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Energy dependence of the ρ Regge trajectory*

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If recent ideas on the renormalization of the Pomeron due to particle-production thresholds are coupled with the exchange-degeneracy constraints of the dual topological expansion, if follows that the nonvacuum Reggeons must also be renormalized. We apply this result to the ρ trajectory.

There has been much interest in exploring the consequences of the dual topological expansion[1-3] for the energy dependence of hadronic total cross sections. One possibility that has received considerable attention is that the total cross sections at moderately high energies are controlled by an "embryonic" Pomeron with intercept below 1, and hence are decreasing functions of energy in this region.[3,4] At higher energy, as thresholds for new kinds of states (strange particles and/or baryons) open up, the decrease of the cross section will presumably cease or even be reversed.[5] By "threshold" here we mean not the absolute kinematic threshold, but rather an effective threshold resulting from the limited momentum transfers characteristic of multiperipheral models (and high-energy data). Such phenomena can be understood, in an imprecise fashion, in terms of an effective Pomeron trajectory that is itself an increasing function of energy, where one regards the various threshold effects as "renormalizing" the Pomeron upwards.[6,7]

In this note we point out that, under the same assumptions, these same renormalization effects should apply to ordinary Regge trajectories, such as the ρ; this would imply that, for example, the power with which the difference of π^+p and π^-p total cross sections fall with energy should itself decrease with energy. We estimate, within the framework of the assumptions referred to above, that the rise of the effective ρ trajectory due to the unfreezing of strangeness and baryon number should be comparable to the rise of the effective Pomeron intercept, or about 0.2 units of angular momentum. Although we are unable either to confirm or refute this rise in the ρ intercept with presently available data on total cross sections, we suggest that in the future a study of the energy dependence of processes governed by ordinary Regge trajectories might furnish an important test of the idea that the energy dependence of the effective Pomeron trajectory can be understood in terms of the production of new kinds of particles rather than, for example, the effects of terms higher than the cylinder term in the topological expansion.

We assume, following Refs. 1–3, that the sum of planar diagrams in which only nonstrange mesons are produced is dominated, at high energy, by the exchange of exchange-degenerate trajectories; let us denote the common intercept of the f and the ρ trajectories in this approximation by $\alpha_\rho^{(0)}$. If one includes cylinder terms (but still considers only the production of nonstrange mesons), the exchange degeneracy is broken; the leading $I=0$, $C=+$ trajectory is pushed up beyond $\alpha_\rho^{(0)}$ to a value we denote by $\alpha_P^{(0)}$, while the ρ trajectory remains at $\alpha_\rho^{(0)}$.

Having defined $\alpha_P^{(0)}$ as the intercept of the Pomeron governed by events in which only nonstrange mesons are produced, we now follow the authors of Refs. 4 and 5 to assume that if one includes diagrams (both planar and cylinder) in which a pair of strange particles are produced, the Pomeron intercept is promoted to a higher value, denoted by $\alpha_P^{(1)}$. Similarly, the production of pairs of baryons[8] raises the Pomeron to a yet higher value, $\alpha_P^{(2)}$. Empirically, strange-pair production is negligible for $s \lesssim 50$ GeV^2 while baryon-pair production is negligible for $s \lesssim 200$ GeV^2,[9] so each of the three Pomerons describes total cross sections well over a different limited range of energy. The value of $\alpha_P^{(2)}$ is given by the solution of

$$J-\alpha_P^{(0)}-g_K{}^2 e^{-b_K J}-g_B{}^2 e^{-b_B J}=0 \,, \tag{1}$$

where $g_K{}^2$ represents the squared coupling of a bare Reggeon (one having no intermediate states involving pairs of strange particles or baryons) to a $K\bar{K}$ system, b_K is an effective threshold (in rapidity units) for the production of $K\bar{K}$ pairs in hadronic collisions, and g_B and b_B are the corresponding parameters for $B\bar{B}$ pairs. The origin of formulas such as (1) is discussed in detail in Refs. 5, 6, and 10. The fit by Dash and Jones[4] yields the following estimates: $\alpha_P^{(0)}=0.85$, $\alpha_P^{(2)}=1.05$, and, if one neglects baryon production, $\alpha_P^{(1)}=1.0$.

What is the effect on the ρ intercept of these additional processes? Since the cylinder does not affect the ρ, the ρ trajectory in the planar + cylinder approximation is identical to the f trajectory in the planar approximation. For the latter, one can use the same reasoning on that outlined above for the Pomeron to see that production of strange mesons and baryons will push the intercept of the planar f to a value above the value $\alpha_\rho^{(0)}$; call this new value $\alpha_\rho^{(2)}$. Because the set of planar diagrams is assumed to be exchange-degenerate, the intercept of the ρ, ω, and A_2 trajectories must also be renormalized upward to the same value $\alpha_\rho^{(2)}$. Notice that this result need not be true outside of the dual topological expansion; only for elastic scattering and hence (at high energies) vacuum exchange does unitarity imply that the production of a new system gives a positive contribution to the absorptive part. *A priori*, threshold effects for nonvacuum Reggeons could have either sign.

The picture that emerges is thus the following: At energies sufficiently low so that only nonstrange mesons are produced, the energy dependence of amplitudes given by ρ exchange is $s^{\alpha_\rho(0)}$; above the effective threshold for $K\bar{K}$ production the effective ρ intercept rises to the value $\alpha_\rho^{(1)}$, and above the threshold for $B\bar{B}$ production the effective ρ rises again, to the value $\alpha_\rho^{(2)}$.

We can estimate the splitting between $\alpha_\rho^{(0)}$ and $\alpha_\rho^{(1)}$ and $\alpha_\rho^{(2)}$ by comparing with the Pomeron case. Just as in Eq. (1), the value of $\alpha_\rho^{(2)}$ is given by the solution of

$$J - \alpha_\rho^{(0)} - \bar{g}_K{}^2 e^{-b_K J} - \bar{g}_B{}^2 e^{-b_B J} = 0 \ , \qquad (2)$$

where $\bar{g}_K{}^2$ and $\bar{g}_B{}^2$ represent the planar couplings; $\alpha_\rho^{(1)}$ is the solution of (2) with $\bar{g}_B{}^2$ set equal to zero. To the extent that the planar and nonplanar couplings are the same, we would expect that[11]

$$\bar{g}_K{}^2 \cong \tfrac{1}{2} g_K{}^2, \quad \bar{g}_B{}^2 \cong \tfrac{1}{2} g_B{}^2 \ . \qquad (3)$$

It might seem, from Eqs. (1)–(3), that $\alpha_\rho^{(2)} - \alpha_\rho^{(0)} = \frac{1}{2}(\alpha_P^{(2)} - \alpha_P^{(0)})$; however, because $\alpha_\rho^{(0)} < \alpha_P^{(0)}$, the exponentials in Eq. (2) are larger than in Eq. (1), and this effect enhances the importance of $\bar{g}_K$ and $\bar{g}_B$. To the extent that this enhancement compensates for the factors of 2 in Eq. (3), we would expect that, roughly,

$$\begin{aligned} \alpha_\rho^{(1)} - \alpha_\rho^{(0)} &\approx \alpha_P^{(1)} - \alpha_P^{(0)} \ , \\ \alpha_\rho^{(2)} - \alpha_\rho^{(0)} &\approx \alpha_P^{(2)} - \alpha_P^{(0)} \ . \end{aligned} \qquad (4)$$

For example, if we adopt for the parameters in (1) the values suggested by Dash and Jones,[4] $g_K{}^2 = 1.1$, $b_K = 2.0$, $g_B{}^2 = 3.6$, $b_B = 3.8$, and take[12] $\alpha_\rho^{(0)} = 0.45$, then Eqs. (2) and (3) yield

$$\alpha_\rho^{(1)} = 0.61, \quad \alpha_\rho^{(2)} = 0.71 \ . \qquad (5)$$

It is well known that the difference of the $\pi^+ p$ and $\pi^- p$ total cross sections is well fitted by a single power of energy. In Fig. 1, we show these data,[13] fitted by these different powers of energy in the three different regions.[14] We would certainly not assert that an energy dependence of the effective trajectory, as given in Eq. (5), is in any way suggested by these data; we merely observe that it is compatible.

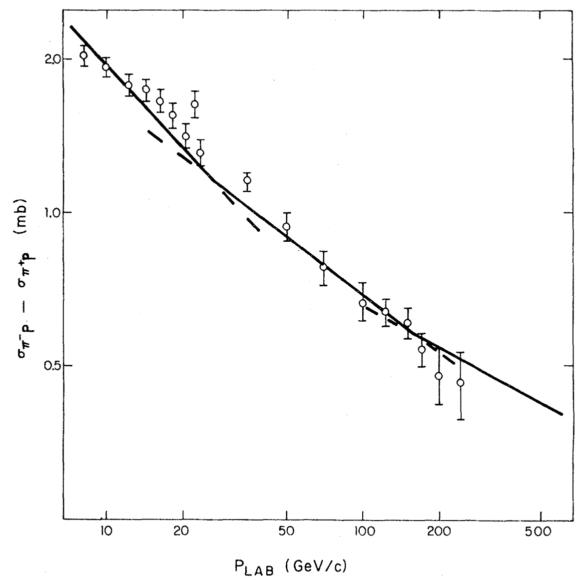

FIG. 1. The differences of the total cross sections for $\pi^- p$ and $\pi^+ p$ (from Ref. 13), together with a fit representing the powers in Eq. (5).

We conclude with a few comments:

(1) The ω trajectory, unlike the ρ, feels the effects of the cylinder. If the intercept of the ω, including the cylinder correction but not including strange particle or baryon exchange, is denoted by $\alpha_\omega^{(0)}$, then the high-energy intercept of the ω would be given by the solution of

$$J - \alpha_\omega^{(0)} - \bar{g}_K{}^2 e^{-b_K J} - \bar{g}_B{}^2 e^{-b_B J} = 0 \ . \qquad (6)$$

Since $\alpha_\omega^{(0)} < \alpha_\rho^{(0)}$, we see from Eqs. (2) and (6) that the shift in the ω intercept as the energy increases (that is, $\alpha_\omega^{(2)} - \alpha_\omega^{(0)}$) should be very slightly bigger than the shift in the ρ intercept ($\alpha_\rho^{(2)} - \alpha_\rho^{(0)}$).

(2) Independent of the details of the parametrization, we expect the effective ρ intercept to increase with energy, just as does the effective Pomeron intercept [in fact, to the extent that the estimates (4) are valid, we would expect $(\sigma_{\pi^+ p} - \sigma_{\pi^- p})/(\sigma_{\pi^+ p} + \sigma_{\pi^- p})$ to be very nearly a pure power of energy]. On the other hand, most absorption models with increasing total cross sections would imply that the effective intercept of the ρ is a decreasing function of energy. Thus a very accurate experimental determination of the direction of the energy dependence of charge-exchange scattering might enable us to ascertain the

relative importance of the unfreezing of new types of particles and higher terms in the topological expansion.

(3) Similar reasoning should apply to the threshold renormalization of the π trajectory. However, because of its low intercept, it would be extremely difficult to extract the effective α_π at high energies, and the effective ρ trajectory provides an easier test of the ideas in this paper.

(4) There has been speculation that there may be more new quantum numbers (charm, etc.) waiting to be unfrozen. If so, we would replace Eq. (1) by

$$0=J-\alpha_P^{(0)}-\sum_i g_i{}^2 e^{-b_i J}-g_B{}^2 e^{-b_B J}\ , \tag{7}$$

where i labels the flavors to be unfrozen ($i=1$ is strangeness, $i=2$ is charm, $i=3$ is ?). If higher symmetries linking these new flavors are broken only by mass differences, we would expect the various g_i to be equal to a common g (see Ref. 15); the b_i are proportional to the logarithm of the mass of the new particles. Now if the solution of Eq. (7) cannot be arbitrarily large (for example, if it cannot be larger than the value of the Pomeron intercept at which the Reggeon calculus breaks down[16]), then there will be a lower limit on the mass scale of any new particles. This reasoning might eventually lead to an understanding of the very large mass scales of the charmed particles and the even more exotically flavored particles.

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[8]Unfortunately, a detailed treatment of baryons in the topological expansion is nonexistent, owing to the ancient problems pursuant to baryon-antibaryon systems in dual models. We assume that the renormalization effects due to the production to $B\bar{B}$ pairs have the same structure as meson production.

[9]M. Antinucci *et al.*, Lett. Nuovo Cimento 8, 121 (1973).

[10]J. Koplik, Nucl. Phys. B82, 93 (1974).

[11]The analogous relation for nonstrange mesons must be approximately true, since it is that relation that gives $\alpha_P^{(0)}\sim 1$.

[12]This value was chosen to yield the best fit, below.

[13]K. J. Foley *et al.*, Phys. Rev. Lett. 19, 330 (1967); A. S. Carroll *et al.*, Phys. Lett. 61B, 303 (1976).

[14]Of course the transitions between the three regions should really be smooth; the corners are an artifact of the kinematic approximations leading to Eq. (1).

[15]This assumption (extended to the production of ordinary mesons) gives reasonable agreement with relative π/K multiplicities observed at high energy.

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