s-channel components of a two-Reggeon cut

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A separation of the two-Reggeon-cut contribution into different s-channel-discontinuity components using a t-channel approach is suggested and compared with the Reggeon-cutting rules obtained from a s-channel approach. The separation agrees with the Abramovskii-Gribov-Kancheli cutting rules and suggests that these rules are model independent.

I. INTRODUCTION

The s-channel structure of a two-Reggeon cut has been the subject of some recent controversy. The cutting rules suggested by Abramovskii, Gribov, and Kancheli[1] (AGK) are based on simple Feynman-diagram models, and the general validity of these rules has been questioned.[2,3] The purpose of this paper is to look at the s-channel structure of a t-channel approach to the two-Reggeon contribution. It is hoped by some (and there is some support for this hope[4]) that an approach based on t-channel unitarity will guarantee that the constraints of s-channel unitarity are satisfied. If this is the case, then it is important to see how the s-channel structure of Reggeon contributions emerges from a t-channel approach. In particular, it is important to understand what a cut Reggeon means in the t channel. There have been attempts to study this problem in Reggeon field theory[5] and in the topological-expansion approach.[6] In both cases the AGK cutting rules have been shown to follow in a natural way, if s-channel unitarity is also imposed.

In this paper we shall consider the two-Reggeon contribution and try to find an unambiguous way of separating out different s-channel discontinuity components. It turns out that this can be done. The separation agrees with the separation suggested by the AGK cutting rules and provides a model-independent proof of these rules, in contrast to the separation suggested by a diffractive excitation model[7] or by simple calculations using the dual resonance model.[2]

We shall use the approach to the two-Reggeon contribution suggested in Ref. 8, which, as was shown in Ref. 9, is equivalent to a t-channel unitarity approach,[4] and shall show that a natural separation into components is suggested.

In Sec. II we shall briefly review the calculation of the two-Reggeon cut in order to establish our notation and to motivate Sec. III. In Sec. III we show that the two-Reggeon contribution is made up of three different parts, each part describing the effects of different number of Reggeons, that is, "no-Reggeon effects," "one-Reggeon effects," and "two-Reggeon effects." Section IV is the conclusion.

II. THE TWO-REGGEON CONTRIBUTION

The starting point for the calculation of the two-Reggeon contribution to the two-particle amplitude is to write the t-channel unitarity equation for two physical intermediate states with spins l_1 and l_2 and t-channel helicities n_1 and n_2 as shown in Fig. 1. The two intermediate states are then put on Regge trajectories by replacing the "on the mass shell" conditions by "on the angular momentum shell" conditions, and finally the spins and helicities are summed over and continued to complex values in the usual Regge way.[8]

The starting equation is (see Fig. 1 for the definitions of the relevant variables)

$$A(s,t+i\epsilon)-A(s,t-i\epsilon)=\frac{i}{(2\pi)^2}\int_{m^2}^{\infty}\frac{dt_1}{2\pi i}\int_{m^2}^{\infty}\frac{dt_2}{2\pi i}\,\frac{(\alpha_1^+-\alpha_1^-)\alpha_1'(t_1)}{(l_1-\alpha_1^+)(l_1-\alpha_1^-)}\,\frac{(\alpha_2^+-\alpha_2^-)\alpha_2'(t_2)}{(l_2-\alpha_2^+)(l_2-\alpha_2^-)}$$
$$\times\int d^4p_1d^4p_2\,\delta(p_1{}^2-t_1)\delta(p_2{}^2-t_2)\delta(p_a+p_b-p_1-p_2)A^+_{n_1n_2}A^-_{n_1n_2}, \tag{2.1}$$

where $A^+_{n_1n_2}$ and $A^-_{n_1n_2}$ are the $a+b\to1+2$ and $1+2\to a'+b'$ scattering amplitudes, respectively, with $i\epsilon$ prescriptions:

$$A^{\pm}_{n_1n_2}\equiv A_{n_1n_2}(s,s_i,t\pm i\epsilon,t_1,t_2). \tag{2.2}$$

The t_1 and t_2 integrals are just generalizations which allow "1" and "2" to lie on Regge trajectories, and $\alpha_i^{\pm} \equiv \alpha_i(t_i \pm i\epsilon)$, with the limit $\epsilon \to 0$ understood.

For the two-Reggeon-cut contribution the relevant part comes from contributions where n_1 and n_2 are both positive or both negative, so with the definition

$$A^+_{n_1n_2}A^-_{n_1n_2} \equiv A^+_{n_1n_2}A^-_{n_1n_2} + A^+_{-n_1-n_2}A^-_{-n_1-n_2}, \tag{2.3}$$

only positive n_1 and n_2 need be considered when the summation over n_1 and n_2 is done.[8] The p_1 and p_2 integrations in (2.1) can be done by writing a dispersion relation for $A^+_{n_1n_2}A^-_{n_1n_2}$ in s_1 and s_2 for fixed s. The result is

$$A(s,t+i\epsilon) - A(s,t-i\epsilon) = \frac{-i}{(2\pi)^4\pi}\int \frac{dt_1dt_2ds_1ds_2\theta(R)}{(-Q)^{1/2}}\,\Omega_{n_1m_1}(\alpha_1)\Omega_{n_2m_2}(\alpha_2)\ln\left(\frac{x-z_1z_2+K^{1/2}}{x-z_1z_2-K^{1/2}}\right)$$
$$\times(\text{disc}_{s_1}\text{disc}_{s_2} + \text{disc}_{s_1}\text{disc}_{u_2} + \text{disc}_{u_1}\text{disc}_{s_2} + \text{disc}_{u_1}\text{disc}_{u_2})A^+_{n_1n_2}(s_1)A^-_{n_1n_2}(s_2), \tag{2.4}$$

where x, z_1, and z_2 are the t-channel center-of-mass-system scattering-angle cosines for $a+b\to a'+b'$, $a+b\to 1+2$, and $1+2\to a'+b'$ which are linearly related to s, s_1, and s_2 (u, u_1, and u_2 are the corresponding u variables), respectively, and

$$K = x^2 + z_1^2 + z_2^2 - 1 - 2xz_1z_2, \tag{2.5}$$

$$Q = -[\lambda(t,t_1,t_2)(s-u)^2 + \lambda(t,m_a^2,m_b^2)(s_1-u_1)^2 + \lambda(t,m_a^2,m_b^2)(s_2-u_2)^2$$
$$-2t(s-u)(s_1-u_1)(s_2-u_2) - \lambda(t,m_a^2,m_b^2)\lambda(t,m_a^2,m_b^2)\lambda(t,t_1,t_2)/t^2], \tag{2.6}$$

with

$$\lambda(a,b,c) \equiv a^2 + b^2 + c^2 - 2ac - 2ab - 2bc.$$

The integration over s_1 and s_2 is such that the associated discontinuities ($\text{disc}_s A(s) \equiv (1/2i)[A(s+i\epsilon) - A(s-i\epsilon)]$) of $A^+_{n_1n_2}(s_1)A^-_{n_1n_2}(s_2)$ are nonzero and such that

$$R = |x| - |z_1z_2| - [(z_1^2-1)(z_2^2-1)]^{1/2} > 0,$$

as indicated by the $\theta(R)$. For large s (or x) in this region the leading s behavior is given by

$$A^+_{n_1n_2}A^-_{n_1n_2} \to B^+_{n_1n_2}(s_1)B^-_{n_1n_2}(s_2)s^{n_1}s^{n_2},$$

where $B^+_{n_1n_2}(s_1)$ and $B^-_{n_1n_2}(s_2)$ do not depend on s. The expression $\Omega_{n_im_i}(\alpha_i)$ is defined by

$$\Omega_{n_im_i}(\alpha_i) = \frac{(\alpha_i^+ - \alpha_i^-)\alpha_i'(t_i)}{(n_i+m_i-\alpha_i^+)(n_i+m_i-\alpha_i^-)}, \tag{2.7}$$

where $m_i = l_i - n_i$ are non-negative integers. It is the poles at $n_i = \alpha_i^+$ for $m_i = 0$ that give the leading two-Reggeon contribution to $A(s,t+i\epsilon)$, as was shown in Ref. 8. For our purposes we may therefore make the replacement

$$\Omega_{n_im_i}(\alpha_i) \to \frac{\alpha_i'(t_i)}{n_i - \alpha_i^+}. \tag{2.8}$$

In addition to large s we are interested in $t \le 0$, so we do a Wick rotation on the t_1 and t_2 integration, that is, we make the replacement

$$\int \frac{dt_1dt_2}{(-Q)^{1/2}} \to i\int_{\lambda(t,t_1,t_2)\le 0} \frac{dt_1dt_2}{Q^{1/2}}. \tag{2.9}$$

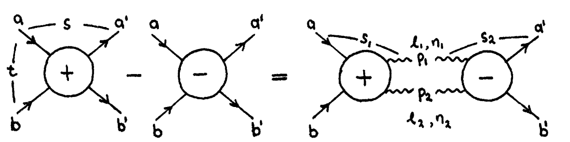

FIG. 1. The two-Reggeon contribution from the t-channel point of view.

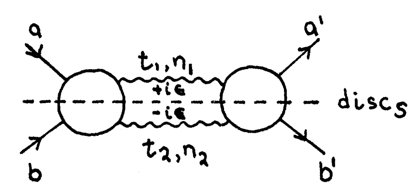

FIG. 2. The s-channel discontinuity with no Reggeon effects included.

Putting all this together we obtain

$$A(s,t)\equiv A(s,t+i\epsilon)$$
$$=\frac{1}{\pi(2\pi)^4}\int_{\lambda(t,t_1,t_2)\le 0}\frac{dt_1dt_2ds_1ds_2\,\theta(R)}{Q^{1/2}}\,\frac{\alpha_1'(t_1)}{n_1-\alpha_1^+}\,\frac{\alpha_2'(t_2)}{n_2-\alpha_2^+}\ln\left(\frac{x-z_1z_2+K^{1/2}}{x-z_1z_2-K^{1/2}}\right)$$
$$\times[\mathrm{disc}_{s_1}B^+_{n_1n_2}(s_1)+\mathrm{disc}_{u_1}B^+_{n_1n_2}(s_1)][\mathrm{disc}_{s_2}B^-_{n_1n_2}(s_2)+\mathrm{disc}_{u_2}B^-_{n_1n_2}(s_2)]$$
$$\times\tfrac{1}{2}[s^{n_1}+\tau_1(-s)^{n_1}]\tfrac{1}{2}[s^{n_2}+\tau_2(-s)^{n_2}]. \tag{2.10}$$

In order to obtain the leading two-Reggeon contribution, non-negative n_1 and n_2 must be summed over and the infinite sums must be converted to contour integrals, which are then pulled to the left, picking up the poles at $n_1=\alpha_1^+$ and $n_2=\alpha_2^+$. We have introduced signatures τ_1 and τ_2 $(=\pm 1)$, which separate the sums over even and odd n_1 and n_2 so that these continuations can be done.

This completes our preliminary discussion, which contains nothing new but which serves to introduce our notation and (we hope) to make the interpretation of our results in the next section clear.

III. COMPONENTS OF THE TWO-REGGEON CONTRIBUTION

One way of obtaining components of the s-channel discontinuity of the two-Reggeon contribution is to interchange orders of taking the discontinuity in s of (2.10) and summing over and continuing in n_1 and n_2. Taking the discontinuity first and then continuing in n_1 and n_2 corresponds to neglecting the effects of the Reggeons contributing to the discontinuity. As we shall see, this will give the s-channel multiperipheral result.[1,8,10] Similarily, continuing in one helicity, taking the discontinuity, and then continuing in the other helicity takes into account the effects of one of the Reggeons contributing to the discontinuity. Finally, continuing in both helicities and then taking the discontinuity will give the total discontinuity as shown in Ref. 8.

For integer n_1 and n_2, the discontinuity in s of $A(s,t)$ in (2.10) comes solely from the

$$\ln\left(\frac{x-z_1z_2+K^{1/2}}{x-z_1z_2-K^{1/2}}\right)$$

term and we obtain[8]

$$\mathrm{disc}_sA(s,t)=\frac{1}{(2\pi)^4}\int_{\lambda(t,t_1,t_2)\le 0}\frac{dt_1dt_2ds_1ds_2\,\theta(R)}{Q^{1/2}}\,\frac{\alpha_1'(t_1)\alpha_2'(t_2)}{(n_1-\alpha_1^+)(n_2-\alpha_2^+)}$$
$$\times[\mathrm{disc}_{s_1}B^+_{n_1n_2}(s_1)\mathrm{disc}_{s_2}B^-_{n_1n_2}(s_2)+\mathrm{disc}_{u_1}B^+_{n_1n_2}(s_1)\mathrm{disc}_{u_2}B^-_{n_1n_2}(s_2)]$$
$$\times\tfrac{1}{2}[s^{n_1}+\tau_1(-s-i\epsilon)^{n_1}]\tfrac{1}{2}[s^{n_2}+\tau_2(-s+i\epsilon)^{n_2}], \tag{3.1}$$

where we have added $\pm i\epsilon$ in the $(-s)^{n_i}$ terms to take into account the meaning of taking the discontinuity (see Fig. 2). Strictly speaking, the terms multiplying $\mathrm{disc}_{s_1}B^+_{n_1n_2}(s_1)\mathrm{disc}_{s_2}B^-_{n_1n_2}(s_2)$ and $\mathrm{disc}_{u_1}B^+_{n_1n_2}(s_1)\mathrm{disc}_{u_2}B^-_{n_1n_2}(s_1)$ should have opposite $i\epsilon$ prescriptions, but we can take that into account by symmetrizing the s-dependent factors in n_1 and n_2.

We can now sum over n_1 and n_2 on the right-hand side of (3.1), replace the sums by contour integrals, and then pull the contours to the left, picking up the poles at $n_1=\alpha_1\equiv\alpha_1^+$ and $n_2=\alpha_2\equiv\alpha_2^+$. This gives

$$[\mathrm{disc}_s]^PA(s,t)=\frac{\pi^2}{8(2\pi)^4}\int_{\lambda(t,t_1,t_2)\le 0}\frac{dt_1dt_2ds_1ds_2\,\theta(R)}{Q^{1/2}}\,\alpha_1'(t_1)\alpha_2'(t_2)$$
$$\times[\mathrm{disc}_{s_1}B^+_{\alpha_1\alpha_2}(s_1)\mathrm{disc}_{s_2}B^-_{\alpha_1\alpha_2}(s_2)+\mathrm{disc}_{u_1}B^+_{\alpha_1\alpha_2}(s_1)\mathrm{disc}_{u_2}B^-_{\alpha_1\alpha_2}(s_2)]$$
$$\times(\xi_1\xi_2^*+\xi_1^*\xi_2)s^{\alpha_1+\alpha_2}, \tag{3.2}$$

where

$$\xi_i=\frac{\tau_i+e^{-i\pi\alpha_i}}{\sin\pi\alpha_i}$$

and $[\mathrm{disc}_s]^P$ denotes this particular way of taking the discontinuity. Since most of the factors in (3.2) will occur over and over again we write (3.2) symbolically as

$$[\mathrm{disc}_s]^PA(s,t)\sim(\xi_1\xi_2^*+\xi_1^*\xi_2). \tag{3.3}$$

With appropriate normalizations we see that (3.2) is just the result obtained from an s-channel-unitarity multiperipheral approach[1,8,10] and gives a positive leading contribution to the two-Pomeron cut, for example. This then supports our interpretation that $[\mathrm{disc}_s]^D$ corresponds to neglecting the effects of 1 and 2 being Reggeons.

Let us now calculate $[\mathrm{disc}_s]^{R_1}$ corresponding to (i) summing over n_1, continuing in n_1, and picking up the pole at $n_1=\alpha_1^+$, (ii) taking the discontinuity in s, and, finally, (iii) summing over n_2, continuing in n_2, and picking up the pole at $n_2=\alpha_2^+$. Step (i) gives

$$A(s,t)=\frac{-1}{4(2\pi)^4}\int_{\lambda(t,t_1,t_2)\le 0}\frac{dt_1dt_2ds_1ds_2\theta(R)}{Q^{1/2}}\frac{\alpha_1'(t_1)\alpha_2'(t_2)}{n_2-\alpha_2^+}$$
$$\times\ln\left(\frac{x-z_1z_2+K^{1/2}}{x-z_1z_2-K^{1/2}}\right)[\mathrm{disc}_{s_1}B^+_{\alpha_1n_2}(s_1)+\mathrm{disc}_{u_1}B^+_{\alpha_1n_2}(s_1)]$$
$$\times[\mathrm{disc}_{s_2}B^-_{\alpha_1n_2}(s_2)+\mathrm{disc}_{u_2}B^-_{\alpha_1n_2}(s_2)]\xi_1s^{\alpha_1}[s^{n_2}+\tau_2(-s)^{n_2}]. \tag{3.4}$$

Taking the discontinuity in s, we find that there are two contributions, one from disc_s of the logarithm and another from $\mathrm{Im}\,\xi_1$. The first term corresponds to a contribution to the two-Reggeon cut, and the other term, first found in Ref. 8, is an additional contribution whose interpretation is not yet clear. Since we want to compare with other results which do not take this additional contribution into account, we shall leave it out for the purposes of this calculation. We thus obtain

$$\mathrm{disc}_sA(s,t)=\frac{-\pi}{8(2\pi)^4}\int_{\lambda(t,t_1,t_2)\le 0}\frac{dt_1dt_2ds_1ds_2\theta(R)}{Q^{1/2}}\alpha_1'(t_1)\alpha_2'(t_2)$$
$$\times[\mathrm{disc}_{s_1}B^+_{\alpha_1n_2}(s_1)\mathrm{disc}_{s_2}B^-_{\alpha_1n_2}(s_2)+\mathrm{disc}_{u_1}B^+_{\alpha_1n_2}(s_1)\mathrm{disc}_{u_2}B^-_{\alpha_1n_2}(s_2)]$$
$$\times s^{\alpha_1}(\mathrm{Re}(\xi_1)\{[s^{n_2}+\tau_2(-s-i\epsilon)^{n_2}]+[s^{n_2}+\tau_2(-s+i\epsilon)^{n_2}]\}$$
$$+\delta i\,\mathrm{Im}(\xi_1)\{[s^{n_2}+\tau_2(-s-i\epsilon)^{n_2}]-[s^{n_2}+\tau_2(-s+i\epsilon)^{n_2}]\}), \tag{3.5}$$

where we have again symmetrized in "1" and "2" so that "2" is both above and below the cutting in s, and also added a term which is strictly zero when n_2 is an integer; since the two canceling terms must be continued separately we must allow for such a term. At this stage the constant δ is arbitrary, but it is clear that it cannot depend on the form of $B^+_{\alpha_1\alpha_2}(s_1)$ or $B^+_{\alpha_1\alpha_2}(s_2)$, so we can determine it by using a particular model for these amplitudes. Finally, with step (iii) we obtain

$$[\mathrm{disc}_s]^{R_1}A(s,t)\sim 2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)-2\delta\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2). \tag{3.6}$$

Similarly, it can be shown that

$$[\mathrm{disc}_s]^{R_2}A(s,t)\sim 2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)-2\delta\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2). \tag{3.7}$$

The full discontinuity, again neglecting the additional terms found in Ref. 8, is easily found to be given by

$$[\mathrm{disc}_s]^FA(s,t)\sim 2\,\mathrm{Re}(\xi_1\xi_2). \tag{3.8}$$

To make comparisons easier we summarize our results in one place:

$$\begin{aligned}
&[\mathrm{disc}_s]^DA(s,t)\sim\xi_1\xi_2^*+\xi_1^*\xi_2=2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)+2\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2),\\
&[\mathrm{disc}_s]^{R_1}A(s,t)\sim 2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)-2\delta\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2),\\
&[\mathrm{disc}_s]^{R_2}A(s,t)\sim 2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)-2\delta\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2),\\
&[\mathrm{disc}_s]^FA(s,t)\sim 2\,\mathrm{Re}(\xi_1\xi_2)=2\,\mathrm{Re}(\xi_1)\mathrm{Re}(\xi_2)-2\,\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2).
\end{aligned} \tag{3.9}$$

It is now easy to see (for example) that, when α_1 and α_2 are Pomeron (P) trajectories with $\tau=+1$ and $\alpha_P(0)=1$, $[\mathrm{disc}_s]^D$ and $[\mathrm{disc}_s]^F$ have opposite sign, since for the leading contribution $\mathrm{Re}(\xi_P)\to 0$ and $\mathrm{Im}(\xi_P)\to -1$.

The different discontinuities in (3.9) are clearly not independent. For example, $[\mathrm{disc}_s]^{R_1}$ or $[\mathrm{disc}_s]^{R_2}$ will also include $[\mathrm{disc}_s]^D$, and the full discontinuity will include all the others. If "1" and "2" are real particles, that is, if $\mathrm{Im}(\xi_1)=\mathrm{Im}(\xi_2)=0$, all the discontinuities are equal, so it is their differences which correspond to the pure Reggeon effects.

The effects of one Reggeon are therefore given by

$$\mathrm{disc}_{R_1}=[\mathrm{disc}_s]^{R_1}-[\mathrm{disc}_s]^D\sim-2(1+\delta)\mathrm{Im}(\xi_1)\mathrm{Im}(\xi_2), \tag{3.10}$$

are similarly

$$\text{disc}_{R_2} = [\text{disc}_s]^{R_2} - [\text{disc}_s]^{D} \sim -2(1+\delta)\text{Im}(\xi_1)\text{Im}(\xi_2). \qquad (3.11)$$

For the two-Reggeon effects we obtain

$$\text{disc}_{R_1 R_2} = [\text{disc}_s]^{F} - \text{disc}_{R_1} - \text{disc}_{R_2} - [\text{disc}_s]^{D} \sim 4\delta\,\text{Im}(\xi_1)\text{Im}(\xi_2), \qquad (3.12)$$

so if we identify disc_{R_1} and disc_{R_2} as the contributions from cutting through the Reggeons R_1 and R_2, respectively, and $\text{disc}_{R_1 R_2}$ as the contribution from cutting through both Reggeons, we obtain for the two-Pomeron cut the relative weightings

$$-1 = 1 - 2(1+\delta) + 2\delta. \qquad (3.13)$$

It now only remains to determine δ. This can be done by appealing to the Mandelstam graphs for $B^{\pm}_{\alpha_1\alpha_2}$ in weakly coupled $\lambda\phi^3$. It is known that this model satisfies all the properties leading to (2.10) and can thus be used to determine δ. It has been shown[1] that this model does satisfy the AGK cutting rules,

$$-1 = 1 - 4 + 2, \qquad (3.14)$$

and thus $\delta = 1$. The AGK cutting rules are therefore valid in general, since, as we already stated, δ does not depend on $B^{\pm}_{\alpha_1\alpha_2}$.

IV. CONCLUSION

Using a t-channel approach to the two-Reggeon cut, we have suggested a way of obtaining different components for the s-channel discontinuity. We have obtained three components, corresponding to

(i) a component not including the Reggeon effects,
(ii) the effects coming from one Reggeon, and
(iii) the effects coming from both Reggeons.

If we identify these components with the contributions obtained from s-channel calculations,[1,2] namely,

(i) discontinuity through no Reggeons (diffractive),
(ii) discontinuity through one Reggeon (absorbed multiperipheral), and
(iii) discontinuity through both Reggeons (polyperipheral),

we obtain the relative weightings for the two-Pomeron cut as $-1 = 1 - 4 + 2$, in agreement with the AGK relative weightings. This then provides a model-independent confirmation of the AGK cutting rules, given that there is at least one model[1,3] which satisfy these rules.

The model independence of δ in Eq. (3.5) is clear since it only affects the s-dependent factors and thus cannot be affected by the particular form of $B^{\pm}_{\alpha_1\alpha_2}$. Of course, it is possible for the coefficient of the s-dependent part, involving integrals over discontinuities of Reggeon-particle amplitudes, to be zero. This will happen, for example, in a planar two-ladder model, but this does not affect δ. In general, the strength of the two-Reggeon cut will be model dependent, but the relative weightings of the different contributions will be in the ratio (3.14).

For completeness, we should mention that other types of terms which are zero for integer n_2 are possible in (3.5), but these can be shown to be zero, again by appealing to a particular model.

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