

## Decay modes of the $e^+e^-$ resonances

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(Received 28 July 1975; revised manuscript received 8 December 1975)

Some order-of-magnitude considerations are presented on the decay processes of the new  $e^+e^-$  resonances, using a form of the  $SU(3)' \times SU(3)''$  or color model. The main problem for this model is radiative decay, which is reduced (but not necessarily below observable intensity levels) by two factors: concentration of most of the radiative strength in near-lying hadronic states around 3–4 GeV; and specific reduction of coupling constants for octet-singlet combinations in the color index. This reduction in  $g^2$  is of order  $10^{-2}$  to  $10^{-3}$ , as evidenced by the observed decays  $\psi(3.7) \rightarrow \psi(3.1) + (2\pi \text{ or } \eta)$ . This reduction appears to be consistent with the small photoproduction cross section for  $\psi$  mesons. Some discussion is sketched of colored pseudoscalar mesons. An anticipated decay mode is  $\psi(3.7) \rightarrow \pi^+ + \psi^\pm$ , where  $\psi^\pm$  is a charged counterpart of the  $\psi(3.1)$  at about the same mass. An alternative form of the color model (extreme  $\bar{\lambda}' \cdot \bar{\lambda}''$  coupling) is indicated to have slightly different dominant decay modes.

### I. INTRODUCTION AND REVIEW

The following note presents some order-of-magnitude considerations on the decay processes of the new  $e^+e^-$  resonances<sup>1</sup> at 3.1, 3.7, and 4.1 GeV. This is a major problem for any scheme of interpretation, as the first two resonances are very narrow while the third appears closer in width to what would ordinarily be expected.

The particular scheme used below is a form of the Han-Nambu model,<sup>2</sup> which has nine basic quarks described by the group  $SU(3)' \times SU(3)''$ . The second index is referred to as "color", and we take all ordinary particles to be color singlets in good approximation. A simple notation for any entity is  $(i, j)$ , where the first index refers to  $SU(3)'$  or ordinary  $SU(3)$  and the second refers to  $SU(3)''$  or color;  $i$  or  $j = 0$  denotes the singlet representation for the respective subgroups, and  $i$  or  $j = 1-8$  denotes the components of the octet representation. Thus, ordinary particles are all of the form  $(i, 0)$  and may be called "uncolored"; particles in the  $SU(3)''$  octet representation are  $(i, j)$  with  $j \neq 0$  and are called "colored."

We take the new resonances all to be vector mesons<sup>3</sup> and write them as  $\psi(3.1)$ ,  $\psi(3.7)$ , and  $\psi(4.1)$ . Their basic narrowness is ensured by the assumption that ordinary strong interactions do not allow  $SU(3)''$  octet-singlet transitions, so that the colored particles of lowest mass are stable against strong decay.

Our particular assignment<sup>4</sup> for the observed resonances is

$$\psi(3.1) = \psi(\omega, \rho^0), \quad \psi(3.7) = \psi(\phi, \rho^0), \quad \psi(4.1) = \psi(\omega, \phi). \quad (1)$$

Here the  $(i, j)$  notation has been adapted in an ob-

vious way from a numerical representation to one that is diagonal in electric charge and octet-singlet mixing. For the  $SU(3)'$  index we have taken ideal nonet mixing; for  $SU(3)''$  some octet-singlet mixing occurs by virtue of an assumed interaction  $H(0, 8)$ .

The interaction  $H(0, 8)$  is what makes the  $\psi(4.1)$  broad relative to  $\psi(3.1)$  and  $\psi(3.7)$  under the Eq. (1) assignment. Since  $H(0, 8)$  preserves  $I''$  spin, it will not convert  $\psi(i, \rho^0)$  into a color-singlet state and therefore it cannot directly affect the decay processes of  $\psi(3.1)$  and  $\psi(3.7)$ . On the other hand, it could mix  $\psi(\omega, 8)$  with uncolored vector-meson states  $\psi(\omega, 0)$ , which are then subject to rapid decay into ordinary particles. By this means, the  $\psi(4.1)$  can have a substantial width for ordinary hadron decay.

It is important to note that this mixing of  $\psi(4.1)$  mostly involves ordinary mesons in the same region, around 4 GeV. These ordinary states would, of course, acquire some color, but this is not currently observable. The main point is that the low-lying vector mesons  $\phi$  and  $\omega$  at masses around 0.8 to 1.0 GeV are still almost entirely uncolored. In simple perturbation terms, their color amplitude is  $H(0, 8)/(M_c - M_o)$ , where the colored-ordinary mass differences  $M_c - M_o \approx (4.1 - 1.0) \text{ GeV} \approx 3 \text{ GeV}$ , while for  $\psi(4.1)$  the effective  $M_c - M_o$  results from a sum over very many states, which in a statistical model<sup>5</sup> come mainly from a region within an average energy range  $T \sim 160 \text{ MeV}$ . This means that if

$$H(0, 8) \sim 10^2 \text{ MeV} \quad (2)$$

the intensity of mixing (amplitude squared) will be substantial for  $\psi(4.1)$ , but will be  $10^{-2}$  to  $10^{-3}$  for the ordinary  $\phi, \omega$  mesons.

The absence of appreciable  $SU(3)''$  mixing for

the ordinary  $\rho^0$ ,  $\phi$ , and  $\omega$  is attested to by the fact that their  $e^+e^-$  partial widths  $\Gamma_{ee}$  agree with the ratio predicted for an ideal  $SU(3)'$  nonet. If much  $SU(3)''$  mixing were present in ordinary mesons, the observed ideal ratios 9:2:1 would not hold for  $\Gamma_{ee}(\rho): \Gamma_{ee}(\phi): \Gamma_{ee}(\omega)$ . The assignment in Eq. (1) leads to predictions in satisfactory agreement<sup>6</sup> with the observed values for  $\Gamma_{ee}(3.1)$ ,  $\Gamma_{ee}(3.7)$ , and  $\Gamma_{ee}(4.1)$ .

## II. RADIATIVE DECAY

The first problem for the present scheme is radiative decay:

$$\psi \rightarrow \gamma + X, \quad (3)$$

where  $X$  is any configuration of low-lying ordinary hadrons. The electromagnetic current operator has elements  $J_\mu(0, 3)$  and  $J_\mu(0, 8)$ , which are responsible for creating the  $\psi$  mesons in  $e^+e^-$  annihilation according to vector-meson dominance. These same current operators should induce ( $j=3, 8$ )  $\rightarrow$  ( $j=0$ ) transition between hadronic states, leading to Eq. (3) for an allowed decay. To proceed by analogy with known cases, the partial width for  $\omega \rightarrow \gamma + \pi^0$  is of order 1 MeV; if we scale this  $M1$  radiation according to  $(E_\gamma^3/E_0^2) \approx E_0 = M$ , then

$$\Gamma_0(\psi \rightarrow \gamma + P) \approx 4 \text{ MeV}, \quad (4a)$$

where  $P$  is some low-lying, ordinary pseudoscalar state. If we now allow for other spin states and transition types, it seems reasonable to increase (4a) by an order of magnitude:

$$\Gamma_0(\psi \rightarrow \gamma + X) \approx 40 \text{ MeV}. \quad (4b)$$

For example, just adding with weights  $(2J+1)$  the spin-1 and spin-2 final states accessible from a vector meson by  $M1$  decay would provide Eq. (4a) with a factor of 9.

The matrix element (i.e., radial overlap) for  $\omega \rightarrow \gamma + \pi_0$  can be taken as roughly maximal. No stronger transition is known; and when it is used to generate (4b), the resulting estimate is approximately  $\alpha M$ , as perturbation theory would suggest for point particles with perfect overlap. This suggests that the radiative matrix elements into low-lying hadron states are mostly exhausted in some sum-rule sense by transitions among those states themselves. Correspondingly, the radial overlap of the  $\psi$  mesons will be small with low-lying states; the radiative sum rule for the  $\psi$  will be mostly concentrated in states of nearby mass, in the range 3–4 GeV.

The simplest modification of Eq. (4b) along these lines is to assume that the total matrix-element sum rule from a meson  $\psi$  is exhausted by the ordinary states statistically distributed in the neigh-

boring mass region. The density of these states varies as  $\exp(-E_\gamma/T)$ , where  $T \approx 160$  MeV; with an  $E_\gamma^3$  dependence appropriate to dominant dipole emission

$$\begin{aligned} \Gamma_0(\psi \rightarrow \gamma + Y) &= \Gamma_0(\psi \rightarrow \gamma + X) \int \exp(-E_\gamma/T) \frac{dE_\gamma}{T} \left(\frac{E_\gamma}{M}\right)^3 \\ &= 6 \left(\frac{T}{M}\right)^3 \Gamma_0(\psi \rightarrow \gamma + X) \\ &\approx 10^{-3} \Gamma_0(\psi \rightarrow \gamma + X) \approx 40 \text{ keV}. \end{aligned} \quad (4c)$$

Here  $Y$  represents ordinary hadron states in the 3–4 GeV region, and the mass  $M$  was the value of  $E_\gamma$  implicit in estimate (4b). A similar estimate<sup>7</sup> has been made independently.

Estimate (4c) is within the bounds of observation for  $\psi(4.1)$  and  $\psi(3.7)$ , and possibly also for  $\psi(3.1)$ . On the other hand, it probably represents an extreme value; a more balanced estimate probably lies somewhere between Eqs. (4b) and (4c). It seems likely, therefore, that some further inhibition exists for these radiative decay processes.

We introduce a specific assumption: namely, that the strong hadron coupling constant  $G_{188}^{188}$  is an order of magnitude smaller<sup>8</sup> than the ordinary  $G_{111}^{888}$ . Here  $G_{ABB}^{abb}$  refers to a three-hadron vertex where the  $SU(3)'$  representation of one hadron is  $a$  and that of the other two is  $b$ ; likewise for  $SU(3)''$  and the indices  $A$  and  $B$ . The ordering of indices does *not* imply any necessary association between  $a$  and  $A$  on a single particle. These are supposed to be bare coupling constants without modification by mixing terms like  $H(8, 0)$  and  $H(0, 8)$ , the effects of which must be considered explicitly.

Radiative decay of the  $\psi$  must proceed through the current operators  $J_\mu(0, 3)$  and  $J_\mu(0, 8)$  to carry off a color index. In the vector-dominance model these currents are carried by  $\psi(\omega, 3)$ ,  $\psi(\phi, 3)$ ,  $\psi(\omega, 8)$ , and  $\psi(\phi, 8)$ , so that all the  $G_{188}^{abb}$  must be small. Suppressing the index  $i$ , we have for the reduction of the emission vertex the factor  $h_\gamma = (G_{188}/G_{111})$  relative to ordinary interactions such as  $\omega \rightarrow \gamma + \pi^0$ ; the decay width is reduced by

$$h_\gamma^2 \approx 10^{-2} \text{ to } 10^{-3} \quad (5)$$

according to our assumption above. This magnitude is confirmed by the discussion in Sec. III below.

A reduction factor like (5) applied to Eq. (4c) would make radiative decay of the  $\psi$  quite negligible. Accepting a mean value between Eqs. (4b) and (4c), we have

$$\begin{aligned} \Gamma(\psi \rightarrow \gamma + Y) &= h_\gamma^2 \Gamma_0(\psi \rightarrow \gamma + Y) \\ &\approx 4 \text{ keV} \times 10^{+1.5}. \end{aligned} \quad (6)$$

Although this estimate is necessarily imprecise at

the present stage, it does indicate that radiative decay may be only a minor mode for all the  $\psi$  mesons. More generally, it may be difficult to distinguish because in the statistical model used in Eq. (4c) the average  $\gamma$ -ray energy is of order  $4T \approx 600$  MeV and is followed by a number of ordinary hadrons including  $\pi^0$  and  $\eta$  mesons, which also emit  $\gamma$ 's. The branching ratio will accordingly be small for single- $\gamma$  emission,  $\psi \rightarrow \gamma +$  (all charged). The problem of radiative decays of  $\psi$  particles has been discussed by many authors, including Greenberg,<sup>9</sup> who reviews various proposals to suppress colored-photon emission.

### III. DECAYS BETWEEN $\psi$

A decay mode open in principle to all but the lowest colored bosons is

$$\psi' \rightarrow \psi + X, \quad (7)$$

where  $X$  consists of ordinary bosons of low mass and  $\psi'$  is a colored boson above the minimum mass (which is 3.1 GeV on present observation). Such a process is in fact prominent for the  $\psi(3.7)$  as follows<sup>10</sup>:

$$\begin{aligned} \psi(3.7) &\rightarrow \psi(3.1) + (\pi^+\pi^-) \quad (32 \pm 4)\% \\ &\rightarrow \psi(3.1) + (\text{other}) \quad (25 \pm 9)\%. \end{aligned} \quad (8)$$

If we take<sup>10</sup> the total decay width as  $\Gamma(3.7) = 225 \pm 56$  keV, and the  $(\pi\pi)$  emission to be entirely in an  $I=0$  state, then the corresponding partial width is

$$\Gamma(\psi(3.7) \rightarrow \psi(3.1) + (\pi\pi)_0) \approx 110 \pm 30 \text{ keV}. \quad (9)$$

Again to proceed by comparison, the best-known analogy is

$$\Gamma(\rho' \rightarrow \rho + (\pi\pi)_0) \approx 300 \text{ MeV}, \quad (10)$$

where  $\rho'$  is the ordinary resonance around 1.6 GeV. The ratio of phase space for the respective cases in Eqs. (8) and (9) is about<sup>11</sup> 0.06, leaving an additional reduction factor of  $h^2 \approx 0.6 \times 10^{-2}$ . This we ascribe to a mechanism like that leading to Eq. (5), with  $G_{188}^{a88} = hG_{111}^{888}$ .

To see how this relates to Zweig's rule for transitions between  $(\phi, j)$  and  $(\omega, j)$ , we write out the relations (suppressing the index  $j$ )

$$\langle \phi | (i=0) | \omega \rangle = \sin\theta \cos\theta (G^{111} - G^{188}), \quad (11)$$

$$\langle \phi | (i=8) | \omega \rangle = (\cos^2\theta - \sin^2\theta)G^{188} - \sin\theta \cos\theta G^{888},$$

where  $\theta$  is the  $(\omega, \phi)$  mixing angle. For ideal mixing  $\sin\theta = (1/\sqrt{2})\cos\theta = 1/\sqrt{3}$ , and both of these relations vanish by virtue of

$$G^{111} = G^{188}, \quad (12a)$$

$$G^{188} = \sqrt{2}G^{888}, \quad (12b)$$

Here we drop Eq. (12) and take the  $(\pi\pi)_0$  to be an  $\epsilon$  meson,  $\epsilon = (0, 0)\cos\theta_\epsilon - (8, 0)\sin\theta_\epsilon$ ; then, with ideal mixing between  $(\phi, \rho^0)$  and  $(\omega, \rho^0)$ ,

$$\begin{aligned} \langle \psi(\phi, \rho^0) | \epsilon | \psi(\omega, \rho^0) \rangle &= \frac{\sqrt{2}}{3} \left[ (G_{188}^{111} - G_{188}^{188}) \cos\theta_\epsilon \right. \\ &\quad \left. + \left( G_{188}^{888} - \frac{1}{\sqrt{2}} G_{188}^{188} \right) \sin\theta_\epsilon \right] \\ &= h_\epsilon G_{111}^{888}. \end{aligned} \quad (13a)$$

The discussion above suggests that

$$h_\epsilon^2 \approx 10^{-2}. \quad (5a)$$

The  $\pi^0\pi^0$  emission to accompany the  $\pi^+\pi^-$  in Eq. (8) totals 16% if the assumption of  $I=0$  is correct. According to Eq. (8) this leaves about  $9 \pm 9\%$  to be accounted for in other ways; the most obvious choice is the emission of an  $\eta$  meson. The observed<sup>10</sup> branching ratio  $(4 \pm 2)\%$  for this mode implies

$$\Gamma(\psi(3.7) \rightarrow \psi(3.1) + \eta) \approx 9 \pm 5 \text{ keV}. \quad (14)$$

In this process the coupling constant is  $(g/M)$ , where  $M$  is some normalizing mass. For the case  $\omega \rightarrow \rho + \pi$  it appears that  $g^2/4\pi M^2 \approx 25 \text{ GeV}^{-2}$ . For  $\psi' \rightarrow \psi + \eta$  one would be inclined to scale  $M$  approximately as  $\frac{1}{2}(M_i + M_f)$ , where  $M_i$  and  $M_f$  are the initial and final vector masses, having in mind something like an axial-vector divergence:

$$\Gamma_0(\psi(3.7) \rightarrow \psi(3.1) + \eta) = \frac{1}{3}(g^2/4\pi M^2) p^3 \approx 3 \text{ MeV}, \quad (15)$$

where  $p = 0.2$  GeV is the  $\eta$  momentum, and an extra factor of  $\frac{1}{3}$  is inserted for SU(3)' coupling. The formula corresponding to Eq. (13a) is now

$$\begin{aligned} \langle \psi(\phi, \rho^0) | \eta | \psi(\omega, \rho^0) \rangle &= \frac{\sqrt{2}}{3} \left[ (G_{188}^{111} - G_{188}^{188}) \sin\theta_\eta \right. \\ &\quad \left. + \left( G_{188}^{888} - \frac{1}{\sqrt{2}} G_{188}^{188} \right) \cos\theta_\eta \right] \\ &= h_\eta G_{111}^{888}. \end{aligned} \quad (13b)$$

Equations (14) and (15) imply that

$$h_\eta^2 \approx 0.3 \times 10^{-2}. \quad (5b)$$

Lastly, one may consider the analogous process

$$\psi(3.7) \rightarrow \psi(\rho, \rho^0) + \pi, \quad (16)$$

where  $\rho = \rho^+, \rho^0, \rho^-$ , and  $\pi = \pi^-, \pi^0, \pi^+$ . The vector meson  $\psi(\rho, \rho^0)$  is not excited in  $e^+e^-$  collision, but is expected<sup>4,12</sup> to be approximately degenerate with  $\psi(\omega, \rho^0) = \psi(3.1)$ . This decay would be copious if allowed: about 20 times more rapid than  $\eta$  emis-

sion. Here

$$\begin{aligned} \langle \psi(\phi, \rho^0) | \pi^i | \psi(\rho^i, \rho^0) \rangle &= (\frac{1}{3})^{1/2} (G_{188}^{188} - \sqrt{2} G_{188}^{888}) \\ &= h_\pi G_{111}^{888}. \end{aligned} \quad (13c)$$

If the decay (16) is not to exceed 40% of the  $\psi(3.1)$  decay, then

$$h_\pi^2 \lesssim 0.4 \times 10^{-3}. \quad (5c)$$

The reduction (5c) is an additional factor of 10 below (5a) and (5b); the simplest suggestion is that for the  $G_{188}$  Zweig's rule is only partially fulfilled; Eq. (12b) is valid and Eq. (12a) is not. It would be desirable to look for process (16). The decay modes of  $\psi(\rho, \rho^0)$  being probably entirely hadronic, the most accessible signature is the emission of a  $\pi^\pm$  with fixed energy  $M(3.7) - M(3.1) = 590$  MeV.

From this discussion, it appears that in the absence of special circumstances, as for (5c),  $h^2 \approx 10^{-2}$  to  $10^{-3}$  in the general case of  $\psi'$  decay into another colored meson by ordinary hadron emission. Without considering the details of various mixing angles, it seems satisfactory at this stage to take Eq. (5) as a general relation for  $(G_{188}^{abb}/G_{111}^{888})^2$ , for all three  $G_{188}^{abb}$  coefficients.<sup>13</sup>

On the assignments in Eq. (1) there are no transitions of the type  $\psi(4.1) \rightarrow \psi(3.1) + X$ , where  $X$  represents ordinary bosons. The  $j$  indices differ for initial and final  $\psi$  and no interaction so far introduced overcomes their orthogonality in the emission of ordinary hadrons. In the next section we consider a second-order term that will permit such decay processes; but this is weak and will not be immediately observable. The remarks of this paragraph apply to the higher resonance<sup>14</sup>  $\psi(\sim 4.9)$ , except under the Ref. 12 assignment.

#### IV. DECAYS $\psi \rightarrow$ ORDINARY HADRONS

The  $\psi(3.1)$  and, presumably,  $\psi(3.7)$  have a substantial fraction of their decay width in the direct process

$$\psi \rightarrow \text{ordinary hadrons} = Y. \quad (17)$$

One contribution is, as in Fig. 1, through conversion to a virtual  $\gamma$  ray by vector dominance and subsequent  $\gamma \rightarrow Y$ . This must be proportional to hadron production in nonresonant  $e^+e^-$  annihilation and can be estimated<sup>3</sup> as  $\Gamma(\psi \rightarrow \gamma \rightarrow Y) = 12$  keV. There remains a substantial direct-coupling term

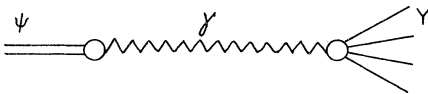


FIG. 1. Decay of  $\psi$  into ordinary hadrons through an intermediate photon state.

for  $\psi(3.1)$ , which is larger than this virtual  $\gamma$  contribution. It is only such direct decay that concerns us below; its partial width is approximately<sup>3</sup>

$$\Gamma(\psi(3.1) \rightarrow Y) \approx 50 \text{ keV}. \quad (18)$$

The expression corresponding to Eq. (18) for  $\psi(3.7)$  is somewhat less certain because of the anticipated but so far unobserved process (16). One expects that the contribution from Fig. 1 will be in proportion to the leptonic width of the resonance, or hence  $\Gamma(\psi' \rightarrow \gamma \rightarrow Y) \approx 5.6$  keV for  $\psi(3.7)$ . This is almost negligible; the chief problem is to divide the residual width of about  $85 \pm 25$  keV between Eqs. (16) and (17) as the decay processes.

For the  $\psi(4.1)$  and its predicted partner  $\psi(\phi, \phi) = \psi(4.9)$  the corresponding partial widths are of order  $10^3$  larger and are typical of ordinary hadron decay. This has already been interpreted above as a reflection of  $SU(3)'$  octet-singlet mixing, which does not affect  $\psi(i, \rho^0)$ , or hence  $\psi(3.1)$  and  $\psi(4.7)$ .

Therefore, to understand Eq. (18) some further mechanism is needed. We suggest the detailed mechanism of a second-order electromagnetic mixing between the  $\psi(i, \rho^0)$  and the  $\psi(i, \phi)$ , denoted by  $H(0, 3)$ . This is illustrated in Fig. 2 and involves only vertices  $G_{888}^{abb}$ , which are not inhibited by any rules introduced in the discussion above. It depends essentially on the fact that a photon, in its subsequent interactions, is independent of whether it relates initially to  $J_\mu(0, 3)$  or  $J_\mu(0, 8)$ , and hence can effect a transition between two states with these characters. The virtual hadrons can remain colored, so weak vertices of type  $G_{188}^{abb}$  are not involved. The amplitude of mixing between (3.1) and (4.1) is, for example

$$A = \frac{H(0, 3)}{4.1 - 3.1} \approx H(0, 3) \times (1 \text{ GeV})^{-1}; \quad (19)$$

if the hadron-decay width of the  $\psi(4.1)$  is  $\Gamma(\omega, \phi) \approx 200$  MeV then by Eq. (18)

$$A^2 = \frac{\Gamma(\psi(3.1) \rightarrow Y)}{\Gamma(\omega, \phi)} \approx \frac{50 \text{ keV}}{200 \text{ MeV}} \sim 2 \times 10^{-4}, \quad (20)$$

or hence

$$H(0, 3) \sim 15 \text{ MeV}. \quad (21a)$$

In the absence of inhibition at any of the vertices

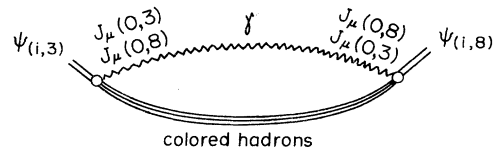


FIG. 2. Second-order electromagnetic mixing of colored 3 and 8 states.

in Fig. 2, one might expect the effective mixing interaction between  $\psi(i, 3)$  and  $\psi(i, 8)$  to be of electromagnetic order

$$H(0, 3) \sim \frac{\delta M^2}{2M} \sim \frac{1}{2}\alpha M \sim 15 \text{ MeV}. \quad (21b)$$

Note that  $H(0, 3)$ , as considered here, has to first order only octet-octet components in  $SU(3)''$ , since octet-singlet components involve  $G_{188}$  and hence are reduced. Equation (18) has also been interpreted<sup>7</sup> in terms of a phenomenological  $H(0, 3)$  with unreduced octet-singlet components.

For  $\psi(3.7)$  the present mechanism mixes in  $\psi(\phi, \phi) = \psi(4.9)$  in the same way; hence we might expect the same order of magnitude for direct hadron emission,

$$\Gamma(\psi(3.7) \rightarrow Y) \sim \Gamma(\psi(3.1) \rightarrow Y). \quad (22)$$

If Eq. (22) is correct, we are left with

$$\Gamma(\psi(3.7) \rightarrow \pi + \psi(\rho, \rho^0)) \approx 35 \pm 25 \text{ keV}. \quad (23)$$

This corresponds to a branching ratio of order 15%, and may be observable. We must emphasize that the division between the two decay widths in Eqs. (22) and (23) cannot be predicted accurately from the present type of simple phenomenology. It is an experimental question of some interest to apportion the approximately 40% of the  $\psi(3.7)$  that does not decay through the  $\psi(3.1)$  in cascade.

The  $G$  parity of all four resonances at 3.1, 3.7, 4.1, and 4.9 GeV is negative. Since only  $SU(3)'$  determines the strong interaction in this version of the Han-Nambu model,  $G = C \exp(i\pi I_2') = C = -1$  for all vector states with  $i = \omega, \phi$ . None of the purely hadronic decay mechanisms discussed above will alter the  $G$  parity, so we expect all hadronic decays of the  $e^+e^-$  resonances to display  $G = -1$ . There are some preliminary experimental indications in this direction<sup>15</sup>. On the other hand, the hadronic products from radiative decay as in Sec. II. above would all have  $G = +1$ . This may provide an ultimate means of determining the fraction of radiative decay.

It is important to note that on the present scheme the distribution of hadronic decay products from all the  $\psi$  resonances, narrow or broad, radiative

or nonradiative, is expected to show no peculiarities. It should be just as anticipated from a mixture of ordinary  $\omega, \phi$ -type mesons in the 4 GeV region: perhaps there would be more  $K\bar{K}$  and  $\eta$  contribution than at lower energies, but nothing strikingly anomalous.

The mechanism of Fig. 2 can be cut in half to yield a first-order process contributing to radiative decay as Fig. 3 shows. The amplitude for this process will be reduced, however, by a factor  $F = H(0, 8)/(M_c - 3.1)$ , where  $M_c$  is the average mass of colored hadrons in the intermediate state. The denominator of  $F$  may be of order 1–2 GeV, because the colored-hadron states will be widely spaced at the bottom of their energy range, while the numerator is given by Eq. (2) as about 0.1 GeV. If we now apply  $F^2$  to the estimate (4c), the effective decay width is

$$\Gamma(\psi \rightarrow \gamma + Y) \sim 0.2 \text{ keV}. \quad (24)$$

This is negligibly small in the present context, even if the reduction is overestimated by an order of magnitude.

## V. PHOTOPRODUCTION OF $\psi$

Although the photoproduction of  $\psi$  mesons on nuclei is not a decay process, it is related to the interactions of colored and uncolored particles and may be usefully commented on here. It seems to be generally agreed that the photoproduction is mainly diffractive and is therefore related to the total cross section  $\sigma(\psi N)$  for  $\psi$ -nucleon scattering. With photon energies of  $E_\gamma \approx 11 \text{ GeV}$ ,  $\approx 18 \text{ GeV}$  and  $\approx 80 \text{ GeV}$ , the respective cross sections<sup>16-18</sup> are

$$\sigma(\psi N) < 1.2 \text{ mb}, < 2.4\text{--}3.6 \text{ mb}, \approx 1 \text{ mb}. \quad (25)$$

At the highest energy it is remarked<sup>18</sup> that there is an appreciable probability of large momentum transfer.

Now  $\psi$ -nucleon scattering must be mediated by uncolored boson exchange that involves a  $G_{188}^{ab}$  vertex with the  $\psi$ . The imaginary part of the forward-scattering amplitude, and hence  $\sigma(\psi N)$  at high energies, will be reduced by a factor of order  $\hbar$  relative to ordinary vector-meson-nucleon scattering. Thus we would predict

$$\sigma(\psi N) \approx \hbar \left\{ \begin{array}{l} \sigma(\phi N) \\ \sigma(\omega N) \end{array} \right\} \approx \frac{1}{2} - 2 \text{ mb}, \quad (26)$$

which is like Eq. (25) in magnitude.

At sufficiently high energies and momentum transfers, we might suppose the mediating field to become like  $(i, \phi)$ , which has strong admixtures of both colored and uncolored components. The net effect should be some increase of effective  $\sigma(\psi N)$ ,

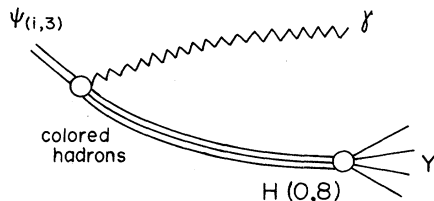


FIG. 3. Radiative decay from the first half of Fig. 2, plus color octet-singlet mixing terms.

with increasing  $s$  and particularly  $|t|$ . Present experimental indications are suggestive but are not sufficiently detailed to test this expectation.

## VI. COLORED PSEUDOSCALARS

By analogy with the ordinary mesons we expect the colored mesons  $\psi$  to be accompanied by a near-lying set of colored pseudoscalars  $\psi_P$ . On a naive quark model these are just the  ${}^3S_0$  configurations corresponding to the  ${}^3S_1$  that make up the vector  $\psi$ ; therefore, the  $SU(3)' \times SU(3)''$  properties of the  $\psi_P$  should be identical with those of the  $\psi$ . This was certainly the case with ordinary mesons. On a phenomenological basis it is not possible to say exactly what the vector-pseudoscalar mass difference is for the colored mesons; but we will assume that the mass patterns are very similar, namely, that for any particular index  $j_0$ , the  $\psi_P(i, j_0)$  are distributed in the neighborhood of the corresponding  $\psi(i, j_0)$ .

We consider the effect of these  $\psi_P$  on the decay processes of the  $\psi$ . If the mass difference  $M_\psi - M_P \gg 0$ , the narrow  $\psi$  would be appreciably broadened. This will provide some information on  $M_P$  for  $\psi_P(i, \rho^0)$  corresponding to the narrow  $\psi(3.1)$  and  $\psi(3.7)$ ; but since the  $\psi(i, \phi)$  are already broad on the present scheme, no restriction on the corresponding  $M_P$  is available.

The colored pseudoscalars that concern us are therefore  $\psi_P(\eta', \rho^0)$ ,  $\psi_P(\eta, \rho^0)$ , and  $\psi_P(\pi^0, \rho^0)$ ; the  $(i, j)$  notation is mixed to emphasize their pseudoscalar character and their color association with  $\psi(i, \rho^0)$ . All the decay processes  $\psi \rightarrow \psi_P + X$  with  $X =$  ordinary hadrons of low mass will involve  $G_{188}^{abb}$  coupling and hence a reduction factor  $h^2 \approx 10^{-2}$  to  $10^{-3}$ .

Consider first the decay of  $\psi(3.7)$ . The ordinary hadronic decay processes are

$$\psi(3.7) \rightarrow \begin{cases} \rho + \psi_P(\pi, \rho^0) & (27a) \\ \omega + \psi_P(\eta', \rho^0) & (27b) \end{cases}$$

$$\omega + \psi_P(\eta, \rho^0). \quad (27c)$$

As with the ordinary pseudoscalars, we take the mass relation of  $\eta'$ -type and  $\eta$ -type pseudoscalars to be arbitrary, so that (27b) and (27c) must be considered equally likely possibilities. Using the customary analogy with  $\omega\rho\pi$  decay, scaling the normalizing mass as  $\frac{1}{2}(M_i + M_j)$  for the vectors, and introducing the reduction factor  $h^2$  as in Eq. (5) yields

$$\Gamma_{\rho, \omega} \approx 10^3 \text{ keV} \times (p/\text{GeV})^3, \quad (28)$$

where  $p$  is the  $\rho$  or  $\omega$  momentum. To make  $\Gamma_{\rho, \omega} \leq 100$  keV as an upper limit of compatibility with current data would correspond to a  $\rho$  or  $\omega$  kinetic

energy  $\leq 100$  MeV. Thus, with  $X^0 = \pi^0, \eta^0$ , or  $\eta'^0$

$$M_P(X^0, \rho^0) \geq 3.7 - (M_{\rho, \omega} + 100 \text{ MeV}) \geq 2.8 \text{ GeV}. \quad (29)$$

Greenberg<sup>9</sup> has pointed out that lower limits to the masses of colored pseudoscalars can be obtained by assuming an upper limit for the width of the decay of  $\psi(3.7)$  with photon emission to colored pseudoscalars. Consider

$$\psi(3.7) \rightarrow \begin{cases} \gamma + \psi_P(\eta', \rho^0) \\ \gamma + \psi_P(\eta, \rho^0). \end{cases} \quad (30)$$

The photon can be associated with the current element  $J_\mu(0, 8)$ , so that the decay vertex is of type  $G_{888}^{abb}$ . If this is on the order of  $G_{111}^{888}$ , we can estimate this decay by comparison with  $\phi \rightarrow \gamma + \eta$ ; viz.,

$$E_\gamma(\psi) = E_\gamma(\phi) \left[ \frac{M_\psi^2 \Gamma_\gamma(\psi)}{M_\phi^2 \Gamma_\gamma(\phi)} \right]^{1/3} \approx 1.2 \text{ GeV}, \quad (31)$$

with  $\Gamma_\gamma(\psi) < 100$  keV; hence

$$M_P(\eta', \rho^0), M_P(\eta, \rho^0) \geq 2.5 \text{ GeV}. \quad (32)$$

The decay  $\psi(3.7) \rightarrow \gamma + \psi_P(\pi^0, \rho^0)$  involves  $J_\mu(3, 0)$  with a vertex suppression of order  $h^2 \sim 10^{-3}$  and can be neglected<sup>19</sup>.

The decay mode for the neutral  $\psi_P$  that comes first to mind is of course

$$\psi_P(X^0, \rho^0) \rightarrow 2\gamma. \quad (33)$$

If the basic process is  $\psi_P \rightarrow \gamma + \psi_V \rightarrow 2\gamma$  by vector-meson dominance, then the first matrix element is proportional to the vector-meson magnetic moment. The decay rate thus scales as  $M_V^{-2} \approx M_\psi^{-2}$  in this case, since the vector meson must be colored. Taking the vector to be an  $\omega$  meson in the ordinary  $\eta \rightarrow 2\gamma$ , and scaling up by  $E_\gamma^3$ , we estimate for each of these three mesons an order of magnitude

$$\begin{aligned} \Gamma(\psi_P \rightarrow 2\gamma) &\approx \left( \frac{M_\omega}{M_\psi} \right)^2 \left( \frac{M_P}{M_\eta} \right)^2 \Gamma(\eta \rightarrow 2\gamma) \\ &\approx 13 \Gamma(\eta \rightarrow 2\gamma) \approx 4 \text{ keV}. \end{aligned} \quad (34)$$

There is no reason, however, that the  $\psi_P$  should not participate in the same hadronic decay processes as discussed for the  $\psi$  in Sec. IV above. A first guess would then be that for direct decay into ordinary hadrons

$$\Gamma(\psi_P \rightarrow Y) \sim 10^2 \text{ keV}. \quad (35)$$

Thus,  $2\gamma$  need not be the predominant decay mode for the  $\psi_P$ .

It is of interest to note that if ideal nonet mixing obtains between the  $\psi_P(\eta', \rho^0)$  and  $\psi_P(\eta, \rho^0)$ , then  $\psi_P(\eta', \rho^0) \rightarrow 2\gamma$  is forbidden. Consider the triangle diagram for  $P \rightarrow 2\gamma$  and recall that the Han-Nambu

quark configuration under nonet symmetry is

$$\psi_P(\eta', \rho^0) = \frac{1}{2}[(\bar{\psi}_1\psi_{-1} + \bar{\psi}_1\psi_1) - (\bar{\psi}_2\psi_2 + \bar{\psi}_2\psi_{-2})]. \quad (36)$$

The electric charges of the four quarks in Eq. (36), are, respectively, 0, -1, +1, and 0. Only the second and third terms contribute to  $2\gamma$  decay with coupling  $(\pm e)^2 = e^2$ ; but they couple to  $\psi_P$  with opposite signs, so the whole matrix element vanishes. The entire width in Eq. (34) is then concentrated in  $\psi_P(\eta, \rho^0) \rightarrow 2\gamma$ . According to the rough numbers in Eqs. (31), (34), and (35), the branching ratio for

$$\psi(3.7) \rightarrow \gamma + \psi_P(X^0, \rho^0) \rightarrow 3\gamma \quad (37)$$

$$B \lesssim \left(\frac{100 \text{ keV}}{225 \text{ keV}}\right) \left(\frac{4 \text{ keV}}{10^2 \text{ keV}}\right) \sim 0.02. \quad (38)$$

Note that the decay of  $\psi(3.1)$  or  $\psi(3.7)$  into an ordinary pseudoscalar plus a colored pseudoscalar gives less stringent limits than Eq. (32).

#### VII. EXTREME $\bar{\lambda}' \cdot \bar{\lambda}''$ COUPLING

An alternative variety of the Han-Nambu scheme to the present one has been considered<sup>20</sup> for colored mesons, and it may be of interest to compare its expected decay modes with the discussion above. The principal feature of  $\bar{\lambda}' \cdot \bar{\lambda}''$  coupling is that  $\psi(3.1)$  and  $\psi(3.7)$  have  $I^G = 1^+$ , while  $\psi(4.1)$  has  $I^G = 0^-$ . In this case, radiative decay as in Eq. (3) yields hadronic decay products with negative G parity. The estimate of widths in (4c) is suitable for decay of the  $\psi(3.1)$ , so that one would not introduce a further reduction as in Eq. (5).

According to the estimate in Sec. II, the average  $E_\gamma \approx 600$  MeV in this decay, which is about 20% of the available energy. If the subsequent hadrons distribute themselves in the standard charge, neutral ratio of 2:1, the decay energy ultimately goes into neutral products about one-half the time instead of one-third, as on the simplest statistical

expectation for hadron decay. This might have some bearing on the decrease of the ratio of charged particle energy to total energy as energy increases in  $e^+e^- \rightarrow$  hadrons.

No particular variation in  $\Gamma_\gamma$  for radiative decay would be expected in going from  $\psi(3.1)$  to  $\psi(3.7)$ . Although about 57% of the neutral  $\psi(3.7)$  decay proceeds through the neutral  $\psi(3.1)$ , this still leaves a width of  $0.4\Gamma(3.7) \approx 90 \pm 25$  keV for other hadronic decays. The direct radiative decay width may be  $\Gamma_\gamma \approx 50$  keV as for  $\psi(3.1)$ , leaving some excess to be accounted for. The  $\bar{\lambda}' \cdot \bar{\lambda}''$  model provides the additional decay mode

$$\psi(3.7) \rightarrow (\pi^+\pi^0)_+ \psi^\mp(3.1), \quad (39)$$

where  $\psi^\mp(3.1)$  are the charged counterparts of the neutral  $\psi(3.1)$ , expected for an  $I=1$  state. The  $\psi^\mp(3.1)$  should undergo radiative decay to (charged) systems of ordinary hadrons in the same way as the neutral  $\psi(3.1)$ , so that the energy fraction to neutral particles is maintained at approximately one-half for  $\psi(3.7)$  as well as for  $\psi(3.1)$ .

Analogous decay modes seem of little relative importance in the decay of  $\psi(4.1)$  because of its large total width  $\Gamma(4.1) \approx 200$  MeV. Radiative hadron decay of some tens of keV will be negligible, and the decay

$$\psi(4.1) \rightarrow \pi^\pm + \psi^\mp(3.1), \quad (40)$$

although uninhibited, is estimated<sup>20</sup> to have a branching ratio of a few percent. This estimate depends on an unknown coupling constant and could perhaps be increased by an order of magnitude; but Eq. (40) should still probably represent a minor decay mode. The major decay peculiarity to be sought for  $\psi(4.1)$  is the presence of colored baryon-antibaryon pairs; in the extreme  $\bar{\lambda}' \cdot \bar{\lambda}''$  coupling limit it is not possible to broaden substantially the  $\psi(4.1)$  by color octet-singlet mixing without doing the same for  $\psi(3.1)$  and  $\psi(3.7)$ .

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In Ref. 8 it is argued that at least some of the  $G_{888}^{abb}$  must be as large as  $G_{888}^{111}$  if vector-meson dominance is to hold for matrix elements of electric charges like

$$\langle (\rho^+, \rho^0) | Q | (\rho^+, \rho^0) \rangle.$$

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