Electromagnetic mass differences of hadrons with $SU(6)_W \times O(3)$ couplings and form factors

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A systematic account of a few typical electromagnetic mass differences of hadrons (N, Σ; K, π) is presented within the framework provided by a broken-$SU(6)_W \times O(3)$ model of hadron couplings. The model, which has specified combinations of couplings of "magnetic" and "charge" origin, is characterized by the supermultiplet form factors at the hadron vertices. The parameters of these functions on the mass shell have been determined recently via a study of the decay widths of the resonances. Using these form factors, suitably extended off the mass shell of the vector meson so as to render the calculations formally free from series and integral divergences, the coupling scheme is found to provide a reasonable description of the mass differences through the twin mechanisms of dominance of magnetic contribution over charge contribution and that of $(L+1)$ wave couplings over $(L-1)$. A formal connection of this approach with the more conventional dispersion-theoretic one can be established through the observation that the subtraction term (necessary for $\Delta I = 1$ cases of mass differences) finds a close parallel to the couplings of magnetic origin (which have extra momentum dependence vis-à-vis the charge couplings) thus making the magnetic couplings relatively more important for the $\Delta I = 1$ cases according to Harari's interpretation. The model is not so successful for $\Delta I = 2$ mass differences which are dominated by the (weaker) charge couplings.

I. INTRODUCTION

Though the age-old problem of electromagnetic mass differences has occasionally received major attention, it can hardly be considered to be solved even today.[1] Basically, there have been two approaches to the problem. The earlier one, which originated through the work of Feynman and Speisman,[2] is characterized by the language of field theory in which the form factors involving extensions off the mass shell play a central role. The newer version owes its origin to the work of Cottingham[3] which established the basic connection between the Compton-scattering amplitude and the electromagnetic self-energy of the particle concerned. Most of the work in the 1960's has been on the latter lines, which are characterized by the language of dispersion relations and Regge poles (tadpoles,[4] A_2 exchange,[5] etc.). The role of subtraction terms in the dispersion-theoretic approach was clarified through the work of Harari,[5] who showed that they are relatively unimportant for the $\Delta I = 2$ cases (adequately covered by the Born term), while they play a significant role for $\Delta I = 1$ mass differences. Calculations of the latter have, therefore, suffered from the uncertainties inherent in the subtraction-parameters in the absence of additional input information. The most comprehensive calculations along these lines have been due to Buccella *et al.*,[6] who were the first to give a unified and systematic account of electromagnetic mass differences, but only with the use of an additional universality hypothesis due to Cabibbo, Horwitz, and Ne'eman[7] which was shown to provide the necessary connections between couplings of the A_2 Regge pole and the particle under study within the framework of U(3) $\times$ U(3).

The potentialities of the Feynman-Speisman[2] language never seem to have been adequately realized, except, in more recent times, when the experimental interest exhibited in the couplings of higher resonances to lower hadrons has provided more meaning and relevance to this approach. The connection of this description to the dispersion-theoretic one is not easy to establish. However, in a general sort of way, it can be said that the subtraction terms (necessary for $\Delta I = 1$ cases) correspond in some sense to the appearance of high-momentum dependence of the higher-resonance couplings in a field-theoretic language. These couplings are, in turn, dominated by the "magnetic" ($\sim\sigma_{\mu\nu}k_\nu$) rather than the "charge" ($\sim\gamma_\mu$) contributions. Therefore, one should expect that the $\Delta I = 1$ mass differences which are dominated by the subtraction terms in dispersion language[5] should be correspondingly dominated by the magnetic couplings in the field-theoretical description. A more precise statement is difficult to make on the correspondence. Both approaches are characterized by the appearance of extra unknown parameters for $\Delta I = 1$ cases: the subtraction terms in dispersion theory and cutoff momenta, especially for magnetic couplings, in field theory. Reduction of parametrization in the latter case is possible only through additional universality assumptions on the coupling structures such as partial symmetry[8] and Regge universality.[9]

The object of this paper is to present the results of calculations of some important electromagnetic mass differences in the spirit of Feynman and Speisman by employing a relativistic coupling model for hadron supermultiplets[10] which has been developed at this institution over the years. In this model, the baryons are classified according to the supermultiplets [$\underline{56}$, even$^+$] and [$\underline{70}$, odd$^-$]. In recent times, however, there has been some evidence of the existence of states belonging to [$\underline{70}$, even$^+$] (Ref. 11) but we do not consider the effects of these states in the present paper. The general philosophy of the approach is in conformity with the duality spirit[12] wherein the effect of Regge exchange is sought to be simulated by an infinite sequence of direct-channel resonances. Further, the framework of the model is versatile enough to allow investigations of a wide class of processes and some of these applications have proved fairly successful.[13–15] In particular, a preliminary application of this model to the n-p mass differences has already yielded some encouraging results.[16] It is, therefore, of interest to present the results of more systematic investigation of electromagnetic mass differences for several typical hadrons ($N, \Sigma; K, \pi$) so as to bring out the general features of the mechanism which produce the desired effects within this coupling scheme.

As explained in the earlier references,[10,14] these couplings have specified mixtures of both magnetic and charge types in accordance with the principle of partial symmetry. Therefore, the over-all results should be expected to reflect the relative contributions of charge and magnetic types for both the $\Delta I=1$ and $\Delta I=2$ cases without the introduction of any new parameters. The "power" form factors[9,17] used in the present calculations have recently been found to have a kinematical basis.[18] This comes about from the Lorentz-contraction effect on the external momenta involved in the transition matrix elements according to an argument due to Licht and Pagnamenta.[19]

In Sec. II, we outline the structures for mesons (those for the baryons having already been presented elsewhere[14]) together with the integral formula for the self-mass (δm). Sec. III briefly explains the assumptions on the off-shell extension of the "power" form factors needed for the evaluation of the self-energy integrals. In Secs. IV and V we present and critically discuss the numerical results. The essential conclusion is that since the coupling scheme is dominated by terms of "magnetic" (rather than "charge") origin, the model is expectedly more successful for the $\Delta I=1$ cases than for $\Delta I=2$ ones.

II. FORMALISM

To first order in e^2, the perturbation theory expression for the self-mass of a baryon [$\delta(m^2)$ for a meson] of mass m is given by the Cottingham formula[3]

$$\delta m = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty} d^4k\, \frac{T_{\mu\nu}(\vec{k}, k_0)}{k^2-i\epsilon}\,\delta_{\mu\nu} \ . \tag{2.1}$$

In the above expression, $\epsilon_\mu\epsilon_\nu T_{\mu\nu}(\vec{k}, k_0)$ is the forward Compton amplitude for the scattering of a virtual photon with three-momentum $\vec{k}$, energy k_0, and polarization vector ϵ_μ from the baryon at rest.

We have to calculate the contribution to $T_{\mu\nu}$ of a particular resonance of spin J and then sum over all the different sequences of resonances relevant to the cases under study. Since, in our model, the electromagnetic interaction is introduced through the VMD (vector-meson-dominance) hypothesis,[14] the two basic ingredients required to evaluate the vertices $\overline{H}_J H\gamma$ (H for hadron) are the $\overline{H}_J HV$ couplings and the γ-V vertex. Most of the material required in this connection has been presented in sufficient details in Refs. 14, 17, and 10, and since we borrow heavily from them in the present discussion we shall, henceforth, denote them as papers I, II, and III, respectively. However, for easy retrieval of this information, we draw attention to the main features of the coupling structures for the different types of particles involved [$N, \Sigma; K, \pi$] as detailed in the above references.

(*a*) *Baryon couplings*. For the case of nucleons, the five basic types of $\overline{B}_J BV$ couplings (A–E) along with the appropriate SU(6)×O(3) factors are as given in I. Also listed therein are the γ-V vertex in the VMD hypothesis and the recipes for making these couplings gauge invariant. The case of the Σ hyperon differs from that of the nucleon in having nonzero couplings to ϕ in addition to ρ and ω, thus giving rise to some extra entries. Since the geometrical SU(6)× O(3) factors for these are adequately given elsewhere[10] we merely refer the reader to paper III for these and related details for meson couplings.

(*b*) *Meson couplings*. For meson states, the case involving transition from a quark-spin $S=0$ (e.g., B meson) to another with $S=0$ (e.g., pion) can only have a "charge-type" coupling of the form

$$A' \equiv \overline{B}_{\mu_1 \ldots \mu_L}\, m_V\left[(p_\mu + P_\mu) - \frac{k(P+p)}{k^2}\, k_\mu\right] \times V_\mu k_{\mu_1}\cdots k_{\mu_L} f_L^{+}\,\Pi \ , \tag{2.2}$$

which has been made gauge invariant on the lines prescribed in I for the corresponding γ-baryon couplings. The transition from the quark-spin triplet ($S=1$) to singlet ($S=0$) is characterized by magnetic-type couplings B' and C' connecting states with $J=L+1$ (e.g., A_2), and $J=L$ (e.g., L_1) to $\pi\gamma$ system, defined by

$$B' \equiv T_{\mu_0\mu_1 \cdots \mu_L} \epsilon_{\mu_0\alpha\mu\sigma} k_\alpha V_\mu (p+P)_\sigma f_L^{(+)} k_{\mu_1} \cdots K_{\mu_L} \Pi \;, \tag{2.3}$$

$$C' \equiv \overline{A}_{\mu_1 \cdots \mu_L} \left(\frac{L}{L+1}\right)^{1/2} \left[(-I_k \delta_{\mu\mu_1} + k_\lambda \theta_{\lambda\mu} k_{\mu_1}) f_L^{(+)} (2M_L) + 2(p^2 - P^2)\left(\delta_{\mu\mu_1} - \frac{k_{\mu_1} k_\mu}{k^2}\right) f_L^{(-)}\right] V_\mu k_{\mu_2} \cdots k_{\mu_L} \Pi \;. \tag{2.4}$$

We should like to draw particular attention to the factors $(2M_L)$ and (p^2-P^2) (appearing in the first and second terms in the square brackets of C') which represent the over-all normalization factors for the $(L\pm 1)$ wave couplings of the concerned meson fields, the factor (p^2-P^2) including the GOR (Gell-Mann–Oakes–Renner)[20,17] effect extended off the mass shell (for details see paper III). Note, further, that whereas the $(L+1)$ wave parts of both B' and C' are gauge invariant as they stand, the $(L-1)$ wave coupling in C' has been made so by introducing the second term. The symbols $f_L^{(+)}, f_L^{(-)}$ here stand for the $(L+1)$, $(L-1)$ wave form factors, respectively. In Table I we list the various (K,π)–γ couplings, taking account of the fact that the $\gamma\pi$ couplings of the $I=0,1$ meson are governed by the G-parity considerations which restrict the possible intermediate states.

(c) Structure of δm integrals. Using the above $\overline{B}_J B V$ structures (in conjunction with VMD), the integral (2.1) for the baryon self-mass δm is expressible as

$$\delta m = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty} \frac{d^4k}{k^2 - i\epsilon}\left(\frac{e}{g_\rho}\right)^2 \Omega_B(\vec{k}, k_0) \;, \tag{2.5}$$

$$\Omega_B(\vec{k}, k_0) = \overline{u}(p) \langle J_\mu J_\mu^\dagger \rangle u(p) \;, \tag{2.6}$$

where

$$J_\mu = \frac{m_\rho^2}{m_\rho^2 + k^2} j_\mu^{(\rho)} + \frac{1}{3}\frac{m_\rho m_\omega}{m_\omega^2 + k^2} j_\mu^{(\omega)} - \frac{\sqrt{2}}{3}\frac{m_\phi m_\phi}{m_\phi^2 + k^2} j_\mu^{(\phi)} \tag{2.7}$$

and the $j_\mu^{(V)}$'s [$V=\rho,\omega,\phi$] are the coefficients of the appropriate V-meson fields (V_μ) in the various couplings given in I for nucleons and similarly for Σ's. As in I, the bracket $\langle\ \rangle$ in (2.6) stands for the product of the propagator for the intermediate state and the $\overline{B}VB_J$ vertices at its ends. To specify the normalization, the contribution to $\Omega_B(\vec{k}, k_0)$ for type-A coupling of a spin $(L+\frac{1}{2})$ baryon to a nucleon is

$$(k_{\mu_1'} \cdots k_{\mu_L'}) \Theta^{\overline{\mu}' \mu_1' \cdots \mu_L'}_{\overline{\mu}\mu_1 \cdots \mu_L}(L+1)(k_{\mu_1} \cdots k_{\mu_L}) \times \overline{u}(p)\left[(i\sigma_{\nu'\mu'} k_{\nu'})\gamma_{\overline{\mu}'}\gamma_{\overline{\mu}} \frac{(M - i\not{P})}{(M^2 + P^2)}(-i\sigma_{\nu\mu'} k_\nu)\right] u(p) \,. \tag{2.8}$$

In (2.8), $\Theta^{(\overline{\mu}')}_{(\overline{\mu})}(J)$ is the projection operator for a boson of spin J (see Appendix B of I for other details).

The structure of the integral for $\delta m^2 (=2m\,\delta m)$ for the case of the mesons is obtained through exactly similar consideration except for replacement of the coupling structures (A–E) (Ref. 14) by (A'–C') and concomitant changes usual for the boson systems.

Provided that the conditions for a Wick rotation are satisfied (i.e., the singularities in the k_0 plane are located just below the positive real axis and just above the negative real axis) we can, following Cottingham,[3] change the integration contour running in Eq. (2.5) from $k_0 = -\infty$ to $k_0 = \infty$ to one running from $k_0 = -i\infty$ to $k_0 = i\infty$. This condition imposes a constraint on the pole structure of the various form factors, governed by the usual Feynman condition $M^2 \to M^2 - i\epsilon$ (see Sec. III).

TABLE I. $\overline{M}_J M\gamma$ couplings of K's and $I=0,1$ mesons. The letters A'–C' correspond quantitatively to the factors indicated in Eqs. [(2.2)–(2.4)]. The operator T_3 common to all the couplings has been suppressed.

Trajectory	J^{PC}	Coupling
(K,π)	$(L)^{-+}$	$(A', 2A')$
(K^*,ω)	$(L+1)^{--}$	$(B', 2B')$
K_B	$(L)^{+-}$	A'
$(K_A, K'^*; A_2, \omega')$	$(L)^{++}; (L)^{--}$	$(C'; 2C')$
(K_T, A_2)	$(L+1)^{++}$	$(B', 2B')$

After this rotation, the variable $x=k^2=\vec{k}^2+k_0{}^2$ is always positive and Eq. (2.5) can be recast as

$$\delta m=-\frac{(4\pi i)}{16\pi^4}\left(\frac{e}{g_\rho}\right)^2\int_0^\infty dx\int_0^{\sqrt{x}}dy\,\frac{(x-y^2)^{1/2}}{x-i\epsilon}\,\Omega_B(x,y^2)\,, \tag{2.9}$$

where we have used the symbol y for the photon energy k_0.

Numerical calculations involving Eq. (2.9) are greatly facilitated by a change in the variables of integration as briefly outlined in the Appendix.

III. OFF-SHELL ASSUMPTIONS AND NORMALIZATION

The on-shell definition of $f_L^{(\pm)}$ have been given in II [Eqs. (1.1) and (1.2)]. Our ansatz for off-shell extrapolation is confined only to the radiation quantum viz. $\mu^2\to -k^2$, while for other particles we propose no extrapolation at all. This extrapolation, in addition to endowing the integrals with the correct pole structure in the k_0 plane for the purpose of a Wick rotation, also ensures that not only does the integral converge for each L value, but so does the summation over L for successive intermediate resonances of each type (the latter arising through a linear dependence of $M_L{}^2$ on L). The numerical values of the reduced coupling constants for the $(L+1)$ waves, the dominant contributors, are

$$(g_B{}^2/4\pi)=1.5,(g_M{}^2/4\pi)=0.16 \tag{3.1}$$

for baryons and mesons, respectively, while for $(L-1)$ waves these are an order of magnitude smaller and make negligible contributions to δm.

The masses of the successive recurrences contained in individual towers are generated, as in I, through the standard linear relation $M_L{}^2=aL+b$, $a\simeq 1$ GeV^2. The comparison given in I between the electroproduction data and the present model provides the (phenomenological) basis for parametrization of all the $\bar{N}^*N\gamma$ types of couplings except $\bar{N}N\gamma$ which was not used as an input for the calculations of that process. Thus, a literal extrapolation of the $\bar{N}^*N\gamma$ couplings used therein to the $\bar{N}N\gamma$ case would unfortunately imply the unpleasant prediction of a nucleonic charge rather too large [about $(m_\rho/m_\pi)^{1/2}$ times the normal value]. We seek to remedy this feature for $\bar{N}N\gamma$ by demanding normalization of the $\bar{N}NV$ form factor so as to produce unit charge via VMD.

For the case of other particles $(\Sigma;K,\pi)$, where no direct guidance from reaction data is available, we use the broad principle of normalization of the elastic form factor to unit charge to fix the reduced widths for the electromagnetic couplings of these particles via VMD, partial symmetry, exchange degeneracy, and the additional principle of Regge universality for the couplings of their recurrences. In principle, this is adequate to fix the reduced coupling constants uniquely for a given class of form factors. However, as decribed above, there is a violation of the universality principle for the nucleon case to the extent that the reduced coupling constant $L=0$ does not fit in with those for $L>0$. We suggest the same amount of violation for $\bar{\Sigma}^*\Sigma\gamma$ couplings versus $\bar{\Sigma}\Sigma\gamma$ couplings as in the N case, viz. the reduced couplings for Σ will be related via SU(3) and SU(6) to those of the N's. For meson couplings (K,π), on the other hand, no such consideration is relevant so that the principle of unit-charge normalization for the elastic form factors of the mesons is enough to specify the reduced coupling constants for higher L states with the form factors of the type given in II without violation of the universality principle in the transitions from $L=0$ to $L>0$.

As to the value of k^2 at which to fix the normalization to unit charge, we use the standard point $k^2=0$ in all the cases except one, viz. $\pi\pi\gamma$. For this isolated case, the unusually low mass of the pion in relation to that of ρ unfortunately makes the form factor particularly sensitive to the extrapolation which involves a long journey from the mass region (m_ρ) used for comparison of the ρ-decay data.[17] To avoid such a long extrapolation, we have little alternative to normalization at the ρ-meson mass shell for the calculation of the pionic charge via $\pi\pi\gamma$ couplings. [A similar ansatz was suggested for electroproduction,[14] where the extrapolation $M^2\to W_m{}^2$ $(=m^2+m_\rho{}^2-2p\cdot k)$ was found to give much better agreement with the experimental results than the simpler choice $M^2\to W^2$ $(=m^2-k^2-2p\cdot k)$.]

IV. NUMERICAL RESULTS

We have evaluated five typical cases of electromagnetic mass differences; three (viz. $n-p$, $\Sigma^--\Sigma^+$, and K^0-K^+) having $\Delta I=1$ while two $(\Sigma^++\Sigma^--2\Sigma^0$ and $\pi^+-\pi^0)$ correspond to $\Delta I=2$. The p-n and K^+-K^0 differences are just the coefficient of $\frac{1}{2}\tau_3$ in the self-energy integrals, arising from $(\rho\omega,\rho\phi)$ interferences only. Similarly, the entire contribution to the $\Delta I=1$ case of Σ and $\Delta I=2$ cases of both Σ and π come from $(\rho\omega,\rho\phi)$ and $(\rho\rho)$ interferences, respectively.

(a) n-p mass difference. In Table II, we present the results of a breakup into contributions of magnetic and charge origin. One finds that for the Born contribution, the charge term dominates, making its expected (albeit small) contribution

TABLE II. Breakup into "magnetic" origin, "charge" origin, and their interference of contributions to n-p mass difference from $L=0,2$ members of N_α. The contributions are given in MeV.

Particle	Magnetic	Charge	Inter.	Sum
$N(938)$	0.142	−0.522	0.358	−0.022
$F_{15}(1690)$	0.144	−0.101	0.170	0.213

with opposite sign. However, from the second recurrence onwards the magnetic and the interference terms soon take over and produce a net positive contribution for all the remaining N_α resonances, the values for the $(L+1)$ wave contributions of the first four members of this tower being −0.022, 0.213, 0.062, and 0.021 MeV, respectively. Exactly the same mechanism is responsible for yielding a positive contribution from the N_γ trajectory as well. Since the contributions of the $(L-1)$ wave couplings is nominal compared to that of the $(L+1)$ wave (a direct consequence of this feature being the dominance of higher-spin trajectories, i.e., $J=L+\frac{3}{2}$, $L+\frac{1}{2}$ over those of low-lying trajectories, i.e., $J=L-\frac{1}{2}$), we have refrained from presenting the corresponding break up into $(L\pm 1)$ waves and their interference. Instead, in Table III, we merely present the *total* contributions of individual *trajectories*. It is clearly seen from Tables II and III that two important factors, viz. (i) the dominance of the magnetic contributions over charge contributions and (ii) preponderance of the $(L+1)$ wave over $(L-1)$, are directly responsible for bringing about the desired sign for the resultant $(n$-$p)$ mass difference.

(b) Σ mass differences. The basic mechanism operative in the case of Σ mass differences is the same as for the nucleonic case except for the differences brought about by the Clebsch-Gordan coefficients and the presence of decimet intermediate states which have no counterpart in the latter case. The masses of the leading members of experimentally unobserved towers (Σ_γ^* and Σ_β^*) have been fixed through using the twin principles of $\underline{10}_d$ and $\underline{8}_d$ mass degeneracy and small $L-S$ coupling. Specifically, the main contributors to $\Sigma^- - \Sigma^+$ are $\Sigma_\alpha(0.590)$, $\Sigma_\delta^*(0.317)$, and $\Sigma_{\text{Roper}}(0.184)$, whereas those to $\Sigma^+ + \Sigma^- - 2\Sigma^0$ are $\Sigma_\gamma(-0.374)$, $\Sigma_\delta^*(-0.153)$, $\Sigma_\beta'(-0.133)$, and $\Sigma_\alpha(-0.124)$ (the numbers in the brackets indicate the corresponding contributions). On the whole, however, the overall results for both the Σ cases in our model are unsatisfactory; $\Sigma^- - \Sigma^+$ has correct sign (1.353) but is too small in magnitude; $\Sigma^+ + \Sigma^- - 2\Sigma^0$ has the wrong sign (−0.897) and is small in magnitude. In Sec. V, we shall discuss some possible reasons for the failure of our model in this case.

(c) K^0-K^+ mass difference: For the K^0-K^+ $(\Delta I=1)$ mass-difference calculations (Table IV), there are, in all, six possible intermediate trajectories. The essential n-p results, viz. the dominance of (i) magnetic over charge contributions and of (ii) the $(L+1)$ waves over $(L-1)$, are maintained in the kaon case, thus providing a similar mechanism for $\delta m_{K^0} > \delta m_{K^+}$. In bringing about the dominance of the $(L+1)$ wave over $(L-1)$, the normalization factor $(4M_L{}^2)$ relevant to the former wave and the off-shell GOR factor (p^2-P^2) relevant to the latter [see Eq. (2.4)] have played an important role. In more specific terms, states of $J=L$, $C=(-)^{L+1}$ [natural parity, such as $K_A, K^{*\prime}$] seem to dominate over both $J=L+1$, $C=(-1)^{L+1}$ (such as K^*, K_T) and $J=L$, $C=(-1)^L$ (unnatural parity such as K_B). Since all the natural-parity states have magnetic couplings, their dominant

TABLE III. Contributions to n-p mass difference of individual trajectories.

Trajectory	Contribution (MeV)
N_α	0.289
N_β'	0.070
N_β''	−0.147
N_γ	0.491
N_γ'	−0.101
N_δ	0.048
Roper	0.060
Total	0.710

TABLE IV. Analysis of K^0-K^+ and π^+-π^0 mass differences. In the absence of relevant experimental data, the mass of the leading member of $K^{*\prime}$ (ω') is taken to be degenerate with the second member of the K^* (ω) trajectory.

Type	Trajectory	Contribution (MeV)
$(K^0$-$K^+)$	K	−0.592
	K^*	0.099
	K_B	0.021
	K_T	0.002
	$(K_A+K^{*\prime})$	3.362
	Total	2.892
$(\pi^+-\pi^0)$	π	22.473
	A_2	−0.054
	ω	0.082
	$(A_1+\omega')$	−18.823
	Total	3.678

contributors, viz. $J=L$ members produce the desired signs in adequate magnitudes. The unnatural-parity states $[C=(-1)^L]$, which are characterized by charge couplings, contribute wrong signs but of smaller magnitudes. The only exception is the Born term whose (charge) coupling is appreciable and considerably offsets the effect of $C=(-1)^{L+1}$ states. [The exception for the Born term is perhaps due to the fact that its coupling does not require a gauge modification because of the equality of masses, in contrast to the higher $(L>0)$ members; see Eq. (2.2).]

We note in passing that the numerical success of this model for the kaon case is largely due to the reinforcement of contributions from even and odd L values (without distinction) involved in $J=L$ states of natural parity. Such a reinforcement is due to a common pattern of SU(6) couplings of the kaon and its excited states to the photon via VMD, without distinction between even and odd values of L, a feature which owes its origin directly to the absence of any role of G-parity selection rule in the kaon case.

(d) $\pi^+\pi^0$ mass difference. Finally, the π^+-π^0 mass difference (Table IV) is governed by the same general mechanism as for the kaon case except for the key role of the G-parity selection rule which brings about some important differences. Thus A_2 and ω, together with their recurrences, make nominal contributions, in conformity with the general pattern of small contribution from $J=L+1$ states of natural parity. Again, the operation of the G-parity selection rule is responsible for the absence of any contribution from ρ and B as well as their recurrences to the π^+-π^0 case. However, the most important consequence of this selection rule is that the $J=L$ states of natural parity now give *opposite contributions for even and odd L, in contrast to the kaon case.* Indeed, there appears to be a close competition between the opposite contributions for odd and even L's leading to a net negative contribution which is able to offset the large positive contribution from the Born term to an observationally more pleasing magnitude. This apparently fortuitous yet interesting result which characterizes our coupling scheme with "power" form factors does not seem to have any simple explanation beyond its over-all consistency with the predictions in the other cases.

V. DISCUSSION AND SUMMARY

Through the twin mechanisms of dominance (i) of magnetic contributions over charge and (ii) of $(L+1)$ wave coupling terms over the $(L-1)$ wave terms, our calculations of electromagnetic mass differences via an $SU(6)_W \times O(3)$ model of hadron couplings,[10] seem to give quite sensible results for the n-p, K^0-K^+, and π^+-π^0 cases, but do not work as well for the Σ case. The unusually low value of the Born term for the n-p case in this model, compared with contemporary calculations,[1] is the result of large destructive interference between the charge and magnetic contributions, each of substantial magnitude. Yet this result is not entirely satisfactory since it implies a less rapid fall with momentum transfer of the magnetic form factors of the nucleon than is claimed to be indicated by the data.[21] However, it should be also noted that the success of the mechanism arises mainly from the contributions of higher resonances and depends only marginally on the fact that the magnitude of the (wrong-sign) contribution of the Born term is small. The lack of success for the Σ case could be partly attributed to the (unexplored) role of some more trajectories, especially $[\underline{70}, \text{even}^+]$ (Ref. 11) mentioned in Sec. I, which abound in Σ states but have fewer N states. These would, however, require additional parameters for their estimation without adequate support from decay data and hence would be of less physical interest.

Our main argument for the estimation of δm in terms of towers of resonant contributions stems from our earlier results on η production,[15] which strongly suggested a simulation of A_2 exchange supposed to be the dominant mechanism (tadpole) for δm calculations.[4,22] Thus the compatibility of these results with those of Buccella *et al.*[6] could be judged in this context, though a direct comparison is not possible beyond what is stated in Sec. I. As to the contribution from the scaling region, our method is probably inadequate to the extent that only single resonant intermediate states have been considered. However, the numerical results would appear to indicate that this region does not dominate the δm contribution, in agreement with at least some claims in the literature. Also, we prefer not to comment on contributions from weak-interaction effects which have been claimed in recent literature. We have merely suggested *one* possible mechanism, viz. a resultant of charge versus magnetic contributions (with dominance of the latter) arising from different towers of resonant intermediate states, with no loose parameters. It seems to work rather well for $\Delta I=1$, but not so much so for $\Delta I=2$.

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APPENDIX

Consider the following double integral occurring in Eq. (2.9):

$$I=\int_0^\infty dx\int_0^{\sqrt{x}} dy\,\frac{(x-y^2)^{1/2}}{x(x+m_\rho^2)^2}f(x,y^2), \tag{A1}$$

the factor $(x+m_\rho^2)^2$ arising from the γ-V vertex [Eq. (2.7)]. Introduce the parabolic coordinates u and v defined by the equation

$$y'^2=4u(\bar{x}+u),\qquad y'^2=-4v(\bar{x}-v), \tag{A2}$$

where y' and $\bar{x}$ are dimensionless quantities given by

$$y'=\frac{y}{m_\rho},\quad \bar{x}=\frac{x}{m_\rho^2}-\frac{1}{4}. \tag{A3}$$

In terms of the new variables, whose ranges of variations are given by $0\leqslant v\leqslant\infty$, $0\leqslant u\leqslant\frac{1}{4}$, the integral (A1) can be recast as

$$I=\int_0^\infty dv\int_0^{1/4} du\,\frac{u+v}{(uv)^{1/2}}\times\frac{(1-4u)^{1/2}(1+\frac{1}{4}v)^{1/2}}{(\frac{1}{4}+v-u)(\frac{5}{4}+v-u)^2}f(u,v). \tag{A4}$$

The further substitution

$$v=(\tfrac{1}{4}-u)\frac{\omega}{1-\omega} \tag{A5}$$

so that

$$0\leqslant\omega\leqslant 1 \tag{A6}$$

leads finally to

$$I=\int_0^1\frac{d\omega}{\sqrt{\omega}}\int_0^{1/4}\frac{du}{\sqrt{u}}(1-4u\omega)^{1/2}(u-2u\omega+\tfrac{1}{4}\omega)\times(\tfrac{5}{4}-u-\omega)^{-2}f\left(\frac{\frac{1}{4}-u}{1-\omega},\frac{(\frac{1}{4}-u)(4u\omega)}{1-\omega}\right), \tag{A7}$$

a form particularly suitable for numerical work.

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