$e^+e^- \to \text{soft pion} + n$$

proceed only through the isovector part of the electromagnetic current and yield only states of even-G parity.

A convenient numerical result can be gotten from (3.1) by summing the right-hand side over all n. The covariant T matrix elements for soft-pion production calculated in the usual way become

$$\begin{aligned}T(e^+e^- \to \pi^\pm + n) &= 4\pi\alpha(sf_\pi)^{-1}\bar{v}\gamma^\mu u\langle n|A_\mu^\mp(0)|0\rangle,\\ T(e^+e^- \to \pi^0 + n) &= 0,\end{aligned} \tag{3.2}$$

where α is the fine-structure constant, $\sqrt{s}$ is the total c.m.s. energy, f_π is the pion decay constant (≈135 MeV in our convention), v is an e^+ spinor, u is an e^- spinor, and $A_\mu^\pm(x) = A_\mu^1(x) \pm iA_\mu^2(x)$ is the usual axial-vector current. Squaring the matrix element in (3.2), summing over all n, and applying[14] asymptotic chiral $SU(3)_L \times SU(3)_R$, which permits the vacuum expectation value of the product of two axial-vector currents to be replaced by the vacuum expectation value of the corresponding product of vector currents, we obtain

$$\lim_{\vec{k} \to 0} k_0 \frac{d^3}{dk^3}\sigma(e^+e^- \to \pi^+(k) + \text{any } n) = \frac{\alpha^2 R}{8\pi^2 f_\pi^2 s}, \tag{3.3}$$

where R is the ratio

$$R = \frac{\sigma(e^+e^- \to \text{any } n)}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

Strictly speaking, (3.3) is not quite correct since (3.2) itself is correct only for n which do not contain baryons. However, since in e^+e^- annihilation something of the order of 5% of the final states contain a baryon-antibaryon pair, it is possible to show that the baryon-pole terms omitted in obtaining (3.3) would not alter the result by much more than 10%. Similarly, the inclusive cross section gotten by summing *only* over states containing mesons should not differ from (3.3) by much more than 5%.

Now in the interval 3.0 GeV $\leq \sqrt{s} \leq$ 3.8 GeV, we have $R \approx 2.5$. In addition, more than 90% of the charged particles found at small momenta are pions. Thus (3.3) yields

$$\lim_{\vec{k}\to 0} k_0 \frac{d^3}{dk^3}\sigma(e^+e^- \to \text{charged particle} + \text{mesons}) = 7.5 \text{ nb GeV}^{-2}c^3 , \tag{3.4}$$

which combined with (3.1) gives

$$\lim_{\vec{k}\to 0} k_0 \frac{d^3}{dk^3}\Gamma(\psi \to \text{charged particle} + \text{mesons}) = \Gamma(\psi \to e^+e^-)0.83 \text{ GeV}^{-2}c^3 . \tag{3.5}$$

Equation (3.5) is specifically restricted to states containing only mesons. To extend (3.5) to states including both mesons and baryons we would need to add baryon-pole terms[13] from final states coupling both to the electromagnetic part of (1.2) and to the nonelectromagnetic part. Even though, as in (3.3), the electromagnetically coupled pole terms are negligible, the overall magnitude of the nonelectromagnetic amplitude is significantly larger than the electromagnetic amplitude, and thus the nonelectromagnetically coupled pole terms would not be negligible.

If (3.4) is compared with the observed charge density in e^+e^- annihilation at $\sqrt{s} = 3.0$ GeV, it turns out that agreement is poor even at $|\vec{k}| \approx 150$ MeV/c, which is the smallest $|\vec{k}|$ for which data are presently available. Thus the soft-pion technique used to arrive at (3.4) can at best be reasonable somewhere within the region $|\vec{k}| < 150$ MeV/c. A similar restriction must therefore be applied to (3.5) and (3.1). It is possible, of course, that the extrapolation from $(k^\mu) = 0$ used to arrive at (3.4) is inaccurate even at $(k_\mu) = (m_\pi, 0, 0, 0)$. If this were true (3.1) and (3.5) would also be expected to fail.

For any other $^3S_1$ or $^3D_1$ $c\bar{c}$ states an equation similar to (3.1) and (3.5) can be gotten. If the preceding arguments are reformulated for $c\bar{c}$ states other than $^3S_1$ or $^3D_1$, we find that since no electromagnetic term coupling to normal hadrons is present we obtain

$$\lim_{\vec{k}\to 0} k_0 \frac{d^3}{dk^3}\Gamma(c\bar{c} \to \pi^+(k) + \text{any } n) = 0 , \tag{3.6}$$

assuming that the branching ratio to states containing baryons is small. Since, as before, this prediction depends on extrapolation to physical momenta from a pion 4-momentum of 0, and is therefore not expected to hold exactly, it is useful to compare the predictions of a model in which the nonelectromagnetic terms in decay Lagrangians for ψ-like states are not effectively $SU(3)_L \times SU(3)_R$ singlets. Suppose, for example, that normal decays of a scalar and pseudoscalar ψ-like states, ψ_+ and ψ_-, respectively, are governed by

$$\begin{aligned} \mathcal{L}_+(x) &= g_+\psi_+(x)S(x) , \\ \mathcal{L}_-(x) &= g_-\psi_-(x)P(x) , \end{aligned} \tag{3.7}$$

where $S(x)$ and $P(x)$ are, respectively, the usual SU(3)-singlet scalar and pseudoscalar densities composed of normal quark fields belonging to the $(3, 3^*) + (3^*, 3)$ representation of $SU(3)_L \times SU(3)_R$. Then by a soft-pion calculation combined with asymptotic $SU(3)_L \times SU(3)_R$, which permits vacuum expectation values of products of scalar densities to be replaced with vacuum expectation values of corresponding products of pseudoscalar densities and vice versa, we obtain

$$\lim_{\vec{k}\to 0} k_0 \frac{d^3}{dk^3}\Gamma(\psi_\pm \to \pi^+(k) + \text{any } n) = \frac{2\Gamma(\psi_\pm \to \text{any } n)}{3(2\pi)^3 f_\pi^{\,2}} . \tag{3.8}$$

The derivation of (3.8) again assumes that branching ratios to states containing baryons are small. Effective Lagrangians of the form (3.7) would be expected to hold in heavy leptonium models[7] of ψ. Thus, in principle, by measuring $\Gamma(\psi_\pm \to \pi^+(k) + \text{any } n)$ it would be possible to distinguish between these models and charmonium.

Finally, a soft-pion theorem can be derived for the decay

$$\psi' \to \psi + \pi^+ + \pi^- . \tag{3.9}$$

In the charmonium model, this reaction proceeds by the exchange of two or more gluons between the pairs (ψ, ψ') and (π^+, π^-). For (3.9) the gluons will not usually carry much 4-momentum, so that the running gauge coupling constant may not be very small and contributions from the exchange of more than two gluons could be significant. Nonetheless, using the structure of the currents to which the gluons couple, a soft-pion theorem similar to those we have already discussed can be derived, yielding the result

$$\lim_{k\to 0} T(\psi' \to \psi + \pi^\pm(k) + \pi^\mp) = 0 \tag{3.10}$$

for the decay matrix element of (3.9). Equation (3.10), in turn, provides an explanation for the experimentally observed pion momentum distributions, as a number of authors have already shown.[15] The derivation we have suggested for (3.10) does not hold for the leptonium model in which the pairs (ψ', ψ) and (π^+, π^-) can couple through scalar particles. Thus these models may have a more difficult time explaining the pion momentum distribution which has been observed.

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[9]In the process $\psi' \to \gamma\chi$ states have been found at energies of roughly 3.41, 3.45, 3.51, and 3.55 GeV. The partial width for $\chi(3.45)$ into normal hadrons seems to be fairly small and only $\chi \to \psi\gamma$ has been seen without question. Thus following arguments of Eichten, Gottfried, Kinoshita, Lane, and Yan, Ref. 6, we identify $\chi(3.45)$ with $^3P_1$. The state $\chi(3.41)$ apparently decays into $\psi\gamma$ and many combinations of normal hadrons; from the angular distribution of the photons in $\psi' \to \gamma\chi(3.41)$ it appears to have $J^{PC} = 0^{++}$ and therefore the assignment $^3P_0$. The state at 3.55 GeV again decays into many channels including $\pi^+\pi^-$, ruling out the possibility that it is an $^1S_0$ ($J^{PC} = 0^{-+}$) radial excitation and leaving only $^3P_2$ ($J^{PC} = 2^{++}$) as a reasonable alternative. Finally, χ (3.51) does not seem to decay into $\pi^+\pi^-$, suggesting an $^1S_0$ ($J^{PC} = 0^{-+}$) assignment. In addition, the line width exhibited in the chain decay $\psi \to \gamma\chi(3.51) \to \psi\gamma\gamma$ is significantly broader than those of the other three states, which would be expected for a state whose wave function does not vanish at the origin and which is permitted to couple to normal hadrons though only two gluons. This collection of data has been reported by W. Braunschweig *et al*. Phys. Lett. 57B, 407 (1975); G. J. Feldman *et al*., Phys. Rev. Lett. 35, 821 (1975); W. Tannenbaum *et al*., *ibid.* 35, 1323 (1975); and G. Goldhaber, rapporteur's talk at the International Conference on the Production of Particles with New Quantum Numbers, Madison, Wisconsin, 1976 (unpublished).

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