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Color-symmetry breaking and the new mesons*

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It is assumed that the ψ (J) and ψ' mesons at 3.1 and 3.7 GeV are color excitations, and that color symmetry is broken badly. A simple symmetry-breaking mechanism leads to the prediction of a total of nine vector and nine pseudoscalar mesons in the 3–4 GeV region. This is in striking contrast with the predictions of both the standard color-excitation model and the charm model of the ψ (J) and ψ'. An estimate is made of the mass of a predicted baryon-resonance multiplet. Several other predictions of the model are compared with those of the charm model and with experiment.

I. BASIC ASSUMPTIONS

Recently it was suggested that the SU(3) color symmetry in the colored-quark model is broken badly, but that an SU(2) subgroup is exact in strong interactions.[1] It was shown that this leads to a possible explanation of the apparent absence of certain baryon-resonance multiplets that are predicted in the standard quark model.

In the model of R1, the observed lightest hadrons are predominantly color-singlet, but have some octet components. One expects predominantly color-octet states to be found at sufficiently high masses. An obvious possibility is that the ψ (J) and ψ' mesons at 3.1 and 3.7 GeV are states with a different color symmetry from that of the lighter mesons. I make such an assumption in this paper, and assume further that the color wave functions for the two types of meson states correspond more closely to $(r\overline{r}+w\overline{w})/\sqrt{2}$ and $b\overline{b}$ than to color-singlet and color-octet states. The symbols r, w, and b denote red, white, and blue quarks. The question of which type of state corresponds to the ψ (J) and ψ', and which to ordinary mesons, will be discussed later. The narrow widths for both nonradiative and radiative mesonic decays of the ψ (J) may result from the Zweig-Iizuka rule, which forbids processes involving disconnected quark diagrams.[2] In this respect, the model resembles the charmed-quark model of the new mesons.

The main purpose of this note is to point out that if the forces between quarks and antiquarks are carried by a color octet of vector gluons, and color symmetry is broken strongly, the predicted meson spectrum is very different from that given before in the literature, either in charm or color-excitation models of the ψ (J) and ψ'. One does not expect a complete color octet of mesons. In order to see this, let us suppose that an important term in the $q\overline{q}$ (quark-antiquark) potential is symmetric in color SU(3). The expectation value of this potential is given by the expression

$$V_8[C(x)-C(q)-C(\overline{q})], \tag{1}$$

where V_8 is a positive energy, $C(i)$ is the eigenvalue of the quadratic Casimir operator of color SU(3) for the representation i, and x is the $q\overline{q}$ representation.[1] The values of this expression for singlet and octet states are $-\frac{8}{3}V_8$ and $\frac{1}{3}V_8$, respectively. This potential is repulsive in the octet state so that there should be no bound states corresponding to color I spin (I') or color hypercharge (Y') different from zero, unless the symmetry-breaking potential contains a large term that is attractive for such states. On the other hand, the $I'=Y'=0$ states under consideration are singlet-octet admixtures. Suppose they are of the types

$$\chi_{rw}=(r\overline{r}+w\overline{w})/\sqrt{2}=(\tfrac{2}{3})^{1/2}\chi_1+(\tfrac{1}{3})^{1/2}\chi_8\,,\qquad \chi_b=(b\overline{b})=(\tfrac{1}{3})^{1/2}\chi_1-(\tfrac{2}{3})^{1/2}\chi_8\,, \tag{2}$$

where χ_1 and χ_8 are color-singlet and color-octet wave functions. The expectation value of Eq. (1) is negative for both χ_{rw} and χ_b, so they both may be bound even though pure octet states are unbound.

Therefore, one expects the ψ (J) to be part of a V (vector) meson nonet under ordinary SU(3). The nine states should have approximately the same color wave function and mass differences comparable to those of the ρ nonet. In contrast, the color-excitation model with exact color symmetry predicts 72 V mesons in the 3–4 GeV region.[3] In the charm model the predictions are still different. If there are n heavy quarks, one expects n^2 V mesons heavier than 3 GeV and $6n$ lighter V mesons containing only one heavy quark. In each of these models a pseudoscalar-meson multiplet is predicted along with each V multiplet.

In Sec. II of this paper a simple model of the quark-quark interactions is made, and some hadron masses are used to determine approximately the parameters of the model. It is shown in Sec. II B that these parameters lead to the prediction of the approximate mass of a baryon-resonance multiplet. In Sec. III predictions concerning various ψ particles are compared to experiment and to the predictions of the conventional charm model.

II. QUARK MASS AND INTERACTION PARAMETERS

A. Values of the parameters

By using a simple model and experimental hadron masses we can make a crude estimate of the masses of the quarks and a prediction of the approximate mass of one of the baryon-resonance multiplets that is predicted in the standard quark model. We consider only the lightest hadron in any multiplet of ordinary SU(3), assuming this hadron to be made primarily of the lightest quarks in the appropriate SU(3) triplet. For example, we will use the pion mass, and hence consider only the nonstrange members of the quarks that dominate the wave functions of the pion nonet. The internal kinetic energy of the quarks is neglected, so that the Hamiltonian may be written $H=M+V$, where M is the quark mass operator, and V is the gluon-exchange potential.

We consider the meson states first. The gluon-exchange potential is taken to be of the form of R1; the potential consists of a color-symmetric term and a symmetry-breaking term that is symmetric in the SU(2) of the colors red and white. The expectation value of the symmetric term is given in Eq. (1). The expectation value of the symmetry-breaking term has the analogous form $V_3[C'(x) - C'(q) - C'(\bar{q})]$, where $C'(i)$ is the eigenvalue of the Casimir operator of red-white SU(2) for the representation i.[4]

I allow the mass m_b of the blue quark to be different from the common mass m_{rw} of the red and white quarks. It is convenient to use the representation of the states χ_{rw} and χ_b of Eq. (2), since the quark mass operator is diagonal in this representation. The Hamiltonian matrix is

$$H=\begin{pmatrix} 2m_{rw}-\frac{5}{3}V_8-\frac{3}{2}V_3 & -\sqrt{2}\,V_8 \\ -\sqrt{2}\,V_8 & 2m_b-\frac{2}{3}V_8 \end{pmatrix}. \tag{3}$$

It is clear from the large ρ-π mass difference that the spin dependence of the forces between quarks is appreciable. I assume that the V_8 and V_3 parameters apply to the states of the most favored quark spin and orbital angular momentum. The eigenvalues of H are identified with the masses of the π and ψ (J).

In order to estimate the parameters I assume that a similar Hamiltonian, of the form $H=M+V$, applies to the nucleon. It is shown in R1 that even if the color-symmetry breaking of V is appreciable, the wave functions of the ground-state baryon SU(6) multiplet are expected to be almost pure color singlets. Each baryon is composed of one red, one white, and one blue quark. If the three quarks in the nucleon are labeled α, β, and γ, the potential energy of the nucleon is $V=V_{\alpha\beta}+V_{\beta\gamma}+V_{\gamma\alpha}$. The expectation value of each term V_{ij} is a sum of color-symmetric (V_8) and symmetry-breaking (V_3) parts. If the average separation between constituents in the nucleon is the same as in mesons, then $V^8_{ij}=V_8[C(x_{ij})-C(i)-C(j)]$, as in Eq. (1). A similar equation applies to the V_3 term. The nucleon mass is given by

$$m_N=2m_{rw}+m_b-4V_8-\tfrac{3}{2}V_3-E\,, \tag{4}$$

where $E=0$ if the average separation between constituents is the same in baryons and mesons. Since a three-particle state is expected to be more tightly bound, I assume that the extra binding energy E is positive but not larger than a few hundred MeV.

I define $m'_{rw}=m_{rw}-\frac{3}{4}V_3$. If the two eigenvalues of Eq. (3) are identified with the π and ψ (J) mesons, and Eq. (4) is used, one may determine m'_{rw}, m_b, V_8, and the π and ψ (J) wave functions in terms of E. The two resulting solutions for V_8 are, in GeV,

$$V_8=0.66-0.44E \pm(14.4+15.9E-5.33E^2)^{1/2}\,. \tag{5}$$

If the upper sign in this equation is taken the mixing between χ_{rw} and χ_b is so great as to contradict the hypothesis made at the beginning of this paper. Consequently, I take the lower sign. This corresponds to the pion and other light mesons being made primarily of red and white quarks and antiquarks.

Very little error is introduced if only linear terms in E are kept. The parameters are then (in GeV)

$$V_8=0.40-0.59E\,, \qquad m'_{rw}=0.465-0.66E\,,\quad m_b=1.63-0.02E\,. \tag{6}$$

The wave function of the ψ (J) is

$$\chi_J=(0.98+0.06E)\chi_b-(0.20-0.30E)\chi_{rw}\,, \tag{7}$$

while χ_π is orthogonal to χ_J. If E is in the range 300–600 MeV, there is very little mixing between χ_b and χ_{rw}.

The meaning of these results is clear. Since V_8 is the only term in the Hamiltonian that mixes χ_b and χ_{rw}, it must be small if the mixing is small. The mass of the ψ (J) is not much smaller than the mass of two blue quarks. The mass of the red and white quarks is not determined because it enters the equations together with V_3. If the extra binding energy E is larger than V_8, it is reasonable to assume that the larger portion of E results from the V_3 potential.

B. Prediction concerning baryon resonances

There are two, three-dimensional, internal degrees of freedom in the three-quark baryon states. The lowest quark-model level in which both degrees of freedom may be excited simultaneously is the second excited ($N=2$) level. Therefore, this level is important. The color-symmetric quark model predicts several multiplets at this level, corresponding to the three SU(6) representations $\underline{56}$, $\underline{70}$, and $\underline{20}$. At present, only multiplets corresponding to the $\underline{56}$ have been detected. The second internal degree of freedom has not been seen.

Of the predicted multiplets that have not been seen, that with the highest spins corresponds to total quark orbital angular momentum 2 and SU(6) multiplet $\underline{70}$. In R1 it was shown that strong color-symmetry breaking, of the type considered here, would lead to a predicted average mass of the ($\underline{70}$, $L=2$) multiplet larger than that of the observed ($\underline{56}$, $L=2$) multiplet. It was shown that the wave functions for both these multiplets would contain one red, one white, and one blue quark, and the expectation value for the symmetry-breaking potential would be the same. Thus the predicted mass difference between these two multiplets is proportional to V_8. The probabilities for the $\underline{56}$ and $\underline{70}$, $L=2$ states of R1 to be in a color singlet are $\frac{1}{2}$ and $\frac{1}{4}$, respectively. This leads to the predicted mass difference $m_{70}-m_{56}=\frac{3}{4}V_8$. Since the V_8 of Eq. (7) is fairly small for any reasonable value of E, the model predicts that the $\underline{70}$ states should be discovered.

III. OTHER ASPECTS OF THE MODEL

It is assumed that the e^+e^- decay of a V meson proceeds through a virtual photon, and that the contribution of each quark to the γ-V coupling is proportional to the product of the quark charge and the amplitude of the quark in the V wave function. If the ρ_0, ω, and φ mesons are pure color singlets, the predicted $\rho_0\to e^+e^- : \omega\to e^+e^- : \varphi\to e^+e^-$ amplitude squared ratios are 9 : 1 : 2. This is consistent with the corresponding observed partial widths of 6.5 ±1.2 keV, 0.75 ±0.20 keV, and 1.35 ±0.15 keV.[5] If these mesons are of the color type $(r\bar{r}+w\bar{w})/\sqrt{2}$ and the charge assignments are those of the Han-Nambu model,[6] the predicted ratios are 2 : 0 : 1 if the $(\mathcal{P},\mathfrak{N},\lambda)$ quark charges of the red and white triplets are (1, 0, 0) and (0,−1,−1); the predicted ratios are 1 : 1 : 0 if the $(\mathcal{P},\mathfrak{N},\lambda)$ charges of the red and white triplets are both (1 : 0 : 0). These are ruled out by the data. The simplest satisfactory charge assignment in the broken color-symmetry scheme is that of Gell-Mann,[7] in which the quark charges are $(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$, and are independent of color. This also leads to the 9 : 1 : 2 prediction.

If the Gell-Mann charge assignments are correct, the predicted ratio of the couplings of a photon to corresponding members of the χ_{rw} and χ_b V multiplets is 2. If the $\psi(3.1)$ and $\psi'(3.7)$ have the same color wave function and correspond respectively to the φ and ω under ordinary SU(3), the predicted $(\psi\to e^+e^-)/(\psi'\to e^+e^-)$ ratio is 2. This is consistent with the observed partial-width ratio (4.8 ±0.6 keV)/(2.1 ±0.3 keV).[8] This identification implies that for the ψ and ψ', the strange quark is lighter than the nonstrange quarks. This causes no difficulty in the model suggested here, since the ψ and ψ' are made of blue quarks and the lighter mesons of red and white quarks. The parameters m_{rw} and m_b refer to the nonstrange red and white quarks and the strange blue quark.

If the observed ψ'' at 4.1 GeV is also of the type $b\bar{b}$ and corresponds to the ρ_0 under ordinary SU(3), the predicted $\psi''\to e^+e^-$ decay partial width would be about 20 keV, rather than the observed value of ~4 keV.[9] This is not a satisfactory assignment for the $\psi''(4.1)$.

It should be noted that one cannot identify either the ordinary vector mesons or the ψ (J) with the color wave function $(r\bar{r}-w\bar{w})/\sqrt{2}$. Such a state contains no color-singlet piece and is not coupled directly to a photon if the Gell-Mann quark-charge assignments are made.

Experimentally, the $\pi_0\to\gamma\gamma$ decay parameter S is in the general region 0.45–0.6.[10] If the modified partial conservation of axial-vector current (PCAC) condition of Adler is used, the predicted S in a quark model is $S=\sum_j g_j Q_j^2$, where g_j is effectively the coupling of the π_0 to the quark j and Q_j is the quark charge.[10] In the color-symmetric quark model, with g_j determined from current algebra, the calculated S is $\frac{1}{2}$. If the pion has the two-color wave function χ_{rw} and the Gell-Mann charge assignments are correct, the calculated S is $\frac{1}{3}$. If the π_0 is a complicated mixture of χ_{rw} and χ_b, the current-algebra calculation of g_j is not reliable.

Another quantity used to test quark models is the ratio $R=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$. At high energies R is predicted to approach $\sum_j Q_j^2$. Above 5 GeV, R is consistent with a constant in the range 5–6.[9,11] This requires the existence of both color and more than one charmlike quantum number. If color and charm do both exist, there is no reason to assume *a priori* that the lowest observed excitations (i.e., the ψ and ψ') correspond to charm, unless one believes that color excitations cannot correspond to physical particles. If this latter is the case, and all particles are color singlets, it will be difficult to verify the validity of the color concept.

In the energy region 2.5–3.5 GeV [excluding the ψ (J) bump], the measured R is consistent with a constant in the range 2–3.[9,11] The value obtained by including the fractionally charged $\mathcal{P}, \mathcal{N}$, and λ quarks, each in three colors, is 2. If only red and white quarks are included, the value is reduced to $\frac{4}{3}$. Therefore, if one assumes that this energy region is asymptotic for the light quarks while heavy quarks make no contribution, the experimental data are consistent with exact color symmetry. They are consistent with the large symmetry breaking postulated here only if the blue quarks are light as far as R is concerned. This is not a strong argument against the present model, since we do not know at what energy and how abruptly a particular quark should contribute to the sum $\sum_i Q_i^2$.

In the charm scheme of the new hadrons, the ψ (J) is a singlet under ordinary SU(3). The predicted $[K\bar{K}^*(890)+\bar{K}K^*(890)]/(\pi\rho)$ branching ratio of the ψ (J) is approximately $\frac{4}{3}$. Experimentally, this ratio is $(0.55 \pm 0.09)/(1.3 \pm 0.3)$.[12] In the scheme of this paper this ratio is not predicted, since ψ (J) is a mixture of a singlet and octet of SU(3). However, if one neglects phase-space factors and identifies the $\psi(J)$ and ψ' with φ and ω states under ordinary SU(3), the $(\psi' \rightarrow \pi\rho)/[\psi \rightarrow (K\bar{K}^* + \bar{K}K^*)]$ partial-width ratio is predicted to be $\frac{3}{2}$. One expects a $\psi' \rightarrow \pi_0\rho_0$ partial width of about 190 eV. The present experimental upper limit is approximately 220 eV.[11]

The most important fact about the broken-color-symmetry interpretation of the ψ (J) is that the predicted meson spectrum is different from that of either the charm model or the weakly-broken-color-symmetry model.

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